Kinematics of β Decay and Parity Nonconservation in Weak Interactions*

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A general description of a Dirac spinor is given in terms of two-component spin wave functions corresponding to its large and small components. These may be used to describe the final states resulting from a β-decay interaction in a form which permits the derivation of angular distributions and correlations by utilization of the quantum-mechanical combination properties of angular momenta. The two-component eigenvectors are such that the one corresponding to the small component of the Dirac spinor has opposite parity from the large component. Hence, this representation is naturally extended to the description of interactions in which parity is not conserved. A general formulation is given for the final-state wave functions corresponding to such interactions, and is used to discuss the β decay of unpolarized and polarized nuclei, pions, muons, and heavy mesons. Appropriate choice of the parameters in this description is shown to lead to the two-component neutrino theory of Lee and Yang, Landau, and Salam.

Finally, a detailed discussion is given of the consequences of parity nonconservation in the π-meson decay modes of a heavy meson, with particular emphasis on the possibilities of its observation in the τ decay mode of a spin-1 heavy meson.

INTRODUCTION

THE problem of the K-meson decay[1] has recently led Lee and Yang[2] to examine the evidence concerning the conservation of parity in weak interactions. They have concluded that conventional β-decay measurements (spectrum, lifetime, e-γ, γ-γ, and e-ν correlations) do not permit any conclusions to be drawn regarding this problem; however, they suggest a number of experiments in which parity nonconservation could be observed. In particular, they have suggested that the effects of the violation of reflection invariance in the β-decay interactions would be observed in the appearance of an asymmetry in the distribution of decay electrons resulting from the β decay of polarized nuclei or polarized μ mesons. Subsequently, such an effect has been observed in the β decay of radioactive cobalt by Wu, Ambler, Hayward, Hoppes, and Hudson,[3] and in the μ-meson decay by Garwin, Lederman, and Weinrich,[4] and by Friedman and Telegdi.[5]

The effect, in both cases, is manifest as an angular distribution of the decay electrons, with respect to the direction of polarization of the decaying particle, of the form

$$W(\theta)=1+a\cos\theta. \qquad (1)$$

In the papers of Lee and Yang, expressions are given for the asymmetry parameter, a, based on a number of possibilities for the form of the β-decay interaction Hamiltonian. In particular, Lee and Yang, Landau, and also Salam[6] propose the two-component neutrino theory,[7] in which the spin of the neutrino is always oriented parallel (or antiparallel) to its direction of flight and in which the antineutrino is the mirror image of the neutrino, as one (and, possibly, the simplest) means of introducing parity nonconservation into β decay while, at the same time, preserving the main features of conventional β-decay theory.[8]

Now, the two-component neutrino theory has a consequence that all the leptons involved in β decay are longitudinally polarized. As a result, those properties of the β decay concerned with angular momenta and their combination (i.e., angular distributions and correlations) are susceptible to simple interpretation in terms of "vector models." However, while such vector models are extremely useful, and have been extensively employed in understanding the qualitative features of the phenomena of interest, they suffer from the well-known deficiency that they do not properly take into account the full quantum-mechanical nature of angular momentum, and are consequently incapable of yielding results which are quantitatively correct. This is, of course, not a serious drawback since we have at our disposal, for the computation of β-decay phenomena, the full force of a formalism based on the Dirac equation for the electron and neutrino, which has been extensively developed over the past 25 years.[8,9] These techniques have been employed by Lee and Yang,[2] by Jackson,

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[1] *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956).
[2] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[3] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
[4] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).
[5] J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).
[6] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957); L. Landau, Nuclear Phys. **3**, 127 (1957); A. Salam, Nuovo cimento **5**, 299 (1957).
[7] H. Weyl, Z. Physik **56**, 330 (1929); W. Pauli, *Handbuch der Physik* (J. Springer, Berlin, 1933), Vol. 24, pp. 226–227.
[8] Lee and Yang, reference 6, show that it is only in the observation of a pseudoscalar quantity, such as $\boldsymbol{\sigma}\cdot\mathbf{p}_e$, that effects of parity nonconservation may become manifest. Hence, in the discussion of β decay, the usual conventions [see *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955)] may be used without essential modification.
[9] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Chap. XIII.

Treiman, and Wyld,[10] and undoubtedly by many others, to examine the consequences of various nonreflection-invariant Hamiltonians for the β decay.

However, in the usual β-decay theories, the formalism tends to obscure the role of angular momenta in determining the angular distributions and correlations of the decay products and their spins. It is the purpose of this paper to present a formulation of the "kinematics" of β decay in terms of conventional angular-momentum operators and in such a form that angular distributions and correlations may be described by the methods of conventional angular-correlation theory.[11] In this formulation, the consequences of parity nonconservation are exhibited in a general and familiar form without the necessity of resort to specific β-decay theories.

In such a kinematical treatment, the dynamics of the interaction are contained in the amplitudes associated with the various possible angular-momentum states of the system under consideration. The results of various specific interactions will be obtained by the appropriate choice of these amplitudes. Thus, we will first consider the kinematics of conventional, parity-conserving β decay. Then, we introduce the consequences of parity nonconservation in a general form and apply the formulation to the β decay of unpolarized and polarized nuclei, π mesons, μ mesons, and K mesons. In each case, the results of the two-component neutrino theory[6,10] represent a special case. Finally, we discuss the problem of parity nonconservation in the π-meson decay modes of the K meson.

A. GENERAL FORMULATION

Our approach is based on the familiar observation[12] that a four-component spinor, representing the solution of the Dirac equation for a spin-$\frac{1}{2}$ particle, may be represented in terms of two two-component wave functions corresponding, respectively, to the large and small components of the Dirac spinor. Thus, writing the Dirac equation in a conventional representation

$$(E-m_0c^2\beta)\psi=c\boldsymbol{\alpha}\cdot\mathbf{p}\psi, \tag{2}$$

the wave function

$$\psi=\begin{pmatrix}u\\w\end{pmatrix} \tag{3}$$

may be written in terms of the two-component spinors u and w, which are connected by

$$w=\left(\frac{c\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m_0c^2}\right)u. \tag{4}$$

For a particle in a state of angular momentum j, if u represents the combination $j=s+l$, then, by virtue of the $\boldsymbol{\sigma}\cdot\mathbf{p}$ operator, the orbital angular momentum l' corresponding to w differs from l by one unit. Thus, the wave functions corresponding to u and w have opposite parity.

In particular, let $\alpha\equiv\begin{pmatrix}1\\0\end{pmatrix}$ and $\beta\equiv\begin{pmatrix}0\\1\end{pmatrix}$ represent the pure spin eigenvectors ($l=0$) of u for a spin-$\frac{1}{2}$ particle with z-component $m=\pm\frac{1}{2}$, respectively. Then, the corresponding spin eigenvectors for the small components, w, may be written

$$\begin{aligned}\alpha'&=-(\tfrac{1}{3})^{\frac{1}{2}}P_1{}^0\alpha+(\tfrac{2}{3})^{\frac{1}{2}}P_1{}^1\beta\\&=-(\cos\theta\alpha+\sin\theta e^{i\varphi}\beta),\end{aligned} \tag{5a}$$

$$\begin{aligned}\beta'&=-(\tfrac{2}{3})^{\frac{1}{2}}P_1{}^{-1}\alpha+(\tfrac{1}{3})^{\frac{1}{2}}P_1{}^0\beta\\&=-\sin\theta e^{-i\varphi}\alpha+\cos\theta\beta.\end{aligned} \tag{5b}$$

The eigenvectors α' and β' represent the appropriate combinations of $s=\frac{1}{2}$ and $l'=1$ into the state $j=\frac{1}{2}$, with $m=\pm\frac{1}{2}$, respectively.[13]

The spinors u and w contain, in addition to the angular-momentum states described above, the radial dependence of the wave function. For the purposes of this paper, since we are interested primarily in angular distributions and correlations, these need not be specified. It is, however, important to note, from Eq. (4), that the relative magnitudes of the small and large components are given by

$$\xi=\frac{2\,\mathrm{Re}(u^*w)}{|u|^2+|w|^2}=\pm\frac{v}{c}. \tag{6}$$

In general, for a given β-decay interaction Hamiltonian, the electron-neutrino wave function is described by

$$\psi_{e\nu}=\psi_eO_i\psi_\nu, \tag{7}$$

where the O_i are operators characteristic of the interaction.[9] Thus, the scalar and vector interactions lead to the singlet electron-neutrino spin combination, while the tensor and axial-vector interactions lead to the triplet spin states. Since conventional, parity-conserving β-decay interactions lead to final states of well-defined parity, the electron-neutrino wave function for an allowed Fermi transition ($\Delta J=0$, no) may be written

$$\psi_0=A_02^{-\frac{1}{2}}(\alpha_e\beta_\nu-\beta_e\alpha_\nu)+B_02^{-\frac{1}{2}}(\alpha_e'\beta_\nu'-\beta_e'\alpha_\nu'), \tag{8}$$

while the allowed Gamow-Teller ($\Delta J=0,\pm1$, no) electron-neutrino wave functions are

$$\psi_1{}^1=A_1\alpha_e\alpha_\nu+B_1\alpha_e'\alpha_\nu', \tag{9a}$$

$$\psi_1{}^0=A_12^{-\frac{1}{2}}(\alpha_e\beta_\nu+\beta_e\alpha_\nu)+B_12^{-\frac{1}{2}}(\alpha_e'\beta_\nu'+\beta_e'\alpha_\nu'), \tag{9b}$$

$$\psi_1{}^{-1}=A_1\beta_e\beta_\nu+B_1\beta_e'\beta_\nu'. \tag{9c}$$

The angular distributions of the decay products from

[10] Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957).

[11] See J. M. Blatt and L. C. Biedenharn, Revs. Modern Phys. 24, 258 (1952).

[12] See J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Press, Cambridge, 1955), Appendix 2.

[13] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1935), p. 76.

unpolarized nuclei are

$$W_0(\theta_e,\varphi_e,\theta_\nu,\varphi_\nu)=|\psi_0|^2=|A_0|^2+|B_0|^2 -2\,\mathrm{Re}A_0B_0^*\cos\varphi_{e\nu}, \quad (10)$$

where

$$\cos\theta_{e\nu}=\cos\theta_e\cos\theta_\nu+\sin\theta_e\sin\theta_\nu\cos(\varphi_e-\varphi_\nu) \quad (10')$$

is the angle between the electron and the neutrino, and

$$W_1=\tfrac{1}{3}\sum_m |\psi_1{}^m|^2=|A_1|^2+|B_1|^2 +\tfrac{2}{3}\,\mathrm{Re}A_1B_1^*\cos\theta_{e\nu}. \quad (11)$$

The nuclear-transition matrix element (squared) and the electron energy spectrum are contained in $|A_1|^2$, which is proportional to the density of final states in phase space and is given by the usual expressions.[9] What is of interest to us, here, is the dependence of W on $\theta_{e\nu}$, the electron-neutrino correlation. This may be obtained in the conventional form[9]:

$$W(\theta_{e\nu})=1+\lambda(v_e/c)\cos\theta_{e\nu}, \quad (12)$$

by associating A and B with the products u_eu_ν and w_ew_ν, respectively, and using Eq. (6) for the relative amplitudes. Comparison of Eqs. (10) and (11) with the results of conventional β-decay theory lead to the sign choices as shown in Table I.[14]

Parity-nonconserving β-decay interactions are characterized by operators O_i, in Eq. (7), which lead to electron-neutrino wave functions of mixed parity. We generalize all such interactions by describing the spinors involved in terms of two-component wave functions of mixed parity. Let

$$x_+=a_x\alpha_x+b_x\alpha_x{}', \quad (13a)$$

$$x_-=a_x\beta_x+b_x\beta_x{}', \quad (13b)$$

represent, respectively, such mixed-parity, $m_s=\pm\frac{1}{2}$ wave functions for the particle, x. Then, the electron-neutrino wave function for the allowed Fermi and Gamow-Teller transitions are, respectively,

$$\phi_0=2^{-\frac{1}{2}}(e_+\nu_--e_-\nu_+), \quad (14)$$

$$\phi_1{}^{\pm1}=e_\pm\nu_\pm, \quad (15a)$$

$$\phi_1{}^0=2^{-\frac{1}{2}}(e_+\nu_-+e_-\nu_+), \quad (15b)$$

TABLE I. Comparison with β-decay theory for sign choice in Eq. (6).

Interaction	Selection rules	λ	Sign of ξ
Scalar	Fermi	-1	$+$
Vector	Fermi	$+1$	$-$
Tensor	Gamow-Teller	$+\frac{1}{3}$	$+$
Axial vector	Gamow-Teller	$-\frac{1}{3}$	$-$

[14] The so-called "relativistic" or parity-forbidden transitions with selectrion rules $\Delta J=0$, yes (F′) and $\Delta J=0,\pm1$, yes (GT′) may be represented, in our formulation, by the wave functions

$$\psi_0{}'=A_0{}'2^{-\frac{1}{2}}(\alpha_e\beta_\nu{}'-\beta_e\alpha_\nu{}')+B_0{}'2^{-\frac{1}{2}}(\alpha_e{}'\beta_\nu-\beta_e{}'\alpha_\nu),$$
$$\psi_1{}^{1\prime}=A_1{}'\alpha_e\alpha_\nu{}'+B_1{}'\alpha_e{}'\alpha_\nu, \quad \text{etc.,}$$

and the corresponding sign associations may be made in the same way.

and the corresponding angular distributions from *unpolarized* nuclei are

$$W_0{}'=(|a_e|^2+|b_e|^2)(|a_\nu|^2+|b_\nu|^2) -4(\mathrm{Re}a_eb_e{}^*)(\mathrm{Re}a_\nu b_\nu{}^*)\cos\theta_{e\nu}, \quad (10a)$$

$$W_1{}'=(|a_e|^2+|b_e|^2)(|a_\nu|^2+|b_\nu|^2) +\tfrac{4}{3}(\mathrm{Re}a_eb_e{}^*)(\mathrm{Re}a_\nu b_\nu{}^*)\cos\theta_{e\nu}. \quad (11a)$$

The results are the same as for conventional β-decay theory provided we connect a and b with u and w in Eq. (6) and adopt the same sign conventions for the products $\xi_e\xi_\nu$ as for ξ in Table I.

B. GENERAL CONSEQUENCES OF PARITY NONCONSERVATION

The introduction of mixed-parity wave functions for a spin-$\frac{1}{2}$ particle [Eqs. (13)] has the consequence that such particles are always polarized along, or opposite to, their directions of motion, the degree of such longitudinal polarization depending on the ratio b/a. This may be seen if we transform the spin vectors α_x and β_x in Eqs. (13) into the vectors $\alpha_x{}^l$ and $\beta_x{}^l$ as observed along the direction of emission of the particle x (i.e., angles θ, ϕ with respect to the original z axis). This is accomplished by the transformation

$$\boldsymbol{\alpha}=2^{-\frac{1}{2}}(1+\cos\theta)^{\frac{1}{2}}\alpha_x{}^l-2^{-\frac{1}{2}}(1-\cos\theta)^{\frac{1}{2}}e^{i\varphi}\beta_x{}^l, \quad (16a)$$

$$\boldsymbol{\beta}=2^{-\frac{1}{2}}(1-\cos\theta)^{\frac{1}{2}}e^{-i\varphi}\alpha_x{}^l+2^{-\frac{1}{2}}(1+\cos\theta)^{\frac{1}{2}}\beta_x{}^l. \quad (16b)$$

Substitution into Eqs. (13) yields

$$\mathbf{x}_+=2^{-\frac{1}{2}}(1+\cos\theta)^{\frac{1}{2}}(a_x-b_x)\alpha_x{}^l -2^{-\frac{1}{2}}(1-\cos\theta)^{\frac{1}{2}}e^{i\varphi}(a_x+b_x)\beta_x{}^l, \quad (13a')$$

$$\mathbf{x}_-=2^{-\frac{1}{2}}(1-\cos\theta)^{\frac{1}{2}}e^{-i\varphi}(a_x-b_x)\alpha_x{}^l +2^{-\frac{1}{2}}(1+\cos\theta)^{\frac{1}{2}}(a_x+b_x)\beta_x{}^l. \quad (13b')$$

A number of conclusions may be drawn directly from Eqs. (13′):

(1) In the extreme of $b_x=\pm a_x$, the particles are *completely* polarized, either opposite to or along their direction of motion. Thus, if we assume a massless spin-$\frac{1}{2}$ particle (the neutrino), with ξ given by Eq. (6), we arrive at the two-component theory.[6,7] Accordingly, $b_\nu=-a_\nu$ and $b_{\bar{\nu}}=a_{\bar{\nu}}$ define the two-component neutrino and antineutrino.

(2) If we associate a and b with u and w for all spinors, then in any interaction which leads to spinor wave functions of the form of Eqs. (13) the emitted particles are longitudinally polarized, the degree of polarization being proportional to v/c of the emitted particle.

(3) In particular, in the case of β decay from *unpolarized* nuclei, we may compute the degree of longitudinal polarization of the emitted electrons:

$$P=[W(\alpha_e{}^l)-W(\beta_e{}^l)]/[W(\alpha_e{}^l)+W(\beta_e{}^l)], \quad (17a)$$

where

$$W(\alpha_e^l)=\frac{1}{(2S+1)}\sum_m|\phi_S^m\cdot\alpha_e^l|^2, \quad (17b)$$

$$W(\beta_e^l)=\frac{1}{(2S+1)}\sum_m|\phi_S^m\cdot\beta_e^l|^2, \quad (17c)$$

in which $\phi_S^m\cdot\alpha^l$ represents the component of ϕ_S^m corresponding to the longitudinal polarization α^l of the emitted electron. Using Eqs. (14) and (15) for the eigenvectors describing, respectively, the allowed Fermi and Gamow-Teller transitions, we obtain, in both cases

$$P=-\frac{2\,\mathrm{Re}a_eb_e^*}{|a_e|^2+|b_e|^2}=-\xi_e. \quad (18)$$

Thus, the nonconservation of parity in the β-decay process leads to a longitudinal polarization of the emitted electrons, *even from unpolarized nuclei*, of a degree determined by v_e/c and sign determined by the form of the interaction. This effect has been observed by Frauenfelder *et al.*[15] on the electrons from the Co^{60} decay.

C. β-DECAY OF ORIENTED NUCLEI

We consider a nucleus in the initial state characterized by the angular-momentum quantum numbers J, m. An allowed decay via the GT selection rules is characterized by the final-state wave function

$$\chi_1=\sum_{m'=-1}^{1}C(J',m-m';1,m'|J,m)\phi_1^{m'}\varphi_{J'}^{m-m'}, \quad (19)$$

in which the $\varphi_{J'}^{m-m'}$ representing the wave functions of the possible states of the product nucleus, $\phi_1^{m'}$ are given by Eqs. (15) and the $C(J',m-m';1,m|J,m)$ are the appropriate angular-momentum combination (Clebsch-Gordan) coefficients corresponding to the permitted transitions.[13]

Our interest is in the angular distribution of the decay electrons

$$\frac{dN_e}{d\Omega_e}=\int|\chi_1|^2d\Omega_\nu$$
$$=\sum_{m'}C^2(J',m-m';1,m'|J,m)|\phi_1^{m'}|^2, \quad (20)$$

in which the cross terms vanish because of the orthogonality of the $\varphi_{J'}^{m-m'}$'s. Equation (20) is easily evaluated, by using Eqs. (5) and (13) for $e_\pm$ and taking advantage of the property

$$\int\nu_i^*\cdot\nu_jd\Omega_\nu=\delta_{ij}. \quad (21)$$

[15] Frauenfelder, Bobene, von Goeler, Levine, Lewis, Peacock, Rossi, and DePasquali, Phys. Rev. **106**, 386 (1957).

The result may be written in the general form

$$\frac{dN_{1e}}{d\Omega_e}(J\to J')$$
$$=|a_e|^2+|b_e|^2-\lambda_1(J,J')2(\mathrm{Re}a_eb_e^*)\frac{m}{J}\cos\theta_e, \quad (22)$$

where

$$\lambda_1(J,J-1)=1, \quad (22a)$$

$$\lambda_1(J,J)=1/(J+1), \quad (22b)$$

$$\lambda_1(J,J+1)=-J/(J+1). \quad (22c)$$

For an allowed Fermi transition,

$$\chi_0=\phi_0\varphi_J^m \quad (19')$$

and

$$dN_{0e}/d\Omega_e=|a_e|^2+|b_e|^2, \quad (22')$$

with $\lambda_0(J,J)=0$. However, only in the case $\Delta J=0$, the transition may be a mixture of F and GT, with the final-state wave function the sum of Eqs. (19) and (19′). In this case, the angular distribution of the decay electrons is

$$\frac{dN_e}{d\Omega_e}(J\to J)$$
$$=(|a_e|^2+|b_e|^2)_{\mathrm{GT}}+(|a_e|^2+|b_e|^2)_{\mathrm{F}}$$
$$-\left\{\frac{2}{J+1}(\mathrm{Re}a_eb_e^*)_{\mathrm{GT}}+2\left(\frac{J}{J+1}\right)^{\frac{1}{2}}\right.$$
$$\left.\times[\mathrm{Re}(a_e)_{\mathrm{GT}}(b_e^*)_{\mathrm{F}}+(a_e)_{\mathrm{F}}(b_e^*)_{\mathrm{GT}}]\right\}\frac{m}{J}\cos\theta_e. \quad (22'')$$

Note the additional asymmetry arising from an interference between the Fermi and Gamow-Teller transitions.

Although Eqs. (22) apply to decay from a pure initial state, Jm, the results for partially polarized β-decaying nuclei are obtained by the substitution of $\langle J_z\rangle$ for m in these equations.

Since, in the general formulation used above, the coefficients a_x and b_x of Eqs. (13) are completely unspecified, the results apply to all possible forms of the β-decay interaction. If we again associate these with the large and small components of the Dirac spinors, through Eqs. (6) with $b_\nu=-a_\nu$ and $b_{\bar\nu}=a_{\bar\nu}$, our results are identical with those of the two-component neutrino theory. Furthermore, assignment of neutrino and antineutrino[6] follows immediately from the experiments, with the aid of Table I, as follows:

(1) In the experiments of Wu *et al.*,[3] the electrons from the decay of polarized Co^{60} ($5^+\to4^+$) were found to be emitted preferentially in the direction opposite to the direction of nuclear polarization. Thus, in Eq. (22), $\xi_{e^-}>0$. Assuming that this decay proceeds through the tensor interaction,[8] we have, from Table I,

$\xi=\xi_e-\xi_{\nu \text{ or } \bar{\nu}}>0$. Thus, we are led to the association

$$n\rightarrow p+e^-+\bar{\nu}, \tag{23}$$

the same conclusion follows from the observations of Frauenfelder *et al.*[15] Correspondingly, in positron decays through the GT interaction

$$p\rightarrow n+e^++\nu, \tag{24}$$

$\xi_\nu=-1$, $\xi>0$ and, therefore, $\xi_{e^+}<0$, and the asymmetries should all be reversed.

(2) A more recent measurement by Wu *et al.*,[3,16,17] on the positrons emitted from polarized Co^{58} nuclei, indicates an asymmetry of the opposite sign and $\sim\frac{1}{3}$ the magnitude of that from Co^{60}. This result is as expected from the above considerations. Indeed, according to Eqs. (22), the Co^{58} ($J=2^+$, $J'=2^+$) asymmetry should be only $\frac{1}{3}$ as great, provided the observations are made on nuclei with the same degree of polarization and on electrons of the same v_e/c, and provided further that the decay arises mainly from the tensor interaction [$(a_e)_F\ll(a_e)_{GT}$ in Eq. (22″)].

(3) However, an appreciable admixture of scalar interaction would produce a significant change in the degree of asymmetry in a $J\rightarrow J$ decay, see Eq. (22″). Thus, for $(a_e)_F=(a_e)_{GT}$ the asymmetry parameter [a in Eq. (1)] would be increased by a factor 3 for Co^{58}. In this regard, a measurement of fundamental interest would be the observation of the asymmetry parameter for the decay of polarized neutrons, reaction (23). In this case, Eq. (22″), together with the two-component neutrino hypothesis, predicts $a_n=-\frac{2}{3}(v_e/c)(\langle J_z\rangle/J)$ for $(a_e)_F=0$, and $a_n=-0.91(v_e/c)(\langle J_z\rangle/J)$ for $(a_e)_F=(a_e)_{GT}$.

(4) The polarization of the electrons emitted from oriented nuclei[10] is computed as outlined in Sec. B(3). For Fermi transitions, the result is again Eq. (18). For pure Gamow-Teller transitions the polarization depends on the angle of emission, as is also the case for a Fermi and Gamow-Teller mixture; in general,

$$W\begin{Bmatrix}\alpha^l\\ \beta^l\end{Bmatrix}=\tfrac{1}{2}|(a_e\mp b_e)_{GT}|^2\left\{1\pm\lambda(J,J')\frac{\langle J_z\rangle}{J}\cos\theta_e\right\}$$
$$+\delta_{J,J'}\left\{\tfrac{1}{2}|(a_e\mp b_e)_F|^2\right.$$
$$\left.\pm\mathrm{Re}(a_e\mp b_e)_{GT}(a_e\mp b_e)_F{}^*\frac{\langle J_z\rangle}{[J(J+1)]^{\frac{1}{2}}}\cos\theta_e\right\}. \tag{18}$$

Here, again, if we go to the relativistic limit, $v_e/c\rightarrow 1$ and use Eq. (6) for $\xi_{e\mp}\rightarrow\pm 1$, the longitudinal polarization of the electrons is complete.

[16] C. S. Wu, post-deadline paper, New York meeting of the American Physical Society, February 2, 1957 (unpublished); Ambler, Hayward, Hoppes, Hudson, and Wu, Phys. Rev. **106**, 1361(L) (1957).

[17] See also Postma, Huiskamp, Miedema, Steenland, Tolhoek, and Gorter, Physica **23**, 259 (1957).

D. DECAY OF THE π MESON

The same approach, as was used above in the discussion of nuclear β decay, may be applied to the decay of the pion:

$$\pi\rightarrow\mu+\nu. \tag{25}$$

Since the pion has spin 0, we have for the wave function of the $\mu-\nu$ (or $\mu-\bar{\nu}$), in the pion rest system [see Eq. (14)]

$$\phi_\pi=2^{-\frac{1}{2}}(\mu_+\nu_--\mu_-\nu_+), \tag{26}$$

where the components are given by Eqs. (13). This differs from the Fermi β decay, discussed above, in that the directions of μ and ν are absolutely correlated ($\theta_\nu=180°-\theta_\mu$) since we are dealing with a two-body decay process. The decay is, of course, isotropic in the pion rest system, since the pion has spin 0.

We may, however, draw some nontrivial conclusions concerning the polarization of the emitted μ and ν (or $\bar{\nu}$). We define the z axis as the direction of emission of the muon. Then

$$\phi_\pi(\theta_\mu=0°,\theta_\nu=180°)=2^{-\frac{1}{2}}\{(a_\mu-b_\mu)(a_\nu-b_\nu)\boldsymbol{\alpha}_\mu\boldsymbol{\beta}_\nu -(a_\mu+b_\mu)(a_\nu+b_\nu)\boldsymbol{\beta}_\mu\boldsymbol{\alpha}_\nu\}. \tag{27}$$

Thus, complete polarization of the μ and ν along their directions of emission are obtained for $b_\nu=-a_\nu$; the choice $b_{\bar{\nu}}=a_{\bar{\nu}}$ leads to complete polarization of the μ and $\bar{\nu}$ along directions opposite to their directions of emission. These choices correspond to the two-component neutrino theory.[6,18]

Equation (27) also indicates naturally the possibility of suppressing the $\pi\rightarrow e+\nu$ decay by the proper choice of interaction.[19] Thus, if we choose an interaction (axial vector) such that ξ_e is opposite in sign to ξ_ν, the ratio of the electron to muon decay probabilities is

$$\frac{\lambda_e}{\lambda_\mu}=\frac{(1-v_e/c)dN_e/dE}{(1-v_\mu/c)dN_\mu/dE}\cong 10^{-4}. \tag{28}$$

E. DECAY OF THE μ MESON

In the following discussion, we consider some consequences which would result from a two-component neutrino theory,[6] since the complications of the three-body decay under consideration make the most general application of the formalism developed in the preceding relatively cumbersome. In considering the alternative possibilities for the muon decay process,

$$\mu\rightarrow e+\nu+\bar{\nu}, \tag{29}$$

$$\mu\rightarrow e+\nu+\nu, \tag{29a}$$

[18] For conventional, parity-conserving β-decay theories, the wave function corresponding to Eq. (26) would contain two separate terms, one obtained by setting $b_\mu=b_\nu=0$ and the second by setting $a_\mu=a_\nu=0$ in Eq. (26). Thus, we would have

$$\phi_\pi(\theta_\mu=0°,\theta_\nu=180°)=2^{-\frac{1}{2}}AB(\alpha_\mu\beta_\nu-\beta_\mu\alpha_\nu),$$

which gives no polarization at all.

[19] B. D'Espagnat, Compt. rend. **228**, 744 (1949); M. A. Ruderman and R. Finkelstein, Phys. Rev. **76**, 1458 (1949).

or

$$\mu \to e + \bar{\nu} + \bar{\nu}, \qquad (29b)$$

a number of interesting conclusions may be drawn concerning energy spectra and angular distributions.

(1) We consider, first, decays in which the two neutrinos are identical. We may immediately conclude, on the grounds of the Pauli exclusion principle, that the electron decay spectrum, for such theories, will always fall to zero at the maximum electron energy (i.e., Michel parameter[20] $\rho=0$), since at the maximum electron energy the two (completely polarized) neutrinos, emitted in the same direction, cannot be in a singlet state; the triplet state is forbidden by the Pauli exclusion principle. The singlet wave function of the identical neutrinos is of the form of Eq. (14).

$$\phi_{2\nu} = 2^{-\frac{1}{2}}(\nu_{1+}\nu_{2-} - \nu_{1-}\nu_{2+}). \qquad (30)$$

Consequently, for the decay of initially polarized μ mesons, the wave function of the emitted electron must be $e_\pm$, for $m_\mu = \pm\frac{1}{2}$, and the electron angular distributions (for all electron energies) are

$$\frac{dN_e}{d\Omega_e} = |e_\pm|^2 = |a_e|^2 + |b_e|^2 \mp 2(\mathrm{Re}\, a_e b_e^*) \cos\theta_e. \qquad (31)$$

Since the decay electrons have high energy, $\xi_e = \pm v_e/c \cong \pm 1$; the sign convention depends on the reaction responsible for the decay and is reversed in going from reaction (29a) to (29b). This has the following consequence for the observation of the asymmetry in the decay of polarized muons obtained from pion decay: if the muons (from pions of a given charge) are polarized in the direction of their emission, the minus sign pertains in Eq. (31) and the experiments[4,5] tell us that ξ_e is positive. In the decay of pions of opposite sign, the muon polarization is reversed, but so is the sign of ξ_e, so that the asymmetry, with respect to the direction of emission of the muon, is the same for pions of both signs.[4]

$$W(\theta_{e\pm}) = 1 - |\xi_e| \cos\theta_e. \qquad (31a)$$

(2) The observed electron spectrum from muon decay indicates a Michel parameter $\rho > 0.5$,[21] thus favoring muon decays involving different two-component neutrinos.[22] However, the arguments developed in (1) above pertain unchanged to the high-energy limit of the electron spectrum in the decay process (29), since in this case the ν and $\bar{\nu}$ are emitted in the same direction, but with opposite polarization, and are thus in the state described by Eq. (30).

(3) The decay process (29), however, leads to an asymmetry parameter which is strongly dependent on the electron energy. Thus, in the limit of very small electron energies, the ν and $\bar{\nu}$ are emitted in opposite directions and are, therefore, in a state of total spin 1 (triplet), whose wave functions are described according to Eqs. (15):

$$\phi_{\nu\bar{\nu}}^{(1,\pm 1)} = \nu_\pm \bar{\nu}_\pm, \qquad (32a)$$

$$\phi_{\nu\bar{\nu}}^{(1,0)} = 2^{-\frac{1}{2}}(\nu_+\bar{\nu}_- + \nu_-\bar{\nu}_+). \qquad (32b)$$

The final, $e+\nu+\bar{\nu}$, wave functions corresponding to the two possible initial muon polarization states are, accordingly, obtained as the appropriate $J=\frac{1}{2}$, $m=\pm\frac{1}{2}$, combinations of the $\mathbf{e}_\pm$ [Eqs. (13)] and $\phi_{\nu\bar{\nu}}^{(1,m)}$ [Eqs. (32)]

$$\chi_\pm = \mp(\tfrac{1}{3})^{\frac{1}{2}} e_\pm \phi_{\nu\bar{\nu}}^{(1,0)} \pm (\tfrac{2}{3})^{\frac{1}{2}} e_\mp \phi_{\nu\bar{\nu}}^{(1,\pm 1)}. \qquad (33)$$

Correspondingly

$$\left(\frac{dN_e}{d\Omega_e}\right)_\pm = \int |\chi_\pm|^2 d\Omega_{\nu\bar{\nu}} = |a_e|^2 + |b_e|^2 \pm \tfrac{2}{3}(\mathrm{Re}\, a_e b_e^*) \cos\theta_e. \qquad (34)$$

Thus, adopting the same sign conventions as in paragraph (1) above,

$$W(\theta_{e\pm}) = 1 + \tfrac{1}{3}|\xi_e| \cos\theta_e, \qquad (35)$$

for $p_e \to 0$, and Eq. (31a) for $p_e \to (p_e)_{\max}$.

The two-component neutrino theory gives,[6] for the electron decay spectrum of polarized μ mesons decaying according to Eq. (29),

$$4\pi(dN_e/d\Omega_e) = 2x^2 dx[(3-2x) + (1-2x)\xi \cos\theta_e], \qquad (36a)$$

where where $x = p_e/(p_e)_{\max}$. In the two extremes considered above ($x=1$ and $x=0$, respectively), Eq. (36a) reduces to Eqs. (31a) and (35).

For a decay into identical neutrinos, on the other hand, the theory gives[6]

$$4\pi(dN_e/d\Omega_e) = 12x^2(1-x)dx(1-\xi\cos\theta_e), \qquad (36b)$$

as expected according to (1) above.

(4) The momentum dependence of Eqs. (36) may be derived in a relatively straightforward fashion by considering some details of the μ-decay interaction. Thus, for a scalar coupling between the electron and neutrino, their directions are correlated, according to Eq. (10a), by the factor $(1-\cos\theta_{e\nu})$. This factor, multiplied into the integrand of the usual three-body phase-space integral,[20] leads to the $x^2(3-2x)$ momentum dependence of Eq. (36a). However, to obtain the asymmetry parameter for the decay of polarized muons, it is necessary to factor the $\nu\bar{\nu}$ wave function into a triplet and singlet part, and consider separately the phase-space integrals, weighted with the appropriate $\theta_{\nu\bar{\nu}}$ correlation functions. These lead to the spectra

$$\frac{dN_e}{dx}(\text{triplet}) = 6x^2(1-x), \qquad (37a)$$

$$\frac{dN_e}{dx}(\text{singlet}) = 2x^3. \qquad (37b)$$

[20] L. Michel, Proc. Phys. Soc. (London) **A63**, 514 (1950).
[21] J. H. Vilain and R. W. Williams, Phys. Rev. **94**, 1011 (1954).
[22] See, however, M. H. Friedman, Phys. Rev. **106**, 387 (1957).

Finally, for the angular correlation, we add Eq. (37a), multiplied by the triplet correlation function [Eq. (35)], to the product of Eq. (37b) with Eq. (31a); this yields Eq. (36a). Equation (36b) may be derived in an analogous fashion, keeping in mind that in the processes 29(a) and 29(b) the neutrinos are identical.

F. DECAY OF A HEAVY MESON

If, as the evidence appears to indicate,[1] the K meson has spin 0, the same arguments, as have been applied in Sec. D to the π-meson decay, apply to the $K_{\mu 2}$-decay process. The nonconservation of parity, if it pertains also in the π-meson decay modes of the K, would permit the $K_{\pi 2}$ and $K_{\pi 3}$ processes to occur from the same K meson with a single lifetime. However, the details of the $K_{\pi 2}$ and $K_{\pi 3}$ decay processes are exactly the same as they would be with parity conservation, since for spin 0 the final 2π and 3π states have definite parity (0^+ and 0^-, respectively) and cannot be mixed.

The situation would, however, be different for a heavy meson of spin 1.[23] Let us consider the decay of such a hypothetical meson, $K' \rightarrow \mu + \nu$. Furthermore, to simplify the discussion, we assume that the neutrino is the two-component spinor to which we have referred in the preceding, with $b_\nu = -a_\nu$ (the discussion for $K' \rightarrow \mu + \bar{\nu}$ proceeds in exactly the same fashion, except that $b_{\bar{\nu}} = a_{\bar{\nu}}$). In this case, since we are dealing with a two-body decay, the wave functions for the decay products are [using Eqs. (13) and (15), with $\theta_\nu = 180° - \theta_\mu$, $\varphi_\nu = 180° + \varphi_\mu$]

$$\phi_{K'}{}^1 = 2^{-\frac{1}{2}} \alpha_\nu{}^l \{ \sin\theta_\mu e^{-i\varphi_\mu} (a_\mu - b_\mu) \alpha_\mu{}^l - (1-\cos\theta_\mu)(a_\mu + b_\mu) \beta_\mu{}^l \} e^{i\varphi_\mu}, \quad (38a)$$

$$\phi_{K'}{}^0 = \alpha_\nu{}^l \{ -\cos\theta_\mu e^{i\varphi_\mu} (a_\mu - b_\mu) \alpha_\mu{}^l + \sin\theta_\mu (a_\mu + b_\mu) \beta_\mu{}^l \}, \quad (38b)$$

$$\phi_{K'}{}^{-1} = -2^{-\frac{1}{2}} \alpha_\nu{}^l \{ \sin\theta_\mu e^{-i\varphi_\mu} (a_\mu - b_\mu) \alpha_\mu{}^l + (1+\cos\theta_\mu)(a_\mu + b_\mu) \beta_\mu{}^l \} e^{-i\varphi_\mu}, \quad (38c)$$

in which $\alpha_\nu{}^l$ is the spin state of a neutrino with spin always in the direction of its motion and $\alpha_\mu{}^l$, $\beta_\mu{}^l$ are the eigenvectors of the μ spin along its direction of motion.

Equations (38) become simpler for $b_\mu = \pm a_\mu$. The choice $b_\mu = -a_\mu$ (or $b_\mu = a_\mu$ for the $\bar{\nu}$ decay) leads to a decay in which there would be no front-back asymmetry, even for polarized K' mesons. However, in this case, the μ-meson spin would always point along its direction of flight. The choice $b_\mu = a_\mu$ leads to an asymmetry in the decay of polarized K' mesons:

$$|\phi_{K'}{}^1|^2 = (1-\cos\theta_\mu)^2, \quad (39)$$

as well as to complete backward polarization of the μ mesons.

[23] It is important to note that definite observation of a $\theta^0 \rightarrow 2\pi^0$ decay mode would forbid the θ to have spin 1, since the Bose statistics forbids the $2\pi^0$ system to exist in the $l=1$ state. Evidence for the $2\pi^0$ decay of the θ^0 now appears to be quite substantial [J. Steinberger (private communication)].

With regard to the π-decay modes of our K' meson, it is possible to draw some inferences which are more directly observable than in the case of spin 0. If parity is conserved in the π decays, it is well known[24] that only a pseudovector (1^-) meson can decay via both the 2π and 3π decay modes; for a vector meson (1^+), the 2π decay mode is forbidden. However, if parity is not conserved in the decay, both the 1^+ and 1^- channels are open to the $K'_{\pi 3}$ decay (the $K'_{\pi 2}$ decay, however, still proceeds only through the 1^- channel). Thus, it would be possible to test the conservation of parity in the π decay by attempting to observe interference effects between the two $K'_{\pi 3}$ decay modes.

In the usual Dalitz analysis[24] of the decay at rest,

$$\tau^\pm \rightarrow \pi^\pm + \pi^+ + \pi^-, \quad (40)$$

one observes the angular distribution of the direction of the relative momentum of the like pions ($\mathbf{q}$) with respect to the direction of emission of the odd pion ($\mathbf{p}$). Since this depends on the scalar product $\mathbf{p}\cdot\mathbf{q}$, such an observation is incapable of testing the conservation of parity.[2] Nevertheless, although in the 3π decay of a heavy meson of any spin a Dalitz analysis of the decay from unpolarized mesons will show no asymmetries, the parity nonconservation will affect the angular distribution. Thus, one obtains for the distribution of the angle between $\mathbf{p}$ and $\mathbf{q}$:

$$W(\theta_{pq}) = |a_0|^2 + |a_2|^2 \tfrac{1}{2}(3\cos^2\theta + 1) - \sqrt{2}(\mathrm{Re}\, a_0 a_2{}^*)(3\cos^2\theta - 1) + |b|^2 (9/2)(\cos^2\theta - \cos^4\theta). \quad (41)$$

The first three terms arise from the 1^+ decay modes with angular momentum $l=1$ for the odd pion and total angular momentum $L=0$ or 2 (amplitude a_0 or a_2) for the other two pions; the last term arises from the lowest 1^- decay mode $l=2$, $L=2$ (amplitude b). We have included two 1^+ decay modes since the second, although inhibited with respect to the first by the angular-momentum barrier, is still important as compared to the lowest 1^- mode.

However, if one could obtain polarized K' mesons an observation involving the pseudoscalar $\mathbf{S}\cdot\mathbf{p}$ or $\mathbf{S}\cdot\mathbf{q}$ would be sensitive to parity nonconservation. Thus, a decay from the state $S=1$, $m=1$ would yield, for the angular distribution of the odd pion with respect to the direction of polarization,

$$W_{1,1}(\theta_{S,p}) = \tfrac{3}{2}|a_0 + (\tfrac{1}{10})^{\frac{1}{2}} a_2|^2 \sin^2\theta + (9/10)|a_2|^2 + \tfrac{3}{4}|b|^2(2-\sin^2\theta) - \tfrac{3}{2}[\mathrm{Re}(a_0 + (\tfrac{2}{5})^{\frac{1}{2}} a_2) b^*] \sin^2\theta \cos\theta. \quad (42)$$

The difficulty, for the observation of the asymmetric term in Eq. (42), and even more for the observation of the last term in Eq. (41), arises, even assuming that the τ meson has spin 1, from the strong inhibition of the 1^-

[24] R. H. Dalitz, Phil. Mag. 44, 1068 (1953); Phys. Rev. 94, 1046 (1954).

decay mode by the angular-momentum barriers. A rough estimate, assuming that the three amplitudes differ only by the appropriate angular-momentum barrier-penetration factors[25] and taking the range of pion interaction to be of the order of the pion Compton wavelength, gives $|a_2|/|a_0|\simeq 0.15$, $|b|/|a_0|\simeq 0.04$, for an energy of ~ 25 Mev of the odd pion. Substitution into Eq. (42) results in

$$W_{1,1}(\theta_{S,p})\approx \sin^2\theta(1-0.1\cos\delta\cos\theta), \qquad (42')$$

where δ is the phase angle between a_0 and b. Integration over the entire energy spectrum would reduce the factor 0.1 to ~ 0.08.

The same type of estimate gives, for the integral angular distribution from unpolarized spin-1 τ mesons,

$$W(\theta_{p,q})\approx 1-0.4\cos\delta'(\cos^2\theta-\tfrac{1}{3}), \qquad (41')$$

where δ' is the phase angle between a_0 and a_2. Since a $\cos^2\theta$ term of this order, as well as the attendant effects on the pion energy distribution, have thus far eluded observation,[1] the possibility of observing a direct effect of parity nonconservation in weak decays involving only π mesons probably awaits the discovery of the appropriate vector K' meson.

Assuming that the nonconservation of parity provides the key to the θ-τ paradox, the question still remains as to the nature of the interaction which gives rise to the decay. In particular, assuming proper Lorentz invariance of such interactions, the Schwinger-Pauli-Lüders theorem, requiring the invariance under the combined operations of time reversal, charge conjugation, and space inversion (TCP), applies,[26] and it is necessary that parity nonconservation be accompanied by the violation of one of the other (or both) invariance properties. The violation of charge-conjugation invariance should have observable consequences in the comparison of K and $\bar{K}$ decay schemes. If time-reversal invariance fails, this would have definite consequences for the $K'_{\pi 3}$ decay discussed above.

For instance, if time reversal holds good and if one assumes no interaction between the outgoing π mesons, the phase angle in Eq. (42′) is 0 or 180°. Deviation of $\cos\delta$ from ± 1 arises also from pion interactions in the final state, however, and these effects would need to be understood before one could attribute such an effect to time-reversal noninvariance. On the other hand, the violation of time-reversal invariance could also be investigated by studying, e.g., the distribution of the plane of the 3π decay with respect to the direction of emission of the K' meson. Such a quantity, i.e., $\mathbf{p}_{K'}\cdot(\mathbf{p}\times\mathbf{q})$, would be observable only if time-reversal invariance fails, provided again that the final-state pion interaction may be neglected. In view of the hypothetical nature of the K' meson, and the experimental difficulty in establishing such effects, even if they existed, it is unlikely that any of these experiments will be performed in the near future. Nevertheless, these questions are of sufficient interest so that such speculations may not be unfruitful.

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[25] J. M. Blatt and V. F. Weisskopf, reference 9, pp. 358–365.
[26] Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).