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Photon Scattering from Protons and Deuterons

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The scattering of dipole photons of 100 to 200 Mev from protons and deuterons is calculated by means of dispersion relations, experimental data on photopion production from nucleons, and several reasonable approximations. A previous calculation by the author using this model is modified by the use of an additional dispersion relation derived by Mathews. The differential cross sections resulting from this modification are smaller than the previously calculated cross sections for scattering angles in the backward direction. Comparison of the photon-deuteron calculations with the results of future experiments will yield information concerning the photon-neutron scattering amplitude.

1. RECENT PREDICTIONS OF PHOTON-PROTON AND PHOTON-DEUTERON SCATTERING

GELL-MANN, Goldberger, and Thirring were the first to apply dispersion relations to the problem of the scattering of photons from protons.[1] These authors derived dispersion relations for both the spin-dependent and spin-independent forward, elastic scattering. The spin-independent relation, together with several reasonable guesses concerning the relative sizes of certain phase shifts, was used by the authors to predict the differential photon-proton scattering cross section at all angles for photon energies up to 250 Mev or so.

Recently, the procedure of Gell-Mann, Goldberger, and Thirring was extended by the author.[2] In I, as in Gell-Mann *et al.*, only the four dipole amplitudes $T_{\mathrm{el},\frac{1}{2}}$, $T_{\mathrm{el},\frac{3}{2}}$, $T_{\mathrm{mg},\frac{1}{2}}$, and $T_{\mathrm{mg},\frac{3}{2}}$ were considered, where the subscripts "el" and "mg" denote electric and magnetic amplitudes, and the subscripts $\frac{1}{2}$ and $\frac{3}{2}$ denote the total angular momentum of the photon-nucleon system in the center-of-mass system. These amplitudes are separated into two parts, i.e.,

$$T_j = T_j^0 + T_j', \tag{1}$$

where the general subscript j denotes any of the four amplitudes mentioned above. The quantities T_j^0 are the "low-energy" parts of the amplitudes T_j, defined rigorously to be the sums of the zero-order and first-order terms in an expansion of the scattering amplitude in powers of the photon energy. It has been shown by several authors that the requirements of relativistic invariance and gauge invariance imply that T_j^0 depend only on the charge, mass, and anomalous moment of the proton.[3] On the other hand, the terms T_j' defined by Eq. (1) may depend in a detailed way upon the mesonic structure of the proton. These terms are referred to as "pionic structure" terms.

In the calculation of I it is assumed that the terms of lowest order in the electric charge (second-order terms) are dominant in the photon scattering amplitudes. In this approximation the imaginary parts of the scattering amplitudes T_j' are determined from unitarity relations and the experimental data on photopion production from protons. The two dispersion relations of Gell-Mann, Goldberger, and Thirring are used to express linear combinations of the real parts of the amplitudes T_j' in terms of photopion production cross sections. Since there are four amplitudes T_j' and only two dispersion relations, it is necessary to make a further assumption in order to determine $\mathrm{Re}T_j'$. The assumption made is that the two amplitudes $T_{\mathrm{el},\frac{1}{2}}'$ and $T_{\mathrm{mg},\frac{3}{2}}'$ which are known to be dominant in photomeson production at energies below 400 Mev, are also dominant for the scattering. The amplitudes $T_{\mathrm{el},\frac{3}{2}}'$ and $T_{\mathrm{mg},\frac{1}{2}}'$ are taken to be zero. The photon scattering amplitudes and differential cross sections are then calculated by making use of the two dispersion relations and the analysis of photomeson production data of Watson, Keck, Tollestrup, and Walker.[4] The results differ considerably from the calculation of Gell-Mann *et al.*

The photon-deuteron scattering problem is also studied in I. The elastic and inelastic photon-deuteron differential scattering cross sections are predicted, using the impulse approximation together with the model used for the photon-proton problem.

The author believes that the assumption in I, that $\mathrm{Re}T_{\mathrm{mg},\frac{1}{2}}'$ is small compared to the "pionic structure" parts of the other amplitudes, is reasonable. On the other hand, the assumption that $\mathrm{Re}T_{\mathrm{el},\frac{3}{2}}'$ may be neglected is certainly not accurate. In the weak pion-nucleon coupling calculations,[5] for example, $\mathrm{Re}T_{\mathrm{mg},\frac{1}{2}}'$ is quite small in the energy range under consideration here, but $\mathrm{Re}T_{\mathrm{el},\frac{3}{2}}'$ is larger than $\mathrm{Re}T_{\mathrm{mg},\frac{3}{2}}'$ (though $\mathrm{Re}T_{\mathrm{el},\frac{1}{2}}'$ is the largest of all the pionic structure parts). In Fig. 3 of I it is shown that the assumption of a reasonable nonzero value of $\mathrm{Re}T_{\mathrm{el},\frac{3}{2}}'$ leads to a considerable difference in the calculated differential

[1] Gell-Mann, Goldberger, and Thirring, Phys. Rev. **95**, 1612 (1954).

[2] R. H. Capps, Phys. Rev. **106**, 1031 (1957), and University of California Radiation Laboratory Report UCRL-3572 (unpublished). Hereafter, this paper is referred to by the symbol I.

[3] F. E. Low, Phys. Rev. **96**, 1428 (1955); M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1955).

[4] Watson, Keck, Tollestrup, and Walker, Phys. Rev. **101**, 1159 (1956).

[5] R. H. Capps and W. G. Holladay, Phys. Rev. **99**, 931 (1955); R. G. Sachs and L. L. Foldy, Phys. Rev. **80**, 824 (1950).

photon-proton cross section, especially at scattering angles larger than 90°.

Recently Mathews has derived dispersion relations for finite-momentum-transfer photon-proton scattering,[6] using techniques similar to those used in the corresponding problem for pion-nucleon scattering.[7] He then uses these equations, together with the photopion production data, to predict photon-proton scattering cross sections which differ considerably from those predicted by Gell-Mann *et al.*, and in I. Because of the many dispersion relations used, Mathews is able to consider many more amplitudes than those considered in I. On the other hand, use of these finite-momentum-transfer equations necessitates an additional assumption, since some of the constants appearing in the finite-momentum-transfer dispersion relations are not determined by invariance properties or by the photoproduction data. In reference 6, the Born approximation is used to determine these terms. If consideration is limited to dipole amplitudes and certain recoil terms are neglected, however, one needs only three or four dispersion relations and may avoid the questionable use of Born approximation. In this paper one of the relations derived by Mathews, and the two relations of Gell-Mann *et al.*, are used to determine the real parts of the quantities $T_{\mathrm{el},\frac{3}{2}}'$, $T_{\mathrm{el},\frac{1}{2}}'$, $T_{\mathrm{mg},\frac{3}{2}}'$, and the photon-proton and photon-deuteron cross sections are recalculated. As expected, the resulting photon-proton results are much the same as those of Mathews, and agree with the experimental results of Yamagata *et al.* to within the accuracy of the experiments.[8]

The principal purpose of the present work is the recalculation of the photon-deuteron cross sections of I. It is believed that the present results are more reliable than those of I, and when suitable experiments are done, comparison of the experimental results with the results of this paper will provide some information concerning the photon-neutron interaction and the electromagnetic structure of neutrons.

2. RECALCULATION OF SCATTERING CROSS SECTIONS

There are several simple ways to calculate the ratio $\mathcal{R}=\mathrm{Re}T_{\mathrm{el},\frac{3}{2}}'/\mathrm{Re}(T_{\mathrm{el},\frac{3}{2}}'+T_{\mathrm{mg},\frac{3}{2}}')$ using the finite momentum-transfer dispersion relations. One way is to take one of the relations derived by Mathews in conjunction with the two derived by Gell-Mann, Goldberger, and Thirring. An appropriate equation, for example, would be the equation for that part of the coefficient of the $\mathbf{e}\cdot\mathbf{e}'$ terms in the scattering amplitude which is independent of momentum-transfer. Another method of determining $\mathcal{R}$ would be to use a simple static, dipole limit of the complete set of equations listed in reference 6. A third method is to compute $\mathcal{R}$ from the calculated results of reference 6. A calculation reveals that the results of all these methods are similar, so the latter method is used here. The ratio $\mathcal{R}$, computed as a function of energy from the calculation of reference 6, is shown in Fig. 1.

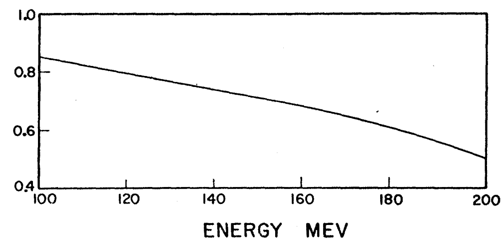

FIG. 1. Calculated value of the ratio $\mathcal{R}=\mathrm{Re}T_{\mathrm{el},\frac{3}{2}}'/\mathrm{Re}(T_{\mathrm{el},\frac{3}{2}}'+T_{\mathrm{mg},\frac{3}{2}}')$ as a function of the incident photon energy in the laboratory system.

In the model presented here (where $\mathcal{R}\neq 0$), the real parts of the expressions for the amplitudes T_j given in Eq. (11) of I must be modified in the following manner:

$$
\begin{aligned}
\frac{k_l}{k}\mathrm{Re}T_{\mathrm{el},\frac{3}{2}}&=\frac{e^2}{m}\left(-\frac{2}{3}-\frac{1}{3}\frac{\lambda k_l}{m}\right)+\frac{e^2}{m}\mathcal{R}[\mathrm{Re}\mathfrak{M}(k)],\\
\frac{k_l}{k}\mathrm{Re}T_{\mathrm{el},\frac{1}{2}}&=\frac{e^2}{m}\left(-\frac{2}{3}+\frac{2}{3}\frac{\lambda k_l}{m}\right)+\frac{e^2}{m}\mathrm{Re}\mathcal{E}(k),\\
\frac{k_l}{k}\mathrm{Re}T_{\mathrm{mg},\frac{3}{2}}&=\frac{1}{3}\frac{e^2k_l}{m^2}(\lambda^2+2\lambda)+\frac{e^2}{m}(1-\mathcal{R})[\mathrm{Re}\mathfrak{M}(k)],\\
\frac{k_l}{k}\mathrm{Re}T_{\mathrm{mg},\frac{1}{2}}&=-\frac{1}{3}\frac{e^2k_l}{m^2}[2(\lambda+1)^2+1].
\end{aligned}
\tag{2}
$$

It has been pointed out to the author by Mathews that two terms were omitted from the low-energy parts of Eqs. (9), (10), and (11) of I.[9] The effect of this omission on the calculated results of I is small compared to the effect of the assumption that $T_{\mathrm{el},\frac{3}{2}}'=0$. However, these terms are included in Eqs. (2).

The values of the dipole amplitudes are calculated from Eqs. (2) and the values of $\mathfrak{M}(k)$ and $\mathcal{E}(k)$ determined in I. The photon-proton differential scattering cross section is then recalculated, following the procedure of I. The results of this calculation, in the center-of-mass system, are shown in Fig. 2. These results are quite similar to those of reference 6. If a multipole analysis of the calculation of reference 6 is made, it is found that the mesonic correction parts of $T_{\mathrm{mg},\frac{1}{2}}$ and of the amplitudes for all higher multipoles are small for energies less than 200 Mev. Thus the present model is quite similar to that of reference 6 in this energy range, and it is to be expected that the results are similar. These results are in agreement with

[6] Jon Mathews, doctoral thesis, California Institute of Technology, 1957 (unpublished).

[7] R. H. Capps and Gyo Takeda, Phys. Rev. **103**, 1877 (1956).

[8] Yamagata, Auerbach, Bernardini, Filosofo, Hanson, and Odian, Bull. Am. Phys. Soc. Ser. II, 1, 350 (1956); T. Yamagata, doctoral thesis, University of Illinois, 1956 (unpublished).

[9] Jon Mathews (private communication).

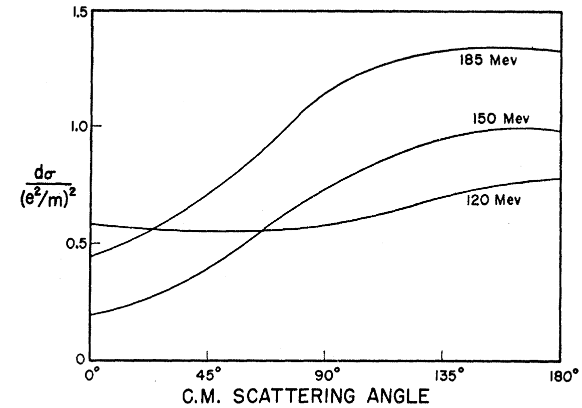

FIG. 2. Calculated photon-proton differential cross sections in the center-of-mass system, shown at incident photon energies in the laboratory system of 120, 150, and 185 Mev.

the experimental results of Yamagata *et al.* within the experimental error.[8]

By using the value of $\mathcal{R}$ taken from reference 6, we also recalculate the elastic and inelastic photon-deuteron cross sections of I. These cross sections are calculated in the impulse approximation, and depend on the photon-proton and the photon-neutron scattering amplitudes. In order that numerical predictions may be made, the photon-neutron amplitudes are calculated from the photon-proton amplitudes in I by making use of the "charge symmetry assumption" discussed in Sec. IIIA of I. The low-energy forms of the neutron amplitudes are given correctly in I, but because of the omissions in the low-energy proton amplitudes discussed above, Eqs. (24) of I should be modified in the following manner:

$$\frac{k_l}{k}(A_n{}^{\mathrm{el}}-A_p{}^{\mathrm{el}})=\frac{e^2}{m},$$
$$\frac{k_l}{k}(A_n{}^{\mathrm{mg}}-A_p{}^{\mathrm{mg}})=\frac{e^2k_l}{2m^2},$$
$$\frac{k_l}{k}(B_n{}^{\mathrm{el}}-B_p{}^{\mathrm{el}})=\frac{e^2\lambda k_l}{2m^2},\tag{3}$$
$$\frac{k_l}{k}(B_n{}^{\mathrm{mg}}-B_p{}^{\mathrm{mg}})=-\frac{e^2k_l}{2m^2}(2\lambda+1).$$

Here, as in I, the symbol A refers to the spin-independent amplitude, B refers to the spin-dependent amplitude, and the subscripts p and n denote protons and neutrons, respectively. The correspondence between the spin-independent and the spin-dependent amplitudes and the amplitudes referring to states of fixed angular momentum is given by

$$A^{\mathrm{el}}=T_{\mathrm{el},\frac{3}{2}}+\tfrac{1}{2}T_{\mathrm{el},\frac{1}{2}},\quad A^{\mathrm{mg}}=T_{\mathrm{mg},\frac{3}{2}}+\tfrac{1}{2}T_{\mathrm{mg},\frac{1}{2}},$$
$$B^{\mathrm{el}}=\tfrac{1}{2}T_{\mathrm{el},\frac{3}{2}}-\tfrac{1}{2}T_{\mathrm{el},\frac{1}{2}},\quad B^{\mathrm{mg}}=\tfrac{1}{2}T_{\mathrm{mg},\frac{3}{2}}-\tfrac{1}{2}T_{\mathrm{mg},\frac{1}{2}}.\tag{4}$$

Equations (4) are valid for either the neutron or the proton amplitudes.

If the proton amplitudes are given by Eq. (2), where the values of $\mathcal{R}$, $\mathfrak{M}$, and $\mathcal{E}$ are determined in the manner discussed above, and the procedure of I is followed, the photon-deuteron elastic and inelastic scattering cross sections may be recalculated. The results of this calculation, in the center-of-mass system, are shown in Figs. 3 and 4.

3. METHODS OF INTERPRETING PHOTON-DEUTERON SCATTERING EXPERIMENTS

The accuracy of the calculated results for photon-deuteron scattering depends on the accuracy of the assumed proton amplitude, the accuracy of the assumed neutron amplitude, and the validity of the impulse approximation for this process. (The computation of the interference factors F and S of I is here considered to be part of the impulse approximation.) Experimentally, it is possible to separate these effects to a certain degree. The accuracy of the photon-proton amplitude may be checked, of course, by photon-proton scattering experiments. A partial check on the impulse assumption may be made by measuring the ratio of the elastic photon-deuteron cross section at one angle to that at another. This angular dependence is, in general, more sensitive to inaccuracy of the impulse approximation than it is to changes in the proton and neutron amplitudes.

If the photon-proton amplitudes given here should agree with future experiments, and if the impulse approximation is accurate for the photon-deuteron scattering problem, then comparison of the results of future photon-deuteron scattering experiments with the curves of Figs. 3 and 4 will provide a test of the accuracy of the photon-neutron amplitudes assumed here. As dis-

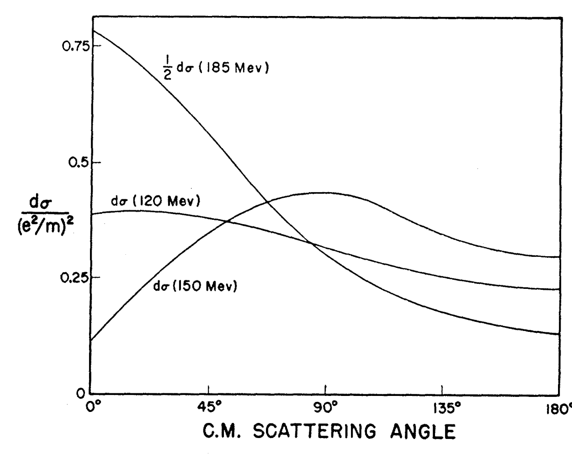

FIG. 3. Calculated photon-deuteron differential elastic cross sections in the center-of-mass system, shown at incident photon laboratory-system energies of 120, 150, and 185 Mev. The curve corresponding to 185 Mev represents one-half the differential elastic cross section.

cussed in Sec. IIIB of I, the "charge symmetry assumption" used to compute the neutron amplitudes cannot apply exactly, since recoil terms involving the nucleon cores are not charge symmetric. Because of the interference between the neutron and proton amplitudes in the scattering from deuterons, there is no simple relation by which the neutron cross section may be determined from proton and deuteron cross sections. However, the following general statements may be made. A larger value of the photon-neutron spin-independent electric dipole amplitude leads to a larger value of the total deuteron inelastic cross section, while the corresponding change in the total deuteron elastic scattering cross section may be either positive or negative, and is probably small. The reason for this effect is that the elastic scattering amplitude is coherent, and the Thomson amplitude for the proton is approximately equal to the mesonic effects in the sum of the proton and neutron spin-independent electric amplitudes. Therefore, the total elastic cross section, which depends on the square of this amplitude, is not very sensitive to a moderate change in the amplitude. On the other hand, the deuteron inelastic cross section is sensitive to a change in any neutron amplitude, since this cross section is primarily incoherent.

From similar arguments it can be seen that a larger value of either of the photon-neutron magnetic dipole amplitudes, or of the spin-dependent electric dipole amplitude, leads to a larger value of both the elastic and inelastic deuteron cross sections. In this case the fractional changes in the coherent parts of the cross sections are greater than the fractional changes in the incoherent parts, since these amplitudes have the same sign for neutrons as for protons. Thus, the total deuteron elastic cross section, which is completely coherent, is particularly sensitive to a change in the magnetic amplitudes or spin-dependent electric amplitude.

The center-of-mass photon-neutron scattering cross section that can be calculated in the model used here is unlike the corresponding proton cross section in that it is about equally large in the forward and backward directions. This difference results from the lack of a negative Thomson term in the neutron spin-independent electric amplitude. Both the electric and magnetic spin-independent amplitudes are positive for the neutron, and thus interfere in such a manner as to increase the cross section at forward angles, and decrease it at backward angles. This asymmetry is approximately balanced, however, by the interference term in the spin-dependent part of the neutron cross section, which increases the cross section at back angles for the neutron, as well as for the proton. For photon energies greater than 120 Mev, the interference between the proton and neutron amplitudes in photon-deuteron scattering is fairly small for scattering angles greater than 90°. Hence, in this angular region, a rough estimate of the neutron cross section may be made by subtracting the proton cross section from the sum of the elastic and inelastic deuteron cross sections. Furthermore, one expects the deuteron cross section to be primarily inelastic in this range of energies and scattering angles. Further discussion of the accuracy of the model, and of methods of obtaining information concerning the photon-neutron scattering from experiments involving deuterons is contained in Sec. IIIB and Sec. IV of I.

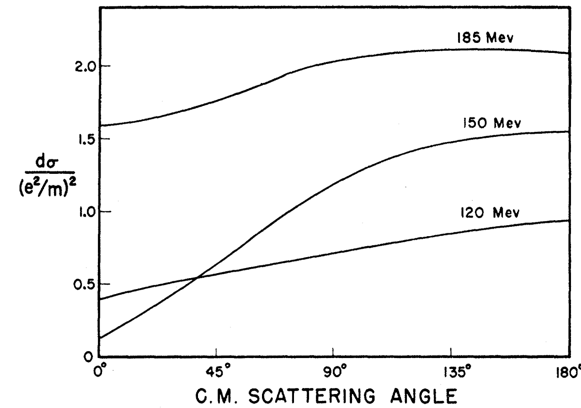

FIG. 4. Calculated photon-deuteron differential cross sections in the center-of-mass-system, shown at incident photon laboratory-system energies of 120, 150, and 185 Mev. These curves represent the sums of the elastic and inelastic cross sections.

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