Mu Decay with Nonconservation of Parity

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The distribution of the decay electrons from polarized μ mesons is calculated, without presumption of parity conservation, time-reversal invariance, or charge-conjugation invariance. The four-component neutrino theory is used. The result is expressed in terms of three parameters, ρ, α, and ζ, of which ρ is the usual Michel parameter. Theoretical inequalities among these parameters are given. An analysis of experimental data is made in terms of these parameters, and the results are found to be in fair agreement with the more stringent requirements of two-component neutrino theory.

RECENT theoretical considerations[1–5] and experimental results[6–9] show that conventional parity is not conserved in weak decays, and many consequences of these phenomena have been discussed.[1–5] In particular, μ decay with nonconservation of parity has been considered from the standpoint of a two-component neutrino theory,[3–5] and explicit formulas for the energy and angular distribution of the electron produced in the decay of a totally polarized μ meson have been given.[3,4]

We have calculated the decay of a spin-$\frac{1}{2}$ μ meson with spin "up" to first order in phenomenological Hamiltonian interaction terms using the conventional four-component neutrino theory,[10] our terms being analogous to those used by Lee and Yang in their discussion of β-decay.[1] These terms comprise all relevant direct-coupling scalars and pseudoscalars, as follows:

$$\begin{aligned} H=&(\bar{\psi}_e\psi_\mu)(C_S\bar{\psi}\psi+C_S'\bar{\psi}\gamma_5\psi)\\ &+(\bar{\psi}_e\gamma_\alpha\psi_\mu)(C_V\bar{\psi}\gamma_\alpha\psi+C_V'\bar{\psi}\gamma_\alpha\gamma_5\psi)\\ &+\tfrac{1}{2}(\bar{\psi}_e\sigma_{\alpha\beta}\psi_\mu)(C_T\bar{\psi}\sigma_{\alpha\beta}\psi+C_T'\bar{\psi}\sigma_{\alpha\beta}\gamma_5\psi)\\ &+(\bar{\psi}_e\gamma_\alpha\gamma_5\psi_\mu)(-C_A\bar{\psi}\gamma_\alpha\gamma_5\psi-C_A'\bar{\psi}\gamma_\alpha\psi)\\ &+(\bar{\psi}_e\gamma_5\psi_\mu)(C_P\bar{\psi}\gamma_5\psi+C_P'\bar{\psi}\psi), \end{aligned} \tag{1}$$

where $\sigma_{\alpha\beta}=-\frac{1}{2}i[\gamma_\alpha,\gamma_\beta]$ and $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$. The symbols ψ and $\bar{\psi}$ are defined to be $\psi=\psi_\nu$, $\bar{\psi}=\psi_\nu{}^\dagger\gamma_4$ for the process $\mu\rightarrow e+\bar{\nu}+\nu$; $\psi=\psi_\nu$, $\bar{\psi}=\tilde{\psi}_\nu\gamma_4$ for $\mu\rightarrow e+2\bar{\nu}$; $\psi=\tilde{\psi}_\nu{}^\dagger$, $\bar{\psi}=\psi_\nu{}^\dagger\gamma_4$ for $\mu\rightarrow e+2\nu$. We use Hermitian γ_α and take a representation with γ_1, γ_2, γ_3, $i\gamma_4$ real so as to simplify charge conjugation to $\psi_\nu{}^C=\tilde{\psi}_\nu{}^\dagger$. (We use † for Hermitian conjugate, $\sim$ for transpose.)

Imposition of various additional invariance requirements yields relations among the ten coupling constants; in particular, if the Hamiltonian is invariant under time reversal, all ten coupling constants must have the same phase modulo π.[1] In no case will subsequent inequalities linking the three shape parameters ρ, α, ζ of Eq. (2) be sharpened by such a restriction on the phases.

We have approximated the electron's mass to be zero, and we have summed over electron polarizations and over all neutrino coordinates.[11] With $\hbar=c=1$, m the mass of μ meson, $\mathbf{p}$ the electron's momentum, $x=2|\mathbf{p}|/m$ the electron's energy in such units that $0\leq x\leq 1$, and with θ the angle between $\mathbf{p}$ and "up", our detailed result for the rate per unit x and per steradian solid angle Ω associated with $\mathbf{p}$ is

$$R(x,\Omega)dxd\Omega=\frac{1}{\tau}12x^2dx\frac{d\Omega}{4\pi}\left\{(1-x)+\frac{2}{9}\rho(4x-3)+\cos\theta\left[\alpha(1-x)+\frac{2}{9}\zeta(4x-3)\right]\right\}. \tag{2}$$

The parameters will be designated as τ=mean life, ρ=Michel parameter,[10] α=asymmetry parameter, ζ=asymmetry Michel parameter. The significance of these parameters is apparent from the variously integrated versions of (2):

$$R(x)dx=\left(\frac{1}{\tau}\right)12x^2dx\left\{(1-x)+\frac{2}{9}\rho(4x-3)\right\};$$

$$R(\Omega)d\Omega=\frac{1}{\tau}\left(\frac{d\Omega}{4\pi}\right)\{1+\alpha\cos\theta\};$$

$$R=1/\tau.$$

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[10] See L. Michel, Proc. Phys. Soc. (London) **A63**, 514 (1950) for the calculation without the parity-nonconserving terms. After the completion of the present calculation, we have been informed by Professor Lee of a new calculation by C. Bouchiat and L. Michel [Phys. Rev **106**, 170 (1957)] taking into account parity-nonconserving terms in the four-component neutrino theory.
[11] We have also neglected all radiative corrections. See A. Lenard, Phys. Rev. **90**, 968 (1953); Behrends, Finkelstein, and Sirlin, Phys. Rev. **101**, 866 (1956); T. Kinoshita and A. Sirlin, Phys. Rev. (to be published), and R. E. Behrends (private communication).

Only the following combinations of coupling constants occur in the parameters: for $\mu\to e+\bar{\nu}+\nu$,

$$\begin{aligned} S&=|C_S|^2+|C_P'|^2+|C_S'|^2+|C_P|^2,\\ s&=2\,\mathrm{Re}(C_S^*C_P'+C_S'^*C_P),\\ V&=|C_V|^2+|C_A'|^2+|C_V'|^2+|C_A|^2,\\ v&=2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A),\\ T&=|C_T|^2+|C_T'|^2,\\ t&=2\,\mathrm{Re}(C_T^*C_T'),\end{aligned}\tag{3}$$

whereas for $\mu\to e+2\bar{\nu}$ and for $\mu\to e+2\nu$,

$$\begin{aligned} S&=2(|C_S|^2+|C_P'|^2+|C_S'|^2+|C_P|^2),\\ s&=4\,\mathrm{Re}(C_S^*C_P'+C_S'^*C_P),\\ V&=2(|C_V'|^2+|C_A|^2),\\ v&=4\,\mathrm{Re}(C_V'^*C_A),\\ T&=t=0.\end{aligned}\tag{4}$$

Indeed, in terms of these combinations the parameters are

$$\begin{aligned} \frac{1}{\tau}&=\frac{m^5}{24(4\pi)^3}(S+4V+6T),\\ \rho&=(3V+6T)/(S+4V+6T),\\ \zeta&=(3v-6t)/(S+4V+6T)\\ \alpha&=\tfrac{1}{3}(3s+4v-14t)/(S+4V+6T).\end{aligned}\tag{5}$$

The restrictions imposed on S, V, T, s, v, t, by their definitions are precisely $S\geq 0$, $|s|\leq S$, $V\geq 0$, $|v|\leq V$, $T\geq 0$, $|t|\leq T$, and, in the cases $2\bar{\nu}$ or 2ν, $T=t=0$. These inequalities and (5) imply that the following are a maximally strong system of inequalities between the shape parameters: for $\mu\to e+\bar{\nu}+\nu$: $0\leq\rho\leq 1$, $|\zeta|\leq\rho$, and $|\alpha-(7/9)\zeta|\leq 1-\rho$ (see Fig. 1); whereas for $\mu\to e+2\bar{\nu}$

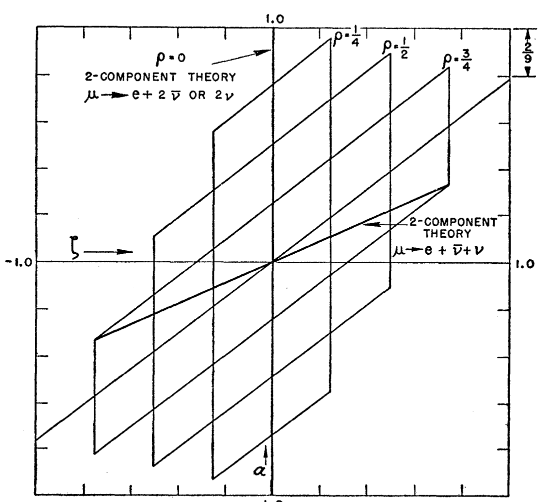

FIG. 1. Inequalities for α and ζ for several values of ρ, for the decay $\mu\to e+\bar{\nu}+\nu$. For each value of ρ, the parameters α and ζ must iie inside the corresponding parallelogram.

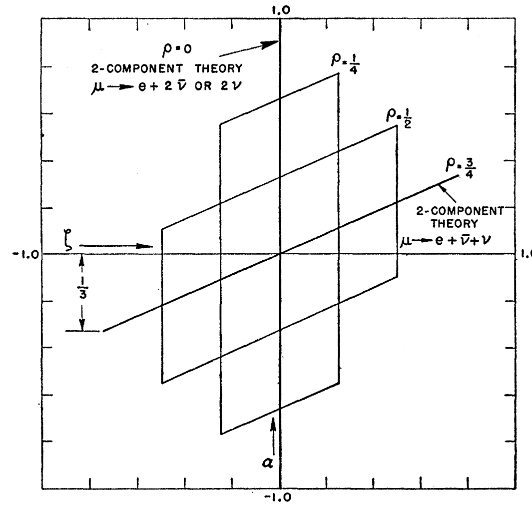

FIG. 2. Inequalities for the decay $\mu\to e+2\bar{\nu}$ or 2ν. For each value of ρ, the parameters α and ζ must lie inside the corresponding parallelogram.

and $\mu\to e+2\nu$: $0\leq\rho\leq\frac{3}{4}$, $|\zeta|\leq\rho$, and $|\alpha-(4/9)\zeta|\leq 1-(4/3)\rho$ (see Fig. 2). If only ρ is given, but not ζ, the conditions on α are, respectively, $|\alpha|\leq 1-(2/9)\rho$; $|\alpha|\leq 1-(8/9)\rho$.

The empirical parameters for a mixture of the cases $2\bar{\nu}$, 2ν, and $\bar{\nu}+\nu$ are readily seen to be the weighted averages of the corresponding parameters proper to the respective cases, with weights proportional to the three branching ratios, as indeed distinct channels do not interfere. Since the inequalities are linear, the $2\bar{\nu}$ or 2ν inequalities apply as well to the empirical parameters for a mixture of $2\bar{\nu}$ and 2ν. Because the cases $2\bar{\nu}$ and 2ν differ effectively from $\bar{\nu}+\nu$ only in involving the additional restriction $T=0$, it follows that the inequalities for identical neutrinos are at least as strong as those for $\bar{\nu}+\nu$: they are, in fact, stronger. Thus the more general $\bar{\nu}+\nu$ inequalities apply to the most general mixture.

Lee and Yang have shown[3] how the two-component neutrino theory may be regarded as a special case of the four-component theory, by choosing the coupling constants in (1) as follows: for the case $\mu\to e+\bar{\nu}+\nu$,

$$\begin{aligned} &C_S=C_S'=C_P=C_P'=C_T=C_T'=0,\\ &C_V'=-C_V,\quad C_A'=-C_A,\end{aligned}\tag{6}$$

whereas for the cases $\mu\to e+2\bar{\nu}$ and $\mu\to e+2\nu$,

$$\begin{aligned} &C_V=C_V'=C_A=C_A'=0,\\ &C_S'=-C_S,\quad C_T'=-C_T,\quad C_P'=-C_P.\end{aligned}\tag{7}$$

Their distributions are, then, necessarily special cases of those described here; see Figs. 1–3. The general conditions under which these special distributions appear in the four-component theory are, for $\mu\to e+\bar{\nu}+\nu$,

$$S=2T,\quad s=2t;\tag{8}$$

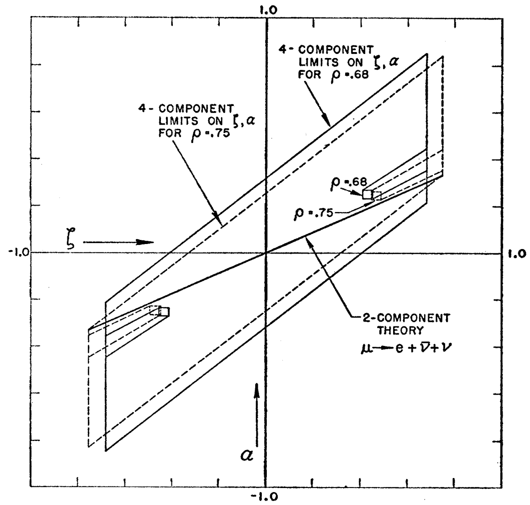

FIG. 3. Comparison of experimental values of α, ζ, with theoretical limits. The direct empirical parameters α', ζ' for $\rho=0.68$, $\rho=0.75$ are represented by the little rectangles. The actual parameters α, ζ are larger in absolute value, because of depolarization effects. The first quadrant applies if the μ's are polarized in the direction back to the parent π, the third quadrant, if they are polarized opposite to this direction.

whereas for $\mu\rightarrow e+2\bar{\nu}$ or 2ν (where $T=0$),

$$V=0. \tag{9}$$

Conditions (8) and (9) are, respectively, more general than the conditions (6) and (7) for two-component theory, but indeed one does not expect to derive a large number of detailed conditions from only three shape parameters.

Professor Lederman has very kindly given us detailed data on the peak-to-valley ratio of electrons for various thicknesses of absorber in his experiment,[7,9] and resolution curves which were obtained by E. Garwin and C. Oxley at Chicago. From this information, we have determined the empirical parameters ζ' and α', which are plotted on Fig. 3. These "empirical parameters" refer directly to the empirical electron distribution expressed by a formula of form (1), where however the direction "up" or $\theta=0$ is chosen arbitrarily in the sense of the peak of the asymmetry, which happens to be the "backward" direction from the μ back to its π origin. Thus, α' is here positive by convention, and our former α is respectively positive or negative according as the μ spins are predominantly parallel or antiparallel to the backward direction; this ambiguity may, however, be resolved by future experiments. A further difficulty in nferring ζ, α from ζ', α' is the lack of precise knowledge as to the extent of polarization of the μ mesons: they are not totally polarized because of imperfect collimation, scattering, capture, etc., and indeed it is not known whether the μ is completely polarized in $\pi-\mu$ decay. But, clearly, $\zeta'=(2f-1)\zeta$; $\alpha'=(2f-1)\alpha$, where f is the fraction of the μ spins parallel to the backward direction, and therefore (ζ,α) must lie in either of the two zones, in the first and third quadrants, indicated on Fig. 3.

Our method of analysis was as follows: If $K(t,x)$ is the resolution function, with $t=$thickness of absorber, then we define

$$u(t)=\int_0^1 K(t,x)(4x-3)x^2dx \Big/ \int_0^1 K(t,x)(1-x)x^2dx.$$

If $P(t)$ is the experimental peak-to-valley ratio and $1+a(t)\cos\theta$ represents the electron distribution from the μ mesons, then $a=1.092(P-1)/(P+1)$, as there is a correction of $\sim 9.2\%$ for gate width and counter solid angle. Then, if we go to u as independent variable,

$$a(t(u))\left(1+\frac{2}{9}\rho u\right)=\alpha'+\frac{2}{9}\zeta' u.$$

We therefore made a least-squares fit of a straight line to the left member, weighting points inversely as the square of their error flags; the latter included only the errors quoted on the peak-to-valley data. For $\rho=0.68$,[12] we find $\alpha'=0.251\pm0.018$, $\zeta'=0.431\pm0.022$. By differentiating our least-squares formulas in order to extrapolate linearly to nearby values of ρ, we find for $\rho=0.75$ that $\alpha'=0.246\pm0.018$, $\zeta'=0.467\pm0.23$. As can be seen on Fig. 3, these values (as well as the values ζ, α one would obtain after the unknown correction for sense and intensity of the polarization) come rather close to the line $\alpha=(4/9)\zeta$ consistent with the two-component decay $\mu\rightarrow e+\bar{\nu}+\nu$. Since the errors quoted represent only those from peak-to-valley data, the consistency with the two-component theory seems fair.

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