Galvanomagnetic Effects in p-Type Silicon

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Measurements have been made on several samples of p-type silicon of the dependences of electrical resistivity and Hall effect on temperature and magnetic field strength. The electrical resistivity follows a $T^{2.7}$ law in the lattice scattering range in the absence of a magnetic field. The Hall coefficient exhibits a small linear negative temperature dependence between about 200°K and 320°K and is almost entirely independent of field strength up to 13 000 gauss at any temperature between 77°K and 320°K. The dependence of the magnetoresistance on the relative directions of current, field, and crystallographic axes has been studied at 77°K and 300°K as a function of the field strength with particular emphasis on obtaining accurate values of the various coefficients which are required for a complete characterization of the magnetoresistance in the limit of zero field. The main features of these results are the relatively large observed values of longitudinal magnetoresistance, which in some cases are nearly as large as the transverse effects. The results of the galvanomagnetic measurements are somewhat inconsistent with the usual valence band model as deduced from cyclotron resonance experiments at 4°K.

INTRODUCTION

IT has been generally accepted on the basis of the results of cyclotron resonance experiments[1,2] at 4°K that the edges of the valence bands in silicon and germanium consist of two sets of warped spherical surfaces of constant energy centered at $k=0$ and degenerate at that point. The energy surface equation corresponding to this situation has the form

$$\epsilon=-\frac{\hbar^2}{2m_0}\{Ak^2\pm[B^2k^4+C^2(k_x^2k_y^2+k_y^2k_z^2+k_z^2k_x^2)]^{\frac{1}{2}}\}, \tag{1}$$

where for silicon at 4°K, $A\cong4.1$, $B\cong1.4$, and $C\cong3.7$, and for germanium $A\cong13$, $B\cong8.7$, and $C\cong11.4$. There are thus two sets of holes in the valence band with two different average effective masses. Measurements of galvanomagnetic effects in p-type germanium[3,4] at temperatures up to around 300°K have in general produced results which can be described at least qualitatively by the valence band model represented by Eq. (1). Very few of the same sorts of measurements have yet been made on silicon.

The present paper presents the results of some measurements of the temperature and magnetic field strength dependences of the electrical resistivity and Hall effect on several samples of p-type silicon. The principal objectives behind these experiments were to obtain some accurate data on galvanomagnetic effects in p-type silicon and then to test the applicability of the two-band warped-sphere model to the interpretation of these data at temperatures quite a bit higher than 4°K. In the body of the paper we give first of all a description of the experiments and results and then discuss the results, mostly in terms of calculations by Lax and Mavroides[5,6] of the various galvanomagnetic coefficients as based on the model represented by Eq. (1). These calculations are the only ones presently available which take into account the warping of the energy surfaces. The work described here represents an extension of that reported earlier.[7]

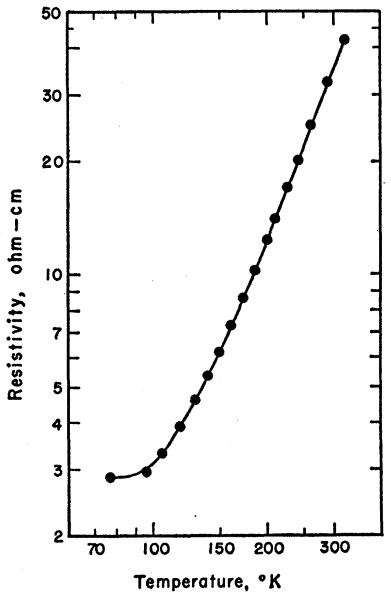

FIG. 1. Resistivity *vs* temperature in p-type silicon, sample *SH2A*.

EXPERIMENTS AND RESULTS

The p-type silicon samples used in these experiments were cut from single crystals regrown several times by the usual Czochralski technique and were boron-doped. The amount of impurity compensation in the samples was not known but was believed to be small, because the as-grown crystals exhibited just the type of slowly

[1] Dresselhaus, Kip, and Kittel, Phys. Rev. **98**, 368 (1955).
[2] Dexter, Zeiger, and Lax, Phys. Rev. **104**, 637 (1957).
[3] Willardson, Harman, and Beer, Phys. Rev. **96**, 1512 (1954).
[4] Goldberg, Adams, and Davis, Phys. Rev. **105**, 865 (1957).
[5] B. Lax and J. G. Mavroides, Phys. Rev. **100**, 1650 (1955), referred to in text as LM.
[6] B. Lax and J. G. Mavroides (private communication).
[7] D. Long and A. Nussbaum, Bull. Am. Phys. Soc. Ser. II, **2**, 57 (1957).

decreasing resistivity profile down their axes of growth that is expected for a silicon crystal in which boron (with a segregation coefficient[8] of 0.9) is the only important impurity. The samples were cut in the form of "bridges,"[9] and electrical contacts were made by alloying aluminum into the silicon in the proper positions for resistivity and Hall effect measurements. The room temperature resistivities and crystallographic orientations of the two pairs of samples to be discussed are listed in Table I. Two types of sample orientation were used in these experiments. In one type the direction of current flow was made parallel to the [100] axis, and the Hall contacts were placed such that the magnetic field would be parallel to the [001] axis when the Hall effect was measured. In the other the Hall contacts were positioned in the same manner, but the current flow was made parallel to the [110] axis. The experiments described below were done over a temperature range of 77°K to 320°K using fields up to about 13 000 gauss as provided by a 12-inch Varian electromagnet.

A log-log plot of resistivity *vs* temperature is given in Fig. 1 for sample *SH2C* and is typical of the results

TABLE I. Orientations and resistivities of p-type silicon samples.

Sample number	Direction of current	Resistivity (ohm cm)
SH2A	[110]	35
SH2C	[100]	35
SH5A	[110]	85
SH5C	[100]	85

obtained on all the samples. The slope of the straight line portion of the curve is 2.7 ± 0.1, so this sample exhibits the same temperature dependence of resistivity in the lattice scattering range as those of similar resistivities studied by Ludwig and Watters.[10] The lattice scattering mobility then follows a $T^{-2.7}$ law, at least between about 150°K and 320°K where nearly all the boron acceptors are expected to be ionized. The deviation from a straight line at lower temperatures in Fig. 1 is undoubtedly due to a combination of ionized-impurity scattering and impurity deionization.

Figure 2 shows a plot of Hall coefficient *vs* temperature for sample *SH2C* at fields of 3000 and 13 000 gauss. The temperature dependences between 150°K and 320°K are almost identical at the two field strengths. The apparently linear negative temperature dependence above about 200°K must be due to a variation of the ratio of Hall mobility to conductivity mobility rather than to an increase with temperature of the hole density, since, as pointed out before, the acceptors should be ionized in this range. In fact a curve of Hall coefficient *vs* temperature on a relatively uncompensated sample of

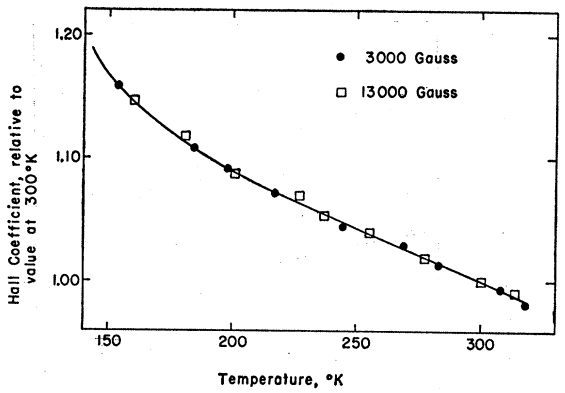

FIG. 2. Hall coefficient (relative to its value at 300°K) *vs* temperature in p-type silicon, sample *SH2A*.

about 200 ohm-cm resistivity exhibited the same sort of negative slope above about 150°K. The data of Fig. 2 are typical of those obtained on all the samples studied and are similar to results of Morin and Maita[11] obtained at an unspecified field strength.

The values of Hall mobility at 300°K, found from the usual relation[5] that $\mu_H=\sigma_0 R_0$, lay between 360 and 390 cm²/volt-sec for the four samples of Table I. Several other workers[11–13] have measured room temperature Hall mobilities of about the same magnitudes on samples of comparable purity. In no case has a Hall mobility greater than 450 cm²/volt-sec been reported.[14] The drift mobility of holes in reasonably pure silicon has been found by several investigators[10,13,15] to lie between about 480 and 510 cm²/volt-sec at room temperature. Since the drift and conductivity mobilities are expected to be equal,[16] the ratio r of Hall to conductivity mobility apparently has a value less than unity at room temperature.[11] From our results the actual value of r at 300°K would be roughly 0.75, and r would approach unity with decreasing temperature according to the curve of Fig. 2. These can be considered "weak field" results, since field dependence was negligible in the Hall effect measurements. It would be desirable to extend these Hall effect measurements over a much wider range of sample purities, and especially to very high purities, to find out whether the linear negative slope in Fig. 2 depends at all on impurity concentration and whether the result that r is less than unity at 300°K is truly an intrinsic property of silicon.

We have measured also the field dependence of the Hall coefficient at several fixed temperatures; in particular, at 77°K, 150°K, and 300°K. The Hall coefficient proved to be independent of field strength to within

[8] J. A. Burton, Physica **20**, 845 (1954).
[9] P. P. Debye and E. M. Conwell, Phys. Rev. **93**, 693 (1954).
[10] G. W. Ludwig and R. L. Watters, Phys. Rev. **101**, 1699 (1956).
[11] F. J. Morin and J. P. Maita, Phys. Rev. **96**, 29 (1954).
[12] P. P. Debye and T. Kohane, Phys. Rev. **94**, 724 (1954).
[13] D. G. Cronemeyer, Phys. Rev. **105**, 522 (1957).
[14] M. Green, Bull. Am. Phys. Soc. Ser. II, **2**, 158 (1957).
[15] M. Zerbst and W. Heywang, Z. Naturforsch. **11a**, 608 (1956).
[16] E. S. Rittner, Phys. Rev. **101**, 1291 (1956), shows that the measured drift mobility is given by the same kind of weighted average of the two-hole mobilities as is the measured conductivity mobility.

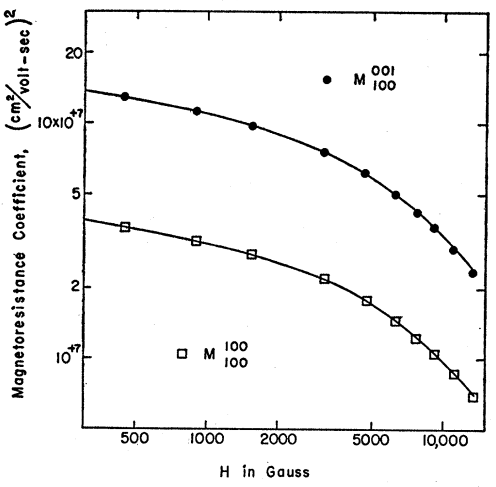

FIG. 3. Magnetoresistance coefficients *vs* magnetic field strength at 77°K in *p*-type silicon, sample *SH5C*. Note that 1(cm²/volt sec)² $\equiv 10^{-16}$ gauss^{-2}.

$\pm 1\%$ up to 13 000 gauss at 150°K and 300°K for al four samples; this behavior is consistent with the results in Fig. 2. At 77°K the Hall coefficient was about 5% larger at 13 000 gauss than at the weakest fields, but this slight increase is felt to be too small to be very significant. Thus, there is no striking field dependence of the Hall effect in *p*-type silicon such as occurs in *p*-type germanium,[3] at least not for the conditions of temperature, sample purity, and field strength which existed in our experiments. Again, it would be desirable to study a wider range of sample purities, particularly at 77°K where impurity scattering is important.

The next results to be presented are those of measurements of magnetoresistance coefficients of *p*-type silicon. We have concentrated on a study of the five coefficients which can be determined from the two kinds of oriented samples that were described previously. Let us define a magnetoresistance coefficient by the relation,

$$M_I{}^H = \Delta\rho/\rho_0 H^2,$$

where the subscript and superscript indicate the crystallographic directions of the current and field, respectively. Then, the coefficients $M_{100}{}^{001}$ and $M_{100}{}^{100}$ were found from measurements on samples in which the current flowed along the [100] axis, and $M_{110}{}^{001}$, $M_{110}{}^{1\bar{1}0}$, and $M_{110}{}^{110}$ were found from measurements on samples with the current along the [110] axis.

The values of the magnetoresistance coefficients for samples *SH5A* and *SH5C* in the limit of zero field at 77°K and 300°K are listed in Table II along with the Hall mobilities at each temperature. The zero-field coefficients were obtained by extrapolating *M vs H* curves back to vanishing field strength. It is not as difficult to obtain accurate results in this way as it is for germanium,[4] because the field-dependence is less pronounced in silicon. The five coefficients were also measured on samples *SH2A* and *SH2C*, and the relative values obtained were essentially identical to those presented in Table II; the actual magnitudes were, however, slightly smaller in this second pair of samples because of the greater importance of impurity scattering and the resultant lower hole mobilities. Our results at 77°K agree fairly well with previous magnetoresistance measurements made by Pearson and Herring[17] at the same temperature and with a field of 4400 gauss.

It is convenient for purposes of comparison with theoretical calculations to express the experimental magnetoresistance results in terms of *magnetoconductivity* coefficients, since these are more directly related to theory. For small enough values of the magnetic field, the current density in a cubic crystal is related to the electric field E and the magnetic field H by an equation of the form[18]

$$\mathbf{j}=\sigma_0\mathbf{E}+\alpha\mathbf{E}\times\mathbf{H}+\beta\mathbf{E}\mathbf{H}^2+\gamma\mathbf{H}(\mathbf{E}\cdot\mathbf{H})+\delta M\mathbf{E}, \quad (2)$$

where M is a diagonal tensor with elements $H_1{}^2$, $H_2{}^2$, and $H_3{}^2$. The σ_0 and the α are, respectively, the ordinary electrical conductivity and a coefficient related to the Hall effect. The three magnetoconductivity coefficients β, γ, and δ can be found from the measured magnetoresistance coefficients and Hall mobilities,[19] and their values for samples *SH5A* and *SH5C* at 77°K and 300°K are given in Table III. Since only three independent coefficients are required to characterize completely the weak-field magnetoresistance in a cubic crystal, our measurement of five M coefficients permits a cross-check on the results.[19]

Plots of the field dependences at 77°K of the five magnetoresistance coefficients of samples *SH5A* and *SH5C* are given in Figs. 3 and 4. In each case the coefficient decreased with increasing field strength. Field dependences were measured also at 300°K, but the coefficients deviated only slightly from constancy at this temperature.

DISCUSSION

In this section we shall give a brief discussion of the experimental results in terms of some current ideas of the valence band structure of silicon. Lax and Mav-

TABLE II. Magnetoresistance coefficients and Hall mobilities in *p*-type silicon samples.

Sample number	Hall mobility (cm²/volt-sec) 77°K	300°K	Magnetoresistance coefficients[a] (cm²/volt-sec)² Coefficient	77°K	300°K
SH5C	9500	380	$M_{100}{}^{001}$	14.2×10^7	7.6×10^5
			$M_{100}{}^{100}$	4.4×10^7	6.7×10^5
SH5A	9500	370	$M_{110}{}^{001}$	14.1×10^7	7.6×10^5
			$M_{110}{}^{1\bar{1}0}$	14.6×10^7	10.7×10^5
			$M_{110}{}^{110}$	3.8×10^7	3.4×10^5

[a] These magnetoresistance coefficients may be converted to units of gauss^{-2} by use of the conversion factor 1 (cm²/volt sec)² $\equiv 10^{-16}$ gauss^{-2}.

[17] G. L. Pearson and C. Herring, Physica **20**, 975 (1954).
[18] F. Seitz, Phys. Rev. **79**, 372 (1950).
[19] G. L. Pearson and H. Suhl, Phys. Rev. **83**, 768 (1951).

roides[5,6] have applied the model of the valence band represented by Eq. (1) to the analysis of galvanomagnetic effects and have obtained expressions for the various coefficients in Eq. (2). They have written Eq. (2) in a slightly different way, but the coefficients in their equation are simply related to σ_0, α, etc. The main assumptions in their theory are (a) that Eq. (1) applies with C not too large relative to the other two constants, (b) that a scalar relaxation time exists which is dependent only on the energy ϵ as $\tau=l\epsilon^{-\lambda}$, where l is a constant and λ is a parameter chosen to fit the scattering mechanism, and (c) that the magnetic field is very weak. Much of the following discussion will be based on the LM model.

We have deduced from experiment that the ratio r of Hall mobility to conductivity mobility is roughly 0.75 at 300°K in p-type silicon and that this ratio becomes larger the lower the temperature. The LM model[5] predicts that $r=1.45$ for acoustical lattice scattering ($\lambda=\frac{1}{2}$) or that $r=1.24$ for $\lambda=0$, provided that the 4°K values of A, B, and C quoted previously are employed in the calculations and that the relaxation time is assumed to be the same for both sets of holes. With this last assumption the ratio r becomes the product of an anisotropy term involving the energy surface shapes and a scattering term involving the relaxation time. The $r=1.24$ result occurs for a choice of λ which makes the scattering term equal to its minimum value of unity and therefore represents the contribution of the band shape term alone.

It is possible to choose a larger C relative to A and B (which is equivalent to increasing the anisotropy or warping of both sets of energy surfaces) in such a way as to lower the theoretical value of r, but the requirement of the LM calculations that the warping not be too pronounced prevents us from being able to make r small enough to agree with the observed room temperature value, even if $\lambda=0$. In any case this would be a rather artificial method of achieving agreement. Shockley[20] has pointed out, however, that energy surfaces of the same general form as those in p-type silicon can give values of r quite a bit smaller than unity if they are warped strongly enough. The Hall effect results thus exhibit the *kind* of behavior which would be expected *if* the energy surfaces were effectively quite strongly warped at room temperature and became less so with decreasing temperature. This point will be discussed further later in this section.

TABLE III. Magnetoconductivity coefficients of Eq. (2) for samples *SH5A* and *SH5C*.

Temperature	Magnetoconductivity coefficients (cm⁴-coul/volt³-sec³) β	γ	δ
77°K	-3.68×10^{7}	$+3.14\times10^{7}$	-0.16×10^{7}
300°K	-1.06×10^{4}	$+1.02\times10^{4}$	-0.73×10^{4}

[20] W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., New York, 1950), p. 338.

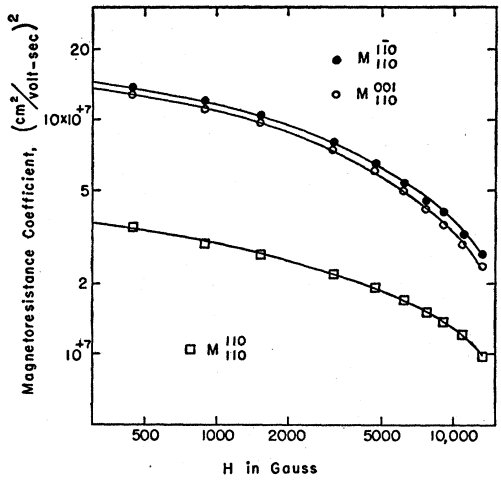

FIG. 4. Magnetoresistance coefficients *vs* magnetic field strength at 77°K in p-type silicon, sample *SH5A*. Note that $1(\text{cm}^2/\text{volt sec})^2 \equiv 10^{-16}$ gauss^{-2}.

There is still some doubt about whether the room temperature result that the Hall mobility is smaller than the conductivity mobility is really an intrinsic property of p-type silicon. Perhaps r would be greater than unity in a crystal of exceptionally high perfection; such an increase of r was once noted in germanium as crystals became better.[20] Nevertheless, all the available evidence to date points to our conclusion about the magnitude of r, and the recently studied silicon crystals should be of higher quality certainly than the old germanium crystals in which anomalously small Hall mobilities were observed.

The weak-field magnetoresistance results can be discussed conveniently in terms of the following two *anisotropy parameters*[4]; magnetoresistance is in general very sensitive to anisotropies in the energy surfaces and in the scattering, and these parameters reflect this sensitivity,

$$a=(2\beta+\delta)/2\beta, \tag{3}$$

$$b=(2\beta+2\gamma+\delta)/2\beta. \tag{4}$$

In an isotropic material $a=1$ and $b=0$. Deviation of experimental values of a and b from the above values is then indicative of a deviation from isotropy either of the energy surfaces or of the scattering or of both. Upon substituting the observed values of β, γ, and δ from Table III into Eqs. (3) and (4) we find that at 77°K, $a=1.02$ and $b=0.17$; and at 300°K, $a=1.35$ and $b=0.39$. These parameters (except for a at 77°K) thus have values quite different from their isotropic ones, and the anisotropy is greater at 300°K than at 77°K. Another interesting quantity is the ratio of b to a, which has an experimental value of 0.17 at 77°K and of 0.29 at 300°K and which is another expression of the anisotropy, the isotropic value of this ratio being zero.

It is seen again that the anisotropy becomes more pronounced with increasing temperature.

Goldberg, Adams, and Davis[4] have found experimentally that in p-type germanium at 77°K, $a=0.90$ and $b=0.17$, and have shown that these values represent an anisotropy too great to be explained by the LM model which predicts that $b=0.10$ when $a=0.90$. In calculating b from the observed value of a, it was not necessary to specify the exact energy dependence of τ, and in fact the relaxation times could be different for the two sets of holes. It is possible to show in exactly the same way and under the same assumptions that $b\cong 1.05\ (1-a)$ for p-type silicon, according to the LM theory[6] and the values of A, B, and C quoted previously. It is obvious that this relation between b and a is not consistent with our experimental results, because it predicts a negative b when a is larger than unity; whereas, the measured b is positive both at 77°K and 300°K. Unfortunately, it is not possible to rectify this disagreement between theory and experiment simply by taking into account the rather large experimental uncertainties[1,2] in the cyclotron resonance values of A, B, and C, that is, by trying to choose values of these constants (within the error limits) to fit the results.

The weak-field Hall and magnetoresistance results, and particularly the changes in them from 77°K to 300°K, are evidently not well described by a model which is based on Eq. (1) and fixed values of the three constants in Eq. (1) and on relatively simple assumptions about scattering. We shall consider now a few possible factors contributing to this discrepancy.

The model utilizes experimental results of cyclotron resonance at 4°K which refer strictly only to the edge of the valence band near $k=0$. As the temperature is raised, more of the holes occupy states which are rather far in energy ($\sim$0.025 ev at room temperature) from the valence band edge. Kane[21] has made calculations for p-type germanium and silicon which indicate that the hole energy does not vary parabolically with wave number away from the band edge, in particular in the low-mass band. The situation is such that the low-mass energy surfaces, which are only slightly anisotropic near $k=0$, change shape to become more nearly like the strongly warped high-mass surfaces at larger energies and wave numbers. In silicon most of the change in the low-mass band occurs within several hundredths of an electron-volt of the top of the valence band; whereas, in germanium most of the change occurs at energies roughly ten times as great. Thus, this band shape change should be important for experiments done between 77°K and 300°K in silicon but not in germanium. We shall not attempt at present to make a quantitative application of these ideas to the experimental results, but shall point out simply that the type of change of band shape with increasing energy described above seems to be in the right direction to give rise to an *apparent* increase in the energy surface anisotropy with increasing temperature as greater fractions of the holes occupy the higher energy states; such an apparent increase of anisotropy can be inferred from both the Hall and magnetoresistance results. These band shape changes may account then for at least a part of the observed behavior.

As the temperature is raised, a greater fraction of the holes will occupy states in the $V3$ band,[1,2,22] which is separated from the two bands represented by Eq. (1) by spin-orbit splitting of the order of 0.04 ev in silicon. The top of the $V3$ band has spherical surfaces of constant energy and so would not likely lead to a greater anisotropy in the magnetoresistance, although its presence could lead to some disagreement between experiment and a theory based on Eq. (1).

It should be emphasized that the scattering processes in p-type silicon actually may be too involved to be described satisfactorily within the assumptions of the LM model, and that the observed Hall effect and magnetoresistance behavior could be due conceivably to scattering effects, at least in part. An indication that the scattering is complicated is given by the anomalous $T^{-2.7}$ temperature dependence of lattice-scattering mobility, the cause for which is not really understood. Anisotropies in the scattering over an energy surface can lead to anisotropy in the magnetoresistance[18] in somewhat the same fashion as energy surface warping does. A rigorous analysis of galvanomagnetic effects for nonisotropic scattering would be difficult to carry out, however, because no relaxation time would be expected to exist in the usual sense.

Because of the difficulty of interpretation of the weak-field galvanomagnetic effects, we shall not attempt to explain the field dependence results in any detail. The lack of field dependence of the Hall coefficient and magnetoresistance at room temperature must mean, however, simply that the hole mobilities are not very large at this temperature, since the criterion for field dependence is essentially that the square of the product of mobility and field strength approach unity.[3] At 77°K the magnetoresistance coefficients decrease with increasing field strength just as would be expected for a semiconductor in which the magnetoresistance is approaching a strong-field saturation value.[3] The almost complete lack of field dependence of the Hall coefficient at 77°K could be explained by saying that $r\cong 1$ at this temperature even in weak fields, so that the magnitude of the Hall coefficient does not change much as H approaches infinity at which point r is exactly unity.[5]

[21] E. O. Kane, J. Phys. Chem. Solids **1**, 83 (1956).

[22] F. Herman, Proc. Inst. Radio Engrs. **43**, 1703 (1955).

A useful consequence of the Hall effect experiments is that it should not ordinarily be necessary in p-type silicon, as it is in p-type germanium,[3] to make Hall measurements in either a very weak or a very strong field in order to obtain consistent results. The Hall effect in p-type silicon seems to be effectively in the weak-field range under normal measuring conditions between 77°K and room temperature.

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New Derivation of Elastic Equations for Trigonal Holoaxial Crystals*

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Based upon potential energy due to interaction between unit cells in a crystal, a new derivation of elastic equations is proposed. The theory is fully developed for the holoaxial class of crystal symmetry. The theory does not require the assumptions that are used in the conventional derivation. Instead, it is shown that a simple translation and a rigid body rotation have no effect on the conventional equations. The derivation achieves two objectives: First, it gives in a more satisfactory fashion physical meaning to each term in the differential equations. Secondly, it provides a general relationship between macroscopic and intermolecular forces. The difficulty of interpreting the nature of the forces acting in the conventional equations is resolved. In addition to the central force, it is found necessary to have a new force, acting in a direction perpendicular to the direction of the central force. It is further found that the forces have centroidal symmetry, in addition to the spherical and trigonal holoaxial symmetries. If only central forces are assumed, the Cauchy relations are easily obtained.

INTRODUCTION

SINCE Voigt[1] and Love,[2] the conventional equations of elasticity have been used exclusively, and the modern theory of elasticity relies upon their derivation and interpretation. It is well known that if only central forces are effective, the Cauchy relations should hold true among elastic coefficients. However, almost any kind of crystal, especially a metal, shows a deviation from the Cauchy relations, indicating noncentral components of forces.[3–6] Several interpretations were attempted by Epstein,[7] Zener,[8] and others to explain the nature of the forces, but none of them seem satisfactory.

The nature of the force in elasticity was also investigated from the standpoint of thermal properties. Namely, in order to clarify the deviation from Debye's theory of specific heat, Born-von Kármán[9–13] and Morse potentials[14–16] were proposed. As far as thermal properties are concerned, these methods seem to work out successfully, particularly owing to the extensive computations by Montroll and others.[17–23] Nevertheless, these do not yet provide better interpretations for the equations of elasticity in general.

On the other hand, modifications of the conventional equations of elasticity were proposed for making a better approximation. Starting with a quadratic form of strain for potential energy and using a further approximation, Epstein[7] presented such a theory. The further approximation was finally formulated by Murnaghan and others in terms of "finite deforma-

* Work supported by the James Knights Crystal Company, Sandwich, Illinois.

[1] W. Voigt, *Lehrbuch der Kristallphysik* (B. G. Teubner, Leipzig, 1928).
[2] A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (Dover Publications, New York, 1944).
[3] C. Zener, *Elasticity and Anelasticity of Metals* (University of Chicago Press, Chicago, 1948).
[4] H. B. Huntington, Phys. Rev. **72**, 321 (1947).
[5] J. K. Galt, Phys. Rev. **73**, 1460 (1948).
[6] Burstein, Smith, and Arenburg, Phys. Rev. **82**, 314 (1951).
[7] P. S. Epstein, Phys. Rev. **70**, 915 (1946).
[8] C. Zener, Phys. Rev. **71**, 323 (1947).
[9] M. Born and T. von Kármán, Physik. Z. **13**, 297 (1912).
[10] M. Born, J. Chem. Phys. **7**, 591 (1939).
[11] M. Blackman, Proc. Roy. Soc. (London) **148**, 365 (1935).
[12] M. Blackman, Proc. Roy. Soc. (London) **148**, 384 (1935).
[13] M. Blackman, Proc. Roy. Soc. (London) **159**, 416 (1937).
[14] J. S. Dugdale and D. K. C. MacDonald, Phys. Rev. **89**, 832 (1953).
[15] J. S. Dugdale and D. K. C. MacDonald, Phys. Rev. **96**, 57 (1954).
[16] P. C. Fine, Phys. Rev. **56**, 355 (1939).
[17] E. W. Montroll, J. Chem. Phys. **10**, 219 (1942).
[18] E. W. Montroll, J. Chem. Phys. **11**, 481 (1943).
[19] E. W. Montroll, J. Chem. Phys. **15**, 575 (1947).
[20] W. V. Houston, Revs. Modern Phys. **20**, 161 (1948).
[21] R. B. Leighton, Revs. Modern Phys. **20**, 165 (1948).
[22] M. Smollett, Proc. Phys. Soc. (London) **A165**, 109 (1952).
[23] L. van Hove, Phys. Rev. **89**, 1189 (1953).