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Measurement of the Product of Viscosity and Density of Liquid Helium with a Torsional Crystal

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The product of the viscosity and the density of liquid helium in the range 1.3°–4.2°K has been measured with a quartz crystal cylinder performing torsional oscillations at 32.4 kc/sec. The measurements are found to be independent of velocity over a wide range.

INTRODUCTION

IN recent years numerous investigations of the viscosity of liquid helium have been carried out, employing the oscillating-disk method.[1–4] One of the difficulties experienced with this technique is the necessity of maintaining a constant temperature over long periods of time. Moreover, the method is not easily adaptable to temperatures lower than those achievable by pumping. We wish to report here some measurements recently obtained with a torsional-crystal method, which eliminates some of the difficulties inherent in the oscillating-disk method and which may, in principle, be employed down to the lowest attainable temperature.

EXPERIMENTAL

In this method a suitably prepared right circular cylinder of piezoelectric quartz, 6 cm long by 0.6 cm in diameter, was excited in its fundamental mode (32.4 kc/sec) of torsional vibration at a constant amplitude. The logarithmic decrement Δ, defined by $\Delta = W^d/2W^v$ (W^d = energy dissipated per cycle, W^v = vibrational energy) was determined by measuring the resistance of the oscillator at resonance.[5] As in the case of the oscillating disk, Δ is related to the product of the viscosity η and the density ρ of the fluid. Indeed, it can be shown that

$$\eta\rho = \left[\frac{M(\Delta-\Delta_0)}{S}\right]^2 \left(\frac{f}{\pi}\right), \tag{1}$$

where Δ_0 is the value of Δ in vacuum and M, S refer, respectively, to the mass and surface area (including the ends) of the crystal, while f is the resonant frequency.[6]

RESULTS

Figures 1 and 2 show the quantity $\eta\rho$ *vs* T obtained by this method over the temperature range 1.3°K (the lowest currently attainable with our equipment) to 4.2°K. Several different "runs" are included in Fig. 1 to illustrate the reproducibility of the data. For the crystal employed in these measurements $\Delta_0 \approx 2.4 \times 10^{-7}$, and this was only about 3% of the value of Δ at 2.19°K. We have been able to make a large number of measurements in the immediate vicinity of the λ point, and it is evident from these data that $\eta\rho$ changes far more rapidly in this region than has heretofore been supposed.*

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[1] A. deTroyer *et al.*, Physica **17**, 50 (1951).

[2] A. C. Hollis-Hallett, Proc. Roy. Soc. (London) **A210**, 404 (1952).

[3] E. L. Andronikashvili, J. Exptl. Theoret. Phys. U.S.S.R. **18**, 429 (1948).

[4] J. G. Dash and R. Dean Taylor, Phys. Rev. **105**, 7 (1957).

[5] See, for example, T. W. Read, Phys. Rev. **58**, 371 (1940).

[6] To obtain Eq. (1) we have assumed lamellar flow and made use of the fact that the characteristic length $(\eta/\rho f)^{\frac{1}{2}}$ occurring in the Navier-Stokes hydrodynamic equation is very small compared to the crystal dimensions. It has come to our attention that this method of measuring viscosity has been described by Mason, Am. Soc. Mech. Engrs. **69**, 359 (1948). The formula given there for $\eta\rho$ is equivalent to Eq. (1).

* *Note added in proof.*—It has been remarked recently [R. D. Taylor and J. G. Dash, Phys. Rev. **106**, 398 (1957)] that the behavior of the quantity $(1/\eta)(\partial\eta/\partial T)$ in the neighborhood of the λ point may be of importance in ascertaining the order of the

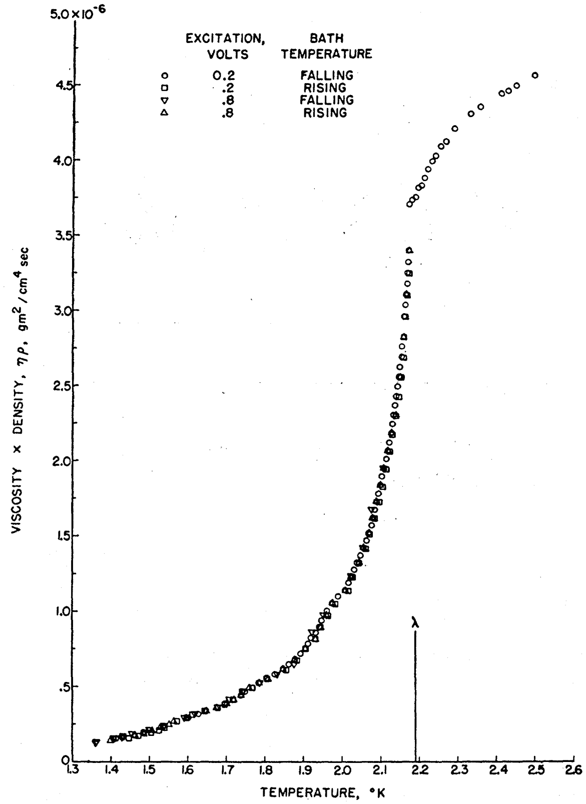

FIG. 1. Values of $\eta\rho$ as measured with the torsional-crystal method. Temperatures were determined according to the "1948" scale for liquid helium. Above the λ point the pressures were corrected to include the height of the fluid above the crystal.

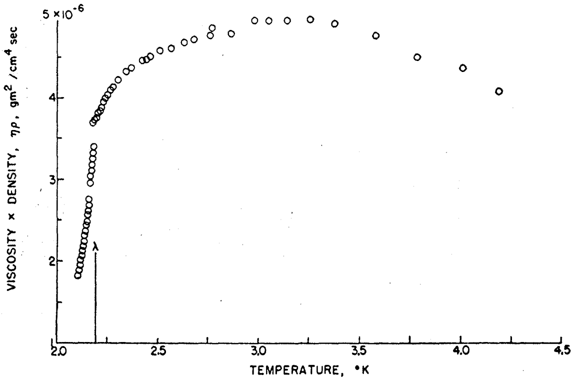

FIG. 2. Values of $\eta\rho$ in the helium I region.

It is also noteworthy that the magnitude of $\eta\rho$ is in good agreement with the values reported with the oscillating disk method. At 32 kc/sec the width of the "boundary layer" $(\eta/\rho f)^{\frac{1}{2}}$ is less than a micron, and we attribute the accuracy of these measurements to extreme care in providing an optically smooth and frost-free surface for the crystal.

Figure 1 also shows that the values of $\eta\rho$ obtained at two different amplitudes of vibration appear to coincide within experimental error. To investigate still further the question of a possible velocity dependence, measurements were made of $\eta\rho$ *vs* applied voltage at several temperatures, 1.34°, 1.40°, 1.94°, and 2.14°K. The voltage was varied from 0.002 volt to 0.5 volt, corresponding to a variation in the rms velocity of a point on rim of the cylinder from 0.1 cm/sec to 25 cm/sec at 1.34°K, and to lower velocities at the higher temperatures. No evidence of a velocity dependence was obtained over the entire range of velocities at these temperatures. The sensitivity of the apparatus was such that even at the lowest velocity $\eta\rho$ was still measurable to within 5%.

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He I-He II transformation. In our measurements this quantity is clearly discontinuous at the λ point. Approaching from above the λ point $(1/\eta)(\partial\eta/\partial T)$ has the value 1.60 ± 0.05 deg^{-1}, while from below $(1/\eta_n)(\partial\eta_n/\partial T)$ has a value *greater* than 30 deg^{-1}, where the subscript n denotes the normal component. Only a lower limit has been assigned here because η_n changes so very rapidly during the last few millidegrees that even the existence of a discontinuity in η_n proper at the λ point cannot be excluded.