Parity Nonconservation in β Decay*

KURT ALDER,† BERTHOLD STECH,‡ AND AAGE WINTHER§
Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California
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The nonconservation of parity in β decay implies that electrons emitted from unoriented nuclei are polarized along their line of flight. Also, if the β process is followed by a γ radiation, the angular distribution of the γ quantum with respect to the electron direction depends on its circular polarization. Formulas relevant for such experiments as well as for the angular distribution of electrons from oriented nuclei are presented. The results, which include Coulomb corrections, are given for scalar, tensor, and pseudoscalar interactions in the cases of allowed and first-forbidden transitions. The formulas show that an experimental determination of the relative sign of the scalar and tensor coupling constants should be possible.

I. INTRODUCTION

SEVERAL recent experiments[1–3] have confirmed the suggestion by Yang and Lee[4] that parity is not conserved in weak interactions. These observations require a generalization of the β decay Hamiltonian through the addition of pseudoscalar terms. Thereby, the most general β interaction consistent with the requirement of invariance under proper Lorentz transformations will contain ten, in general complex, coupling constants.

Some information on these new coupling constants can be derived from the study of the angular distribution of the electrons emitted from polarized nuclei.[1] Since at the present time, however, only a few nuclei can be polarized, it may be interesting to study the effect of nonconservation of parity in other experimental situations, which do not require the polarization technique.

The correlation between the electron direction and orientation of the nucleus implies that the daughter nucleus in the decay of unoriented nuclei will be partly polarized along the electron direction. If the β decay is followed by a γ transition, this polarization can be studied by means of the β-γ angular correlation when at the same time the circular polarization of the γ quantum is detected. Such an experiment gives exactly the same information about the coupling constants as do the experiments with oriented initial nuclear states.

Similar considerations suggest that also the electron itself will be polarized. When the electron is emitted from unoriented nuclei, the polarization vector will be parallel to the electron momentum. The magnitude of the electron polarization depends on other combinations of the coupling constants than those appearing in the above-mentioned processes. Therefore a comparison of the results of the two experiments may be used advantageously for a determination of the coupling constants. Especially it should be possible to determine the relative sign of the scalar and tensor interaction constants.

The reality of the coupling constants, which is suggested by the requirement of time-reversal invariance, may be studied in these experiments as well as in ordinary β-γ angular-correlation measurements. The imaginary parts of the coupling constants enter in these cases through the interference of different partial waves of the electron in the Coulomb field of the nucleus.[5]

Previous interpretation[6] of β-decay experiments indicated that the β-decay law contains only scalar (S), tensor (T), and perhaps pseudoscalar (P) interactions. Although these arguments are now somewhat weakened by the introduction of parity-nonconserving terms, explicit results are given in this paper for S, T, and P interactions only. A discussion of this point will be given in Sec. III.

Section II contains formulas appropriate for the analysis of the experiments on the electron distribution from oriented nuclei, β-γ polarization correlation, and

TABLE I. Geometrical factor $h_k(I_i,M_i)$ defined in Eq. (A9).

k	$h_k(I_i,M_i)$
0	1
1	$\sqrt{3}\dfrac{M_i}{[I_i(I_i+1)]^{\frac{1}{2}}}$
2	$(\sqrt{5})\dfrac{2[3M_i^2-I_i(I_i+1)]}{[(2I_i+3)(2I_i+2)2I_i(2I_i-1)]^{\frac{1}{2}}}$
3	$(\sqrt{7})\dfrac{4M_i[5M_i^2-3I_i(I_i+1)+1]}{[(2I_i+4)(2I_i+3)(2I_i+2)2I_i(2I_i-1)(2I_i-2)]^{\frac{1}{2}}}$

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† Previous address: University of Michigan, Ann Arbor, Michigan.
‡ On leave from the University of Heidelberg, Heidelberg, Germany.
§ On leave from the Institute for Theoretical Physics, Copenhagen, Denmark.
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[2] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).
[3] J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).
[4] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[5] Other ways of investigating the reality of the coupling constants have been proposed by Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957).
[6] See, e.g., C. S. Wu, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North Holland Publishing Company, Amsterdam, 1955), Chap. 11.

electron polarization. The expressions include the effect of the Coulomb interaction and are given for allowed and first-forbidden transitions. The formulas also apply for ordinary β-γ correlations with complex β-decay coupling constants. Details of the derivation of the formulas are contained in the appendix.

In Sec. III a discussion of the results is given.

II. FORMULAS AND RESULTS

The general β interaction containing pseudoscalar terms has the following form[4]:

$$\begin{aligned}H_\beta=&(\bar\psi_p\psi_n)(C_S\bar e\nu+C_S'\bar e\gamma_5\nu)\\&+\textstyle\sum_\lambda(\bar\psi_p\gamma_\lambda\psi_n)(C_V\bar e\gamma_\lambda\nu+C_V'\bar e\gamma_\lambda\gamma_5\nu)\\&+\textstyle\sum_{\lambda<\mu}(\bar\psi_p i\gamma_\lambda\gamma_\mu\psi_n)(C_T\bar e i\gamma_\lambda\gamma_\mu\nu+C_T'\bar e i\gamma_\lambda\gamma_\mu\gamma_5\nu)\\&+\textstyle\sum_\lambda(\bar\psi_p i\gamma_\lambda\gamma_5\psi_n)(C_A\bar e i\gamma_\lambda\gamma_5\nu+C_A'\bar e i\gamma_\lambda\nu)\\&+(\bar\psi_p\gamma_5\psi_n)(C_P\bar e\gamma_5\nu+C_P'\bar e\nu)+\text{Herm. conj.}\end{aligned}\tag{1}$$

The wave functions ψ_n and ψ_p represent the initial and final state of the nucleus in a β^- decay. The γ matrices are defined in the usual way: $\gamma_k=i\beta\alpha_k$, $\gamma_4=\beta$, $\sigma_k=-\gamma_5\alpha_k$, where $k=1$, 2, 3, and $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$. $\bar\psi$ is an abbreviation for $\psi^\dagger\gamma_4$. The electron and neutrino wave functions are denoted by e and ν, respectively.

The angular correlations and polarizations presented in this section have been obtained by expanding the Hamiltonian (1) in multipole components. The results have been classified according to forbiddenness and are given in a tabular form convenient for comparison with experiments. For a derivation of the formulas the reader is referred to the Appendix. The results throughout this section are given for β^- decays. In order to obtain the corresponding results for β^+ decays, the following formal substitution should be performed, $Z\to-Z$, $C_S\to-C_S^*$, $C_S'\to C_S'^*$, $C_T\to C_T^*$, $C_T'\to-C_T'^*$, $C_P\to-C_P^*$, and $C_P'\to C_P'^*$.

(a) Electron Distribution from Oriented Nuclei

For β particles emitted from oriented nuclei, the angular distribution with respect to the orientation is found to be

$$W(\theta)d\Omega_e=\sum_{k,L,L'}h_k(I_i,M_i)F_k(L,L',I_f,I_i)(-1)^{L+L'+k}\times b_k(L,L')P_k(\cos\theta)d\Omega_e.\tag{2}$$

In this formula the spins of the initial and final nuclei are denoted by I_i and I_f, respectively. The initial nucleus is assumed to be in a definite substate with magnetic quantum number M_i; i.e., the axis of polarization is along the z axis. The dependence on M_i appears in the geometrical factor $h_k(I_i,M_i)$. In practice an appropriate weighted average over h_k with respect to M_i must be performed.[7] P_k is the usual Legendre polynomial of order k, depending on the angle θ between the axes of polarization and the electron direction. The F coefficient is the familiar geometrical coefficient which appears in the theory of angular correlations. It depends on the multipole orders L and L' of the β radiation.[8,9] The function h_k is defined in the Appendix [Eq. (A9)] and is given for $k=0$, 1, 2, and 3 in Table I. Since the coefficients $F_k(L,L',I_f,I_i)$ have previously been tabulated for even k only, we have also included a table of these coefficients for low spins and multipole orders and for $k=0$, 1, 2, and 3 (Table II). The quantity $b_k(L,L')$ is the characteristic parameter for the β process in question and is independent of the spins involved. The expression for $b_k(L,L')$ is derived in the Appendix [see Eq. (A11)] and is given explicitly for allowed transitions in Table III and for first-forbidden transitions in Table IV.[10] The exact expression for b in terms of Coulomb wave functions is rather involved and the results are therefore presented in the approximation[11] $(\alpha Z)^2\ll1$, α being the fine-structure constant.

The absolute transition rate $N(E,\theta)$ is obtained from Eq. (2) by the relation

$$N(E,\theta)dpd\Omega_e=\frac{1}{2\pi^4}W(\theta)F_0(Z,E)p^2q^2dpd\Omega_e,\tag{3}$$

where E is the energy of the electron including the rest mass. The electron and neutrino momenta are denoted by p and q, respectively. $F_0(Z,E)$ is the Fermi function, as defined in reference 11. We use throughout the paper the units usual in β-decay theory: $\hbar=c=m=1$.

As an illustration we give the important example of an allowed transition, where Eq. (2) reduces to[12]

$$\begin{aligned}W(\theta)=&\tfrac14[|\textstyle\int\beta|^2(|C_S|^2+|C_S'|^2)\\&+|\textstyle\int\beta\boldsymbol\sigma|^2(|C_T|^2+|C_T'|^2)]\\&+\frac{p}{E}\cos\theta\Big\{\frac{M_i}{2[I_i(I_i+1)]^{\frac12}}|\textstyle\int\beta||\int\beta\boldsymbol\sigma|\\&\qquad\times\mathrm{Re}\,(C_SC_T'^*+C_S'C_T^*)\\&+\frac{M_i[I_i(I_i+1)-I_f(I_f+1)+2]}{8I_i(I_i+1)}|\textstyle\int\beta\boldsymbol\sigma|^2\\&\qquad\times(C_TC_T'^*+C_T'C_T^*)\Big\}.\end{aligned}\tag{4}$$

[7] This weighted average is often denoted by $(2I_i+1)^{\frac12}F_k(I_i)$; see, e.g., S. R. de Groot and H. A. Tolhoek, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North Holland Publishing Company, Amsterdam, 1955), Chap. 19.

[8] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

[9] M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL-5324 (unpublished). These references contain tables of F_k for even k.

[10] For even k, the parameters $b_k(L,L')$ are identical with those given in reference 8.

[11] Even for heavy nuclei the general behavior of the b's is well described by this approximation. See E. Konopinski, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North Holland Publishing Company, Amsterdam, 1955), Chap. 10.

[12] For transitions with spin change this formula has been given in reference 4.

TABLE II. Numerical table of the F coefficients. Separate tables are given for $k=1$, 2, and 3. The values for $k=2$ are taken from reference 9. The coefficient $F_k(L,L',I_1,I_2)$ is symmetric in the multipole orders L and L', which are listed at the top of each column. The first two columns give the values of I_1 and I_2. For $k=0$, one has $F_0(L,L',I,I)=\delta_{L,L'}$.

(a) $F_1(L,L',I_1I_2)$ for integral spins

I_1	I_2 \ LL' = 01		11	12	22	23	33
0	1	0	−1.225	0	0	0	0
0	2	0	0	0	−0.707	0	0
0	3	0	0	0	0	0	−0.500
1	1	1.732[a]	−0.612	0.612	−0.612	0	0
1	2	0	−1.061	0.473	−0.589	0.421	−0.472
1	3	0	0	0	−0.667	0.356	−0.458
1	4	0	0	0	0	0	−0.484
2	1	0	0.612	0.822	−0.204	0.730	−0.408
2	2	1.732_k	−0.353	0.725	−0.354	0.676	−0.354
2	3	0	−1.000	0.548	−0.500	0.585	−0.375
2	4	0	0	0	−0.645	0.436	−0.420
2	5	0	0	0	0	0	−0.474
3	1	0	0	0	0.408	0.873	−0.102
3	2	0	0.708	0.775	0	0.828	−0.177
3	3	1.732[a]	−0.250	0.750	−0.250	0.756	−0.250
3	4	0	−0.968	0.581	−0.452	0.647	−0.323
3	5	0	0	0	−0.632	0.478	−0.395
4	1	0	0	0	0	0	0.306
4	2	0	0	0	0.471	0.797	0.059
4	3	0	0.750	0.750	0.083	0.836	−0.083
4	4	1.732[a]	−0.194	0.760	−0.194	0.786	−0.194
4	5	0	−0.948	0.600	−0.422	0.680	−0.290
5	3	0	0	0	0.500	0.756	0.125
5	4	0	0.774	0.735	0.129	0.832	−0.032

(b) $F_1(L,L'I_1I_2)$ for half-integral spins

I_1	I_2 \ LL' = 01		11	12	22	23	33
1/2	1/2	1.732[a]	−1.000	0	0	0	0
1/2	3/2	0	−1.118	0.387	−0.671	0	0
1/2	5/2	0	0	0	−0.683	0.138	−0.488
1/2	7/2	0	0	0	0	0	−0.490
3/2	1/2	0	0.500	0.866	−0.500	0	0
3/2	3/2	1.732[a]	−0.447	0.693	−0.447	0.566	−0.447
3/2	5/2	0	−1.025	0.520	−0.537	0.528	−0.415
3/2	7/2	0	0	0	−0.654	0.404	−0.436
3/2	9/2	0	0	0	0	0	−0.479
5/2	1/2	0	0	0	0.333	0.943	−0.333
5/2	3/2	0	0.671	0.794	−0.075	0.806	−0.261
5/2	5/2	1.732[a]	−0.293	0.741	−0.293	0.727	−0.293
5/2	7/2	0	−0.982	0.567	−0.473	0.622	−0.346
5/2	9/2	0	0	0	−0.638	0.460	−0.406
7/2	1/2	0	0	0	0	0	0.250
7/2	3/2	0	0	0	0.447	0.828	0
7/2	5/2	0	0.732	0.761	0.049	0.835	−0.098
7/2	7/2	1.732[a]	−0.218	0.873	−0.218	0.774	−0.218
7/2	9/2	0	−0.957	0.592	−0.435	0.666	−0.305
9/2	3/2	0	0	0	0	0	0.335
9/2	5/2	0	0	0	0.488	0.774	0.098
9/2	7/2	0	0.609	0.742	0.087	0.835	−0.044
9/2	9/2	1.732[a]	−0.174	0.763	−0.174	0.794	−0.174

(c) $F_2(L,L'I_1I_2)$ for integral spins

I_1	I_2 \ LL' =	11	12	22	23	33
0	1	0.707	0	0	0	0
0	2	0	0	−0.598	0	0
0	3	0	0	0	0	−0.866
1	1	−0.354	−1.061	−0.354	0	0
	2	0.418	−0.935	−0.299	−0.535	−0.717
1	3	0	0	−0.495	−0.463	−0.650
1	4	0	0	0	0	−0.783
2	1	0.071	0.474	0.354	−0.632	−0.424
2	2	−0.418	−0.612	0.128	−0.571	−0.179
2	3	0.346	−0.949	−0.124	−0.592	−0.274
2	4	0	0	−0.448	−0.530	−0.470
2	5	0	0	0	0	−0.736
3	1	0	0	−0.101	0.378	0.530
3	2	0.120	0.655	0.341	−0.175	0.329
3	3	−0.433	−0.433	0.227	−0.436	0.144
3	4	0.313	−0.940	−0.045	−0.571	−0.085
3	5	0	0	−0.421	−0.556	−0.368
4	1	0	0	0	0	−0.177
4	2	0	0	−0.171	0.505	0.448
4	3	0.144	0.722	0.309	0	0.433
4	4	−0.439	−0.335	0.265	−0.347	0.269
4	5	0.294	−0.931	0	−0.548	0.017
5	2	0	0	0	0	−0.299
5	3	0	0	−0.206	0.546	0.361
5	4	0.160	0.757	0.285	0.092	0.453
5	5	−0.442	−0.274	0.283	−0.286	0.328

(d) $F_2(L,L'I_1I_2)$ for half-integral spins

I_1	I_2 \ LL' =	11	12	22	23	33
1/2	3/2	0.500	−0.866	−0.500	0	0
1/2	5/2	0	0	−0.535	−0.378	−0.802
1/2	7/2	0	0	0	0	−0.818
3/2	3/2	−0.400	−0.775	0	−0.632	−0.600
3/2	5/2	0.374	−0.949	−0.191	−0.587	−0.441
3/2	7/2	0	0	−0.468	−0.505	−0.546
3/2	9/2	0	0	0	0	−0.757
5/2	3/2	0.100	0.592	0.357	−0.338	0.150
5/2	5/2	−0.428	−0.507	0.191	−0.498	0.027
5/2	7/2	0.327	−0.945	−0.078	−0.583	−0.164
5/2	9/2	0	0	−0.433	−0.546	−0.413
5/2	11/2	0	0	0	0	−0.719
7/2	3/2	0	0	−0.143	+0.463	0.500
7/2	5/2	0.134	0.694	0.325	−0.071	0.401
7/2	7/2	−0.436	−0.378	0.249	−0.387	0.218
7/2	9/2	0.303	−0.935	−0.020	−0.559	−0.028
7/2	11/2	0	0	−0.411	−0.564	−0.332
9/2	3/2	0	0	0	0	−0.250
9/2	5/2	0	0	−0.191	0.530	0.401
9/2	7/2	0.153	0.742	0.296	0.052	0.447
9/2	9/2	−0.440	−0.302	0.275	−0.314	0.303
9/2	11/2	0.288	−0.927	0.016	−0.537	0.052
11/2	5/2	0	0	0	0	−0.334
11/2	7/2	0	0	−0.218	0.556	0.327
11/2	9/2	0.165	0.769	0.275	0.123	0.454
11/2	11/2	−0.442	−0.251	0.289	−0.263	0.348

(e) $F_3(L,L',I_1I_2)$ for integral spins

I_1	I_2 \ LL' =	12	22	23	33
0	2	0	1.414	0	0
0	3	0	0	0	1.080
1	2	0.632	0	−0.316	0.707
1	3	0	0.926	−0.289	0.540
1	4	0	0	0	0.874
2	2	−0.414	−0.808	−0.084	0.354
2	3	0.507	−0.463	−0.211	−0.180
2	4	0	0.749	−0.295	0.175
2	5	0	0	0	0.768
3	2	0.111	0.505	0.242	−0.471
3	3	−0.463	−0.617	0.068	−0.540
3	4	0.449	−0.599	−0.139	−0.397
3	5	0	0.658	−0.290	0
4	2	0	−0.101	−0.100	0.471
4	3	0.154	0.617	0.226	−0.180
4	4	−0.481	−0.490	0.126	−0.493
4	5	0.416	−0.658	−0.095	−0.473
5	2	0	0	0	−0.118
5	3	0	−0.154	−0.136	0.540
5	4	0.181	0.667	0.205	−0.016

[a] The F-coefficient actually vanishes for $L=0$. The definitions of F_k and b_k have therefore been modified, so that the product b_kF_k is correct.

TABLE II—*Continued*

(f) $F_3(L,L'I_1I_2)$ for half-integral spins

I_1	$I_2 \backslash LL'=$	12	22	23	33
1/2	3/2	0.775	1.414	0	0
1/2	5/2	0	1.095	−0.258	0.913
1/2	7/2	0	0	0	0.957
3/2	3/2	−0.346	−0.894	−0.283	0.447
3/2	5/2	0.555	−0.313	−0.262	0.091
3/2	7/2	0	0.821	−0.295	0.319
3/2	9/2	0	0	0	0.814
5/2	3/2	0.076	0.383	0.227	−0.671
5/2	5/2	−0.445	−0.704	0.013	−0.517
5/2	7/2	0.474	−0.547	−0.171	−0.319
5/2	9/2	0	0.697	−0.293	0.074
7/2	3/2	0	−0.064	−0.069	0.226
7/2	5/2	0.136	0.574	0.236	−0.304
7/2	7/2	−0.474	−0.547	0.103	−0.551
7/2	9/2	0.431	−0.634	−0.115	−0.441
9/2	3/2	0	0	0	−0.075
9/2	5/2	0	−0.130	−0.121	0.517
9/2	7/2	0.169	0.647	0.215	−0.147
9/2	9/2	−0.486	−0.444	0.143	−0.492

In this equation $|\int\beta|$ and $|\int\beta\sigma|$ denote the usual Fermi- and Gamow-Teller matrix elements. For the definition of the sign of these matrix elements, which is important for the interference term, the reader is referred to the appendix.

The angular distribution (4) is given in the $(\alpha Z)^2\ll 1$ approximation. A more exact formula is obtained from (4) by multiplying the isotropic term with $(f_1^2+g_{-1}^2)$ and replacing the factor p/E in the angular-dependent term by $[1+(Z\alpha/p)^2]^{-\frac{1}{2}}2g_{-1}f_1$. The functions f_1 and g_{-1} are the usual Coulomb wave functions[13] of the electron (or positron) taken at the nuclear surface. It turns out, however, that this change of Eq. (4) is quite unimportant, even for high Z.

For first-forbidden transitions, the angular distribution can be obtained by means of Tables I, II, and IV. We have neglected the interference terms with B_{ij} which are usually an order of magnitude smaller than the terms evaluated.

As an illustration, the angular distributions for the unique transitions ($L=2$) will be given. One finds from Eq. (2) and Table IV:

$$\begin{aligned}W(\theta)=|\int B_{ij}^{\beta}|^2\Big\{&(|C_T|^2+|C_T'|^2)\frac{1}{48}(p^2+q^2)\\&-(C_TC_T'^*+C_T'C_T^*)h_1(I_i,M_i)F_1(2,2,I_f,I_i)\\&\times\frac{p}{E}\Big[\frac{1}{80}p^2+\frac{1}{48}q^2\Big]P_1(\cos\theta)\\&+(|C_T|^2+|C_T'|^2)h_2(I_i,M_i)F_2(2,2,I_f,I_i)\\&\times\frac{7}{240}p^2P_2(\cos\theta)\\&-(C_TC_T'^*+C_T'C_T^*)h_3(I_i,M_i)F_3(2,2,I_f,I_i)\\&\times\frac{p}{E}\frac{1}{80}p^2P_3(\cos\theta)\Big\}.\end{aligned}\quad(5)$$

[13] See, e.g., reference 11.

Although forbidden transitions in general contain several unknown matrix elements, the degree of asymmetry is not necessarily smaller than for allowed transitions.

(b) β-γ Polarization Correlation

A γ quantum following a β transition will in general be circularly polarized if it is measured in coincidence with the β particle. The polarization correlation has the form

$$\begin{aligned}W(\theta,\tau)=\sum_{k,\lambda,\lambda',L,L'}&(-\tau)^k\delta_\lambda\delta_{\lambda'}F_k(\lambda,\lambda',I_{ff},I_f)\\&\times F_k(L,L',I_i,I_f)b_k(L,L')P_k(\cos\theta),\end{aligned}\quad(6)$$

where θ measures the angle between the directions of γ ray and electron. The γ polarization is indicated by the quantum number τ, which takes on the values $\tau=+1$ and $\tau=-1$ for right- and left-hand circular polarization, respectively.[14] The γ ray will in general be a mixture of different multipoles λ and λ′ with intensities $|\delta_\lambda|^2$ and $|\delta_{\lambda'}|^2$. The factors F_k are the F coefficients given in Table II. The initial and final states in the β transition have the spins I_i and I_f, while the spin of the final state after the γ transition is denoted by I_{ff}. The parameters $b_k(L,L')$ characteristic for the β processes are the same as those occurring in the previous section.

For allowed transitions, one finds from Eq. (6) and Table III:

$$\begin{aligned}W(\theta,\tau)=&\tfrac{1}{4}\textstyle\sum_\lambda|\delta_\lambda|^2\big[|\int\beta|^2(|C_S|^2+|C_S'|^2)\\&+|\int\beta\sigma|^2(|C_T|^2+|C_T'|^2)\big]\\&+\tau\cos\theta\frac{p}{E}\frac{\sqrt{3}}{24}\Big[\sum_{\lambda\lambda'}\delta_\lambda\delta_{\lambda'}F_1(\lambda,\lambda',I_{ff},I_f)\Big]\\&\times\Big\{\frac{I_f(I_f+1)-I_i(I_i+1)+2}{[I_f(I_f+1)]^{\frac{1}{2}}}|\int\beta\sigma|^2\\&\times(C_TC_T'^*+C_T'C_T^*)\\&-4|\int\beta||\int\beta\sigma|\mathrm{Re}(C_SC_T'^*+C_S'C_T^*)\Big\}.\end{aligned}\quad(7)$$

TABLE III. Particle parameters $b_k(L,L')$ for allowed transitions.[a] The parameter $b_k(L,L')$ for given L and L' is the product of the matrix element given in the second column and the factor S_k given in the third column. The electron energy and momentum are denoted by E and p, respectively. For $L\neq L'$, the quantity listed is actually $b_k(L,L')+b_k(L',L)$.

L	L'	Matrix element	S_k
0	0	$\|\int\beta\|^2$	$S_0=\frac{1}{4}\{|C_S|^2+|C_S'|^2\}$
1	1	$\|\int\beta\sigma\|^2$	$S_0=\frac{1}{6}\{|C_T|^2+|C_T'|^2\}$ $S_1=\frac{1}{6}\{C_TC_T'^*+C_T^*C_T'\}(p/E)$
0	1	$\|\int\beta\|\cdot\|\int\beta\sigma\|$	$S_0=0$ $S_1=\frac{1}{6}\,\mathrm{Re}\{C_SC_T'^*+C_S'C_T^*\}(p/E)$

[a] The b_k [see Eq. (A11)] is actually infinite for $L=0$. The definition has therefore been modified so that the product b_kF_k is correct (see Table II).

[14] The term right-handed is here used for a γ quantum having its spin in the direction of the momentum.

TABLE IV. Particle parameters $b_k(L,L')$ for first-forbidden transitions. The parameter $b_k(L,L')$ for given L and L' is the sum of the products of the matrix elements given in the second column and the corresponding shape-factors S_k given in the third column. The quantity ξ is equal to $\alpha Z/2R$, where R is the nuclear radius. The electron energy and momentum are denoted by E and p, respectively, while q is the neutrino momentum. The nuclear charge number is equal to Z, and α is the fine-structure constant. For $L\neq L'$ the quantity listed is actually $b_k(L,L')+b_k(L',L)$.

L	L'	Matrix element	S_k
0	0	$\lvert\int\beta\gamma_5\rvert^2$	$S_0=\frac{1}{4}\{\lvert C_P\rvert^2+\lvert C_P'\rvert^2\}$
		$\lvert\int(1/i)\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rvert^2$	$S_0=\frac{1}{4}\{\lvert C_T\rvert^2+\lvert C_T'\rvert^2\}\left[\frac{1}{9}(p^2+q^2)-\frac{2}{9}\frac{p^2q}{E}+\frac{2}{3}\xi\left(-q+\frac{p^2}{E}\right)+\xi^2\right]$
		$\lvert\int\beta\gamma_5\rvert\times\lvert(1/i)\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rvert$	$S_0=\frac{1}{2}\,\mathrm{Re}\{C_PC_T^*+C_P'C_T'^*\}\left[\frac{p^2}{3E}-\frac{1}{3}q+\xi\right]$
0	1	$\lvert\int i\beta\mathbf{r}\rvert\times\lvert(1/i)\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rvert$	$S_1=-\frac{1}{6}\,\mathrm{Re}\{C_SC_T'^*+C_S'C_T^*\}\frac{p}{E}\left[\frac{1}{3}p^2+\frac{1}{g}q^2-\frac{4}{g}Eq+\frac{2}{3}\xi(-q+2E)+\xi^2\right]$
			$-\frac{1}{36}\,\mathrm{Im}\{C_SC_T'^*+C_S'C_T^*\}\frac{Z\alpha}{E}[Ep^2-qE^2-\frac{1}{3}q+\xi(3E^2+1)]$
		$\lvert\int\beta\boldsymbol{\sigma}\times\mathbf{r}\rvert\times\lvert(1/i)\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rvert$	$S_1=-\frac{1}{6}\{C_TC_T'^*+C_T^*C_T'\}\frac{p}{E}\left[-\frac{1}{g}q^2+\frac{1}{g}qE+\frac{1}{3}E\xi+\xi^2\right]$
		$\lvert\int\beta\alpha\rvert\times\lvert(1/i)\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rvert$	$S_1=-\frac{1}{6}\{C_TC_T'^*+C_T^*C_T'\}\frac{p}{E}[\frac{1}{3}E-\frac{1}{3}q+\xi]$
		$\lvert\int i\beta\mathbf{r}\rvert\times\lvert\int\beta\gamma_5\rvert$	$S_1=-\frac{1}{6}\,\mathrm{Re}\{C_SC_P'^*+C_S'C_P^*\}\frac{p}{E}[E-\frac{1}{3}q+\xi]-\frac{1}{6}\,\mathrm{Im}\{C_SC_P'^*+C_S'C_P^*\}\frac{Z\alpha}{E}[\frac{1}{2}+\frac{1}{2}E^2]$
		$\lvert\int\beta\boldsymbol{\sigma}\times\mathbf{r}\rvert\times\lvert\int\beta\gamma_5\rvert$	$S_1=-\frac{1}{6}\,\mathrm{Re}\{C_TC_P'^*+C_T'C_P^*\}\frac{p}{E}[-\frac{1}{3}E+\frac{1}{3}q+\xi]+\frac{1}{6}\,\mathrm{Im}\{C_TC_P'^*+C_T'C_P^*\}\frac{Z\alpha}{E}[-\frac{1}{6}+\frac{1}{2}E^2]$
		$\lvert\int\beta\boldsymbol{\alpha}\rvert\times\lvert\int\beta\gamma_5\rvert$	$S_1=-\frac{1}{6}\,\mathrm{Re}\{C_TC_P'^*+C_T'C_P^*\}\times\frac{p}{E}$
1	1	$\lvert\int i\beta\mathbf{r}\rvert^2$	$S_0=\frac{1}{4}\{\lvert C_S\rvert^2+\lvert C_S'\rvert^2\}\left[\frac{1}{3}(p^2+q^2)-\frac{2}{9}\frac{p^2q}{E}+\frac{2}{3}\xi\left(\frac{p^2}{E}-q\right)+\xi^2\right]$
			$S_1=\frac{1}{6}\{C_SC_S'^*+C_S^*C_S'\}\frac{p}{E}[-\frac{2}{3}q\xi+\xi^2]$
			$S_2=\frac{1}{3}\{\lvert C_S\rvert^2+\lvert C_S'\rvert^2\}\frac{p^2}{E}\left[\frac{1}{3}q-\frac{E}{2}-\xi\right]$
		$\lvert\int\beta\boldsymbol{\sigma}\times\mathbf{r}\rvert^2$	$S_0=\frac{1}{4}\{\lvert C_T\rvert^2+\lvert C_T'\rvert^2\}\left[\frac{1}{6}(p^2+q^2)+\frac{2}{9}\frac{p^2q}{E}+\frac{2}{3}\xi\left(\frac{p^2}{E}+q\right)+\xi^2\right]$
			$S_1=\frac{1}{6}\{C_TC_T'^*+C_T^*C_T'\}\frac{p}{E}[\frac{1}{4}p^2+(1/12)q^2+\frac{1}{3}qE+\xi(E+\frac{2}{3}q)+\xi^2]$
			$S_2=\frac{1}{6}\{\lvert C_T\rvert^2+\lvert C_T'\rvert^2\}\frac{p^2}{E}[(q/3)+\frac{1}{4}E+\xi]$
		$\lvert\int\beta\boldsymbol{\alpha}\rvert^2$	$S_0=\frac{1}{4}\{\lvert C_T\rvert^2+\lvert C_T'\rvert^2\}$ $S_1=\frac{1}{6}\{C_TC_T'^*+C_T^*C_T'\}p/E$
		$\lvert\int i\beta\mathbf{r}\rvert\times\lvert\int\beta\boldsymbol{\sigma}\times\mathbf{r}\rvert$	$S_0=\frac{1}{2}\,\mathrm{Re}\{C_SC_T^*+C_S'C_T'^*\}\left[\frac{2}{3}\frac{p^2}{E}\xi+\xi^2\right]$
			$S_1=\frac{1}{3}\,\mathrm{Re}\{C_SC_T'^*+C_S'C_T^*\}\frac{p}{E}[-\frac{1}{6}q^2-\frac{1}{6}qE+\frac{1}{2}\xi E+\xi^2]-\frac{1}{6}\,\mathrm{Im}\{C_SC_T'^*+C_S'C_T^*\}$
			$\times\frac{Z\alpha}{E}\left[\frac{3}{4}p^2\left(\frac{E}{3}+\frac{1}{9}q+\xi\right)+(-\frac{1}{3}q+\xi)\right]$

TABLE IV—*Continued*

L	L'	Matrix element	S_k
			$S_2=-\frac{1}{6}\,\mathrm{Re}\{C_SC_T^*+C_S'C_T'^*\}\frac{p^2}{E}[\frac{1}{3}E+q+\xi]-\frac{3}{4}\,\mathrm{Im}\{C_SC_T^*+C_S'C_T'^*\}Z\alpha p\left[\frac{p^2}{3E}+\frac{1}{9}q+\xi\right]$
		$\lvert\int i\beta\mathbf{r}\rvert\times\lvert\int\beta\boldsymbol{\alpha}\rvert$	$S_0=\frac{1}{2}\,\mathrm{Re}\{C_SC_T^*+C_S'C_T'^*\}\left[\frac{p^2}{3E}-\frac{1}{3}q+\xi\right]$
			$S_1=\frac{1}{3}\,\mathrm{Re}\{C_SC_T'^*+C_S'C_T^*\}\frac{p}{E}[-\frac{1}{3}q+\xi]-\frac{1}{12}\,\mathrm{Im}\{C_SC_T'^*+C_S'C_T^*\}\frac{Z\alpha}{E}p^2$
			$S_2=-\frac{1}{3}\,\mathrm{Re}\{C_SC_T^*+C_S'C_T'^*\}\frac{p^2}{E}-\frac{1}{4}\,\mathrm{Im}\{C_SC_T^*+C_S'C_T'^*\}Z\alpha p$
		$\lvert\int\beta\boldsymbol{\sigma}\times\mathbf{r}\rvert\times\lvert\int\beta\boldsymbol{\alpha}\rvert$	$S_0=\frac{1}{2}\{\lvert C_T\rvert^2+\lvert C_T'\rvert^2\}\left[\frac{1}{3}\left(q+\frac{p^2}{E}\right)+\xi\right]$
			$S_1=\frac{1}{3}\{C_TC_T'^*+C_T^*C_T'\}\frac{p}{E}[\frac{1}{2}E+\frac{1}{3}q+\xi]$
			$S_2=\frac{1}{6}\{\lvert C_T\rvert^2+\lvert C_T'\rvert^2\}p^2/E$
2	2	$\lvert\int iB_{ij}{}^{\beta}\rvert^2$	$S_0=\{\lvert C_T\rvert^2+\lvert C_T'\rvert^2\}[(1/48)(p^2+q^2)]$
			$S_1=\{C_TC_T'^*+C_T^*C_T'\}[(1/48)q^2+(1/80)p^2]p/E$
			$S_2=\{\lvert C_T\rvert^2+\lvert C_T'\rvert^2\}(7/240)p^2$
			$S_3=\{C_TC_T'^*+C_T^*C_T'\}(1/80)p^3/E$

The polarization correlation for first-forbidden transitions can be obtained from Eq. (6) and the Tables II and IV.

(c) Electron Polarization

The nonconservation of parity gives rise to a polarization of the β particles emitted from unoriented nuclei. The polarization is described by a polarization vector $\mathbf{P}$, which is defined as twice the mean value of the spin vector. It is given by the following expression:

$$\mathbf{P}=\frac{\mathbf{p}}{\lvert\mathbf{p}\rvert}\sum_L C(L)/\sum_L b_0(L,L). \tag{8}$$

The coefficients $C(L)$ are calculated in the Appendix and can be found in Table V for allowed and all first-forbidden transitions.

As an example, we give the formula for allowed transitions. In this case Eq. (8) reduces to

$$\mathbf{P}=\frac{\mathbf{p}}{E}\frac{\lvert\int\beta\rvert^2(C_SC_S'^*+C_S'C_S^*)+\lvert\int\beta\boldsymbol{\sigma}\rvert^2(C_TC_T'^*+C_T'C_T^*)}{\lvert\int\beta\rvert^2(\lvert C_S\rvert^2+\lvert C_S'\rvert^2)+\lvert\int\beta\boldsymbol{\sigma}\rvert^2(\lvert C_T\rvert^2+\lvert C_T'\rvert^2)}. \tag{9}$$

As can be seen from Tables IV and V, the degree of polarization in forbidden transitions is not necessarily smaller than in allowed transitions.

III. DISCUSSION

In the tables given in this paper, we have omitted the vector (V) and axial-vector (A) couplings. With the generalized β interaction, which includes parity-nonconserving terms, the absence of the Fierz interference terms in the β spectra is no more sufficient to exclude these couplings. The Fierz terms have now the form:

$$\mathrm{Re}\{C_SC_V^*+C_S'C_V'^*\}\frac{1}{E}\quad\text{and}\quad\mathrm{Re}\{C_TC_A^*+C_T'C_A'^*\}\frac{1}{E}.$$

These quantities can be made small or zero in many ways by a suitable choice of the constants. From the electron-neutrino angular-correlation measurements[15] one may, however, still obtain the unambiguous limits

$$\frac{\lvert C_A\rvert^2+\lvert C_A'\rvert^2}{\lvert C_T\rvert^2+\lvert C_T'\rvert^2}<0.2\quad\text{and}\quad\frac{\lvert C_V\rvert^2+\lvert C_V'\rvert^2}{\lvert C_S\rvert^2+\lvert C_S'\rvert^2}\lesssim 0.5. \tag{10}$$

The present evidence for ruling out the vector and axial-vector couplings is therefore very weak. It might however be supported by theoretical arguments: when time-reversal invariance is fulfilled for the β interaction, all the coupling constants are necessarily real. If in addition the neutrino field is strictly massless, so that the interaction is invariant under the replacement of ν by $\gamma_5\nu$, then the relation $C=-C'$ holds for all interactions.[16] The latter statement is equivalent to the two-component theory of the neutrino.[17,18] These arguments

[15] J. S. Allen and W. K. Jentschke, Phys. Rev. **89**, 902(A) (1953); B. M. Rustad and S. L. Ruby, Phys. Rev. **97**, 991 (1955); Maxon, Allen, and Jentschke, Phys. Rev. **97**, 109 (1955); W. P. Alford and D. R. Hamilton, Phys. Rev. **95**, 1351 (1954).

[16] The recent experiments on the angular distribution of electrons from oriented nuclei have excluded the possibility $C=+C'$.

[17] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[18] L. Landau (to be published).

TABLE V. Parameters $C(L)$ for allowed and first-forbidden transitions. For a given L, $C(L)$ is found as the sum of the products of the matrix elements given in the second column and the corresponding energy-dependent factor given in the third column. The quantity ξ is equal to $\alpha Z/2R$, where R is the nuclear radius. The electron energy and momentum are denoted by E and p, respectively, while q is the neutrino momentum. The nuclear charge number is equal to Z, and α is the fine-structure constant.

L	Matrix element	
		Allowed transitions
0	$\lvert\int\beta\rvert^2$	$\frac{1}{4}(C_S C_S'^* + C_S^* C_S')p/E$
1	$\lvert\int\beta\boldsymbol{\sigma}\rvert^2$	$\frac{1}{4}(C_T C_T'^* + C_T^* C_T')p/E$
		First forbidden transitions
0	$\lvert\int\beta\gamma_5\rvert^2$	$\frac{1}{4}(C_P C_P'^* + C_P^* C_P')p/E$
	$\lvert\int(1/i)\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rvert^2$	$\frac{1}{4}(C_T C_T'^* + C_T^* C_T')(p/E)[\frac{1}{9}(p^2+q^2) - (2/9)qE + \frac{2}{3}\xi(-q+E) + \xi^2]$
	$\lvert\int\beta\gamma_5\rvert\cdot\lvert\int(1/i)\beta\boldsymbol{\sigma}\cdot\mathbf{r}\rvert$	$\frac{1}{2}\,\mathrm{Re}(C_P C_T'^* + C_P' C_T^*)\frac{p}{E}[\frac{1}{3}(-q+E)+\xi] - \frac{1}{2}\,\mathrm{Im}(C_P C_T'^* + C_P' C_T^*)\frac{p}{E}\frac{1}{3}\frac{Z\alpha}{p}$
1	$\lvert\int i\beta\mathbf{r}\rvert^2$	$\frac{1}{4}(C_S C_S'^* + C_S^* C_S')(p/E)[\frac{1}{3}(p^2+q^2) - (2/9)qE + \frac{2}{3}\xi(-q+E) + \xi^2]$
	$\lvert\int\beta\boldsymbol{\sigma}\times\mathbf{r}\rvert^2$	$\frac{1}{4}(C_T C_T'^* + C_T^* C_T')(p/E)[\frac{1}{6}(p^2+q^2) + (2/9)qE + \frac{2}{3}\xi(q+E) + \xi^2]$
	$\lvert\int\beta\boldsymbol{\alpha}\rvert^2$	$\frac{1}{4}(C_T C_T'^* + C_T^* C_T')p/E$
	$\lvert\int\beta\boldsymbol{\alpha}\rvert\cdot\lvert\int\beta\boldsymbol{\sigma}\times\mathbf{r}\rvert$	$\frac{1}{2}(C_T C_T'^* + C_T^* C_T')(p/E)[\frac{1}{3}(E+q)+\xi]$
	$\lvert\int i\beta\mathbf{r}\rvert\times\lvert\int\beta\boldsymbol{\sigma}\times\mathbf{r}\rvert$	$\frac{1}{2}\,\mathrm{Re}(C_S C_T'^* + C_S' C_T^*)\frac{p}{E}[\frac{2}{3}E\xi + \xi^2] + \frac{1}{2}\,\mathrm{Im}(C_S C_T'^* + C_S' C_T^*)\frac{p}{E}\frac{2}{9}\frac{Z\alpha}{p}$
	$\lvert\int i\beta\mathbf{r}\rvert\times\lvert\int\beta\boldsymbol{\alpha}\rvert$	$\frac{1}{2}\,\mathrm{Re}(C_S C_T'^* + C_S' C_T^*)\frac{p}{E}[\frac{1}{3}(-q+E)+\xi] + \frac{1}{2}\,\mathrm{Im}(C_S C_T'^* + C_S' C_T^*)\frac{p}{E}\frac{1}{3}\frac{Z\alpha}{p}$
2	$\lvert\int iB_{ij}{}^{\beta}\rvert^2$	$\frac{1}{4}(C_T C_T'^* + C_T^* C_T')(p/E)[1/12(p^2+q^2)]$

imply that no cancellations occur in the Fierz terms and therefore lower limits than those given in Eq. (10) can be set on the vector and axial-vector couplings.[19] For these reasons, only a general scalar-tensor-pseudoscalar coupling has been treated explicitly.

The effects of nonconservation of parity given in the previous sections are all approximately proportional to v/c, where v is the velocity of the electron. They attain their maximum values if the parity-nonconserving coupling constants are equal in magnitude to the parity-conserving constants. Indeed, when $v=c$, the two component theory of the neutrino predicts a complete polarization of the electron for any STP combination even in forbidden transitions.

In the electron polarization experiment, choosing a zero-zero transition, the scalar coupling constants can be studied separately. In the same way, a β transition with $\Delta j=1$(no) or $\Delta j=2$(yes) in any of the discussed experiments will give information about the tensor couplings. A study of a transition where scalar and tensor couplings interfere [see, e.g., Eq. (7)] may then finally be used to determine the relative sign of the scalar and the tensor coupling constants. In forbidden transitions the formulas also contain terms which depend on the imaginary parts of the coupling constants. As can be seen from Table IV, these terms also appear in the expressions for ordinary β-γ correlations ($k=2$). They may in principle be studied by precise measurements of the energy dependence, and one could thus obtain information about the time-reversal invariance.

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IV. APPENDIX

For calculations of angular correlations in β decay it is convenient to expand the β interaction in multipole components.[20] The Hamiltonian, Eq. (1), then takes the form

$$H^{\beta}=\sum_{L=0}^{\infty}\sum_{M=-L}^{L}\sum_{\Lambda=L-1}^{L+1}\sum_{\alpha=1}^{8}(-1)^{L+M+p(\alpha)}(\psi_p{}^{\dagger}O_{\Lambda,\alpha}{}^{L,-M}\psi_n)\times[C_{\alpha}e^{\dagger}O_{\Lambda,\alpha}{}^{L,M}\nu+C_{\alpha}'e^{\dagger}O_{\Lambda,9-\alpha}{}^{L,M}\nu]. \quad \text{(A1)}$$

The indices Λ and α refer to the different types of interaction, while L and M indicate the multipolarity. The operators $O_{\Lambda\alpha}{}^{LM}$ and the signs $(-1)^{p(\alpha)}$ are given in Table VI.

[19] On arguments for the STP interaction, see B. Stech and J. H. D. Jensen, Physik 141, 175 (1955).

[20] See, e.g., M. E. Rose and L. C. Biedenharn, reference 8.

TABLE VI. Multipole operators for scalar, vector, tensor, axial vector, and pseudoscalar β interactions. The first five columns contain the quantities entering in Eq. (A1). The vector spherical harmonics $\mathbf{\Phi}_{L\Lambda}{}^{M}$ are defined in reference 8. Column seven gives the Cartesian notation of the operators for typical values of multipole order L and interaction type Λ, α. The relation between the matrix elements of these operators, which are used in Tables III–V, and the reduced matrix elements of the multipole operators is given in Eq. (A12).

α	$(-1)^{p(\alpha)}$	C_α	$O_{\Lambda\alpha}{}^{LM}$	C_α'	Λ	L	O_{Cart}
1	+	C_S	$i^\Lambda\beta Y_{LM}\delta_{\Lambda L}$	C_S'	0	0	$(1/4\pi)^{\frac{1}{2}}\beta$
					1	1	$(3/4\pi)^{\frac{1}{2}}i\beta\mathbf{r}/r$
2	+	C_V	$i^\Lambda Y_{LM}\delta_{\Lambda L}$	C_V'	0	0	$(1/4\pi)^{\frac{1}{2}}1$
3	−	C_T	$i^\Lambda\beta\boldsymbol{\sigma}\mathbf{\Phi}_{L\Lambda}{}^{M}$	C_T'	0	1	$(1/4\pi)^{\frac{1}{2}}\beta\boldsymbol{\sigma}$
					1	0	$(1/4\pi)^{\frac{1}{2}}(1/i)\beta\boldsymbol{\sigma}\cdot\mathbf{r}/r$
					1	1	$(3/8\pi)^{\frac{1}{2}}\beta\boldsymbol{\sigma}\times\mathbf{r}/r$
					1	2	$(3/16\pi)^{\frac{1}{2}}iB_{ij}{}^{\beta}/r$
4	−	C_A	$i^\Lambda\boldsymbol{\sigma}\mathbf{\Phi}_{L\Lambda}{}^{M}$	C_A'	0	1	$(1/4\pi)^{\frac{1}{2}}\boldsymbol{\sigma}$
5	+	C_V	$i^\Lambda\gamma_5\boldsymbol{\sigma}\mathbf{\Phi}_{L\Lambda}{}^{M}$	C_V'	0	1	$-(1/4\pi)^{\frac{1}{2}}\boldsymbol{\alpha}$
6	−	C_T	$i^\Lambda\beta\gamma_5\boldsymbol{\sigma}\mathbf{\Phi}_{L\Lambda}{}^{M}$	C_T'	0	1	$-(1/4\pi)^{\frac{1}{2}}\beta\boldsymbol{\alpha}$
7	−	C_A	$i^\Lambda\gamma_5 Y_{LM}\delta_{\Lambda L}$	C_A'	0	0	$(1/4\pi)^{\frac{1}{2}}\gamma_5$
8	+	C_P	$i^\Lambda\beta\gamma_5 Y_{LM}\delta_{\Lambda L}$	C_P'	0	0	$(1/4\pi)^{\frac{1}{2}}\beta\gamma_5$

The different experiments discussed in Sec. II are all described by the general density matrix

$$\rho=\sum_\nu\langle I_fM_f\mathbf{p}\sigma|H^\beta|I_iM_i\nu\rangle\langle I_fM_f'\mathbf{p}\sigma'|H^\beta|I_iM_i\nu\rangle^*. \quad \text{(A2)}$$

In this expression the electron is described by its momentum $\mathbf{p}$ and spin component σ along $\mathbf{p}$. The summation is extended over the states of the unobserved neutrino. The wave function of the electron, which at large distances behaves as a distorted plane wave with momentum $\mathbf{p}$ plus an incoming spherical wave, is expanded in partial waves according to the relation[21]

$$|\mathbf{p}\sigma\rangle=(4\pi)^{\frac{1}{2}}\sum_{\kappa,m}[2l(\kappa)+1]^{\frac{1}{2}}\langle l\tfrac{1}{2}0\sigma|j\sigma\rangle e^{-i\Delta(\kappa)} \times D_{m,\sigma}{}^{j}(\mathbf{z}\to\mathbf{p})\,|\kappa m\rangle, \quad \text{(A3)}$$

with

$$|\kappa m\rangle=i^{l(\kappa)}\begin{pmatrix} g_\kappa\chi_\kappa{}^m \\ if_\kappa\chi_{-\kappa}{}^m\end{pmatrix}.$$

We have used the same notation as that used in reference 8 except that the two upper components of the wave functions are here the large ones. The Coulomb phase shift $\Delta(\kappa)$ is given by

$$\Delta(\kappa)=\tfrac{1}{2}\arg\left(\frac{-\kappa+iZ\alpha/p}{\gamma+iZ\alpha E/p}\right)-\arg\Gamma(\gamma+iZ\alpha E/p) +\tfrac{1}{2}\pi[l(-\kappa)-\gamma], \quad \text{(A4)}$$

where

$$\gamma=[\kappa^2-(Z\alpha)^2]^{\frac{1}{2}}.$$

The rotation matrix D depends on the Eulerian angles describing the rotation from a fixed coordinate system (indicated by $\mathbf{z}$) to a system where the z axis is along the momentum $\mathbf{p}$.[22]

[21] Rose, Biedenharn, and Arfken, Phys. Rev. 85, 5 (1952).
[22] See, e.g., A. R. Edmonds, CERN Report 55-26, 1955 (unpublished).

When one introduces the quantum numbers j_ν, κ_ν, and m_ν for the neutrino, and performs the summation over the magnetic quantum numbers m_ν the density matrix (A2) may be expressed in the form

$$\rho=4\pi\sum_{\substack{\kappa,\kappa',\kappa_\nu,k,s,\mu\\ L,L',\alpha,\alpha',\Lambda,\Lambda'}}(2k+1)(2l+1)^{\frac{1}{2}}(2l'+1)^{\frac{1}{2}} \times(2j+1)^{\frac{1}{2}}(2j'+1)^{\frac{1}{2}}(-1)^{j_\nu+k+M-\sigma'+L+L'} \times\begin{pmatrix} I_i & L & I_f\\ -M_i & M & M_f\end{pmatrix}\begin{pmatrix} I_i & L' & I_f\\ -M_i & M' & M_f'\end{pmatrix} \times\begin{pmatrix} L & L' & k\\ -M & M' & \mu\end{pmatrix}\begin{pmatrix} \frac{1}{2} & j & l\\ -\sigma & \sigma & 0\end{pmatrix}\begin{pmatrix} \frac{1}{2} & j' & l'\\ -\sigma' & \sigma' & 0\end{pmatrix} \times\begin{pmatrix} j & j' & k\\ -\sigma & \sigma' & s\end{pmatrix}D_{\mu,s}{}^{k*}(\mathbf{z}\to\mathbf{p})e^{i[\Delta(\kappa)-\Delta(\kappa')]} \times\begin{Bmatrix} L & L' & k\\ j' & j & j_\nu\end{Bmatrix}B_{\Lambda\alpha}{}^{L}(\kappa,\kappa_\nu)B_{\Lambda'\alpha'}{}^{L'*}(\kappa',\kappa_\nu). \quad \text{(A5)}$$

In this formula the following abbreviation has been introduced:

$$B_{\Lambda\alpha}{}^{L}(\kappa,\kappa_\nu)=(-1)^{p(\alpha)}\langle I_f\|O_{\Lambda,\alpha}{}^{L}\|I_i\rangle \times(C_\alpha\langle\kappa\|O_{\Lambda,\alpha}{}^{L}\|\kappa_\nu\rangle+C_\alpha'\langle\kappa\|O_{\Lambda,9-\alpha}{}^{L}\|\kappa_\nu\rangle). \quad \text{(A6)}$$

The Wigner notation for the vector-addition coefficients and Racah coefficients has been used.[22] The reduced

matrix elements are defined by

$$\langle \kappa m | O_{\Lambda\alpha}{}^{LM} | \kappa_\nu m_\nu \rangle = (-1)^{j-m} \begin{pmatrix} j & L & j_\nu \\ -m & M & m_\nu \end{pmatrix} \langle \kappa \| O_{\Lambda\alpha}{}^{L} \| \kappa_\nu \rangle. \quad \text{(A7)}$$

By the present definition of $O_{\Lambda\alpha}{}^{LM}$ the reduced matrix elements $\langle \kappa \| O_{\Lambda\alpha}{}^{L} \| \kappa_\nu \rangle$ are real; similarly the matrix elements $\langle I_f \| O_{\Lambda\alpha}{}^{L} \| I_i \rangle$ are real by an appropriate choice of the phases of the nuclear wave functions.[8] The electron distribution from oriented nuclei is given by the trace of ρ with regard to M_f and σ and is found to be

$$W(\mathbf{p},\mathbf{z}) = \sum_{M_f M_f' \sigma\sigma'} \rho\, \delta_{M_f M_f'}\, \delta_{\sigma\sigma'} = \pi^{-1} \sum_{k,L,L'} h_k(I_i,M_i) F_k(L,L',I_f,I_i) (-1)^{L+L'+k} \times b_k(L,L') P_k(\cos\theta). \quad \text{(A8)}$$

We have here introduced the following abbreviations:

$$h_k(I_i,M_i) = (2k+1)^{\frac{1}{2}} (2I_i+1)^{\frac{1}{2}} \times (-1)^{I_i - M_i} \begin{pmatrix} I_i & I_i & k \\ M_i & -M_i & 0 \end{pmatrix}, \quad \text{(A9)}$$

$$F_k(L,L',I_f,I_i) = (-1)^{I_f+I_i-1} (2k+1)^{\frac{1}{2}} (2I_i+1)^{\frac{1}{2}} \times (2L+1)^{\frac{1}{2}} (2L'+1)^{\frac{1}{2}} \times \begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} L & L' & k \\ I_i & I_i & I_f \end{Bmatrix}, \quad \text{(A10)}$$

and

$$b_k(L,L') = 4\pi^2 \begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix}^{-1} \sum_{\substack{\kappa,\kappa',\kappa_\nu, \\ \alpha,\alpha',\Lambda,\Lambda'}} (2I_i+1)^{-1} \times (2L+1)^{-\frac{1}{2}} (2L'+1)^{-\frac{1}{2}} (2j+1)^{\frac{1}{2}} (2j'+1)^{\frac{1}{2}} \times (2l+1)^{\frac{1}{2}} (2l'+1)^{\frac{1}{2}} (-1)^{i+i'+i_\nu-\frac{1}{2}} \times \begin{Bmatrix} L & L' & k \\ j' & j & j_\nu \end{Bmatrix} \begin{Bmatrix} l & l' & k \\ j' & j & \frac{1}{2} \end{Bmatrix} \begin{pmatrix} l & l' & k \\ 0 & 0 & 0 \end{pmatrix} \times e^{i[\Delta(\kappa)-\Delta(\kappa')]} B_{\Lambda\alpha}{}^{L}(\kappa,\kappa_\nu) B_{\Lambda'\alpha'}{}^{L'*}(\kappa',\kappa_\nu). \quad \text{(A11)}$$

These functions are given for some cases of interest in the Tables I–IV. The $b_k(L,L')$ are given there in terms of the nuclear matrix elements in the usual Cartesian notation $|\int \beta\boldsymbol{\sigma}|$, etc. The exact definition of these matrix elements including their signs in terms of the reduced matrix elements of Eq. (A6) is given by

$$\left| \int O_{\text{Cart}} \right| = (2I_i+1)^{-\frac{1}{2}} \langle I_f \| O_{\Lambda\alpha}{}^{L} \| I_i \rangle, \quad \text{(A12)}$$

where O_{Cart} is obtained from the last column of Table VI.

The integrated transition probability is proportional to the average value, S, of $W(\mathbf{p},\mathbf{z})$ over all M_i:

$$S = (2I_i+1)^{-1} \sum_{M_i M_f M_f' \sigma\sigma'} \rho\, \delta_{M_f M_f'}\, \delta_{\sigma\sigma'} = \pi^{-1} \sum_L b_0(L,L). \quad \text{(A13)}$$

In order to evaluate the β-γ correlation, we define a "density matrix" ρ_γ for the γ transition, $I_f \rightarrow I_{ff}$:

$$\rho_\gamma = \sum_{M_{ff}} \langle \mathbf{q},\tau,I_{ff}M_{ff} | H_\gamma | I_f M_f \rangle \times \langle \mathbf{q},\tau,I_{ff}M_{ff} | H_\gamma | I_f M_f' \rangle^*, \quad \text{(A14)}$$

where $\mathbf{q}$ is the wave vector for the γ-quantum and $\tau = \pm 1$ stands for right- and left-circular polarization respectively. By performing the summations over the magnetic quantum-numbers, one obtains

$$\rho_\gamma = \sum_{k\lambda\lambda'} (2k+1)^{\frac{1}{2}} (\tau)^k (-1)^{I_f+M_f} F_k(\lambda,\lambda',I_{ff},I_f) \times (2I_f+1)^{-\frac{1}{2}} \begin{pmatrix} I_f & I_f & k \\ -M_f & M_f' & \mu \end{pmatrix} \delta_\lambda \delta_{\lambda'} D_{\mu,0}{}^{k*}(\mathbf{z}\rightarrow\mathbf{q}). \quad \text{(A15)}$$

The quantity δ_λ is the usual transition amplitude for the 2^λ-pole radiation (see reference 8). The β-γ correlation formula may now be written

$$W(\mathbf{p},\mathbf{q},\tau) = \sum_{M_i M_f M_f' \sigma\sigma'} (2I_i+1)^{-1} \rho\, \rho_\gamma\, \delta_{\sigma\sigma'} = \pi^{-1} \frac{2I_i+1}{2I_f+1} \sum_{k,\lambda,\lambda',L,L'} (-\tau)^k \delta_\lambda \delta_{\lambda'} \times F_k(\lambda,\lambda',I_{ff},I_f) F_k(L,L',I_i,I_f) \times b_k(L,L') P_k(\cos\theta), \quad \text{(A16)}$$

where θ is the angle between the β particle and the γ quantum.

The polarization vector $\mathbf{P}$ of the electrons is defined as the trace of the product of ρ with the Pauli spin-vector matrix $\langle \sigma' | \boldsymbol{\sigma} | \sigma \rangle$ divided by the trace of ρ. For unoriented initial states, one finds

$$\mathbf{P} = S^{-1} \sum_{M_i M_f M_f' \sigma\sigma'} \rho \langle \sigma' | \boldsymbol{\sigma} | \sigma \rangle \delta_{M_f M_f'} (2I_i+1)^{-1} = \frac{\mathbf{p}}{|\mathbf{p}|} \sum_L C(L) / \sum_L b_0(L,L), \quad \text{(A17)}$$

where

$$C(L) = -4\pi^2 \sum_{\substack{\Lambda\Lambda'\alpha\alpha' \\ \kappa\kappa'\kappa_\nu}} (2L+1)^{-1} (2I_i+1)^{-1} \times e^{i[\Delta(\kappa)-\Delta(\kappa')]} B_{\Lambda\alpha}{}^{L}(\kappa,\kappa_\nu) \times B_{\Lambda'\alpha'}{}^{L*}(\kappa',\kappa_\nu) \delta_{jj'} \delta_{l',l\pm1}. \quad \text{(A18)}$$

The parameters $b_0(L,L)$ are defined in Eq. (A11).