Measurements of Proton Strength Functions*

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HE complex-potential model of the nucleus¹ has been successful in explaining the variation with atomic weight of the neutron strength function² (the average reduced partial width of nuclear energy levels divided by their spacing). A maximum in the S-wave neutron strength function is well established at $A \approx 55$. A previous search from A = 44 to A = 64revealed no corresponding maximum in the S-wave proton strength function.³ We have extended these measurements using a different technique and find a maximum at $A \approx 75$. Independently from our work Weisskopf and Margolis⁴ have found that adding a Coulomb correction to an intrinsic proton potential well, which differs in depth only very slightly from that used for neutrons, shifts this maximum from $A \approx 55$ to about 70.

The yield from (p,n) reactions has been used to obtain proton strength functions for $37 \leq A \leq 133$. The quantity calculated is defined by

$$\langle (\gamma^2)_{AV}/\bar{D}\rangle_{AV} \equiv Y_{\text{thick}} \left\{ 4\pi^2 \sum_l \left[(2l+1) \int_{E_0}^E S\lambda P_l dE \right] \right\}^{-1}.$$

Here Y_{thick} is the yield from a target thick compared to the range of the incident protons, l is the angular momentum of the incident protons, E_0 is the threshold energy, S is the reciprocal of the stopping power in atoms cm⁻² Mev⁻¹, $2\pi\lambda$ is the wavelength of the protons, and $P_l = 1/A_l^2$ is their Coulomb penetrability. Measurements on thick targets of twenty-eight elements were made using proton energies up to 4 Mev from the Argonne Van de Graaff accelerator. The "long counter" employed was calibrated against a standard RaBe source. Figure 1 shows the results of these measurements. The peak at $A \approx 70$ (or ≈ 75 if a correction is added for the incident proton energies) is in agree-



FIG. 1. Graph of $\langle (\gamma^2)_{AV}/\bar{D} \rangle_{AV}$ vs atomic weight of target nuclide. The several points at one value of A indicate determinations at several bombarding energies.



FIG. 2. Graph of $\langle (\gamma^2)_{AV}/\bar{D} \rangle_{AV}$ vs X as defined in the figure. $B = Ze^2/R$ is the barrier height. The same type of symbols represent determinations at different bombarding energies but for the same target nuclide. The symbols are repeated for every fourth target element used.

ment with the calculated prediction for an S-wave maximum.

Figure 2 shows the data with the Coulomb correction included in the abscissa. A nuclear radius of 1.45×10^{-13} $A^{\frac{1}{2}}$ cm was used here as well as in the penetrability calculations. A charge distributions of constant density and a radius of $1.20 \times 10^{-13} A^{\frac{1}{3}}$ cm was assumed in computing the Coulomb correction to a 44-Mev intrinsic potential by using the WKB approximation. These assumptions are more or less equivalent to those of reference 4. The curve was calculated by assuming a resonant S-wave strength function¹ and a "black nucleus" strength function for P- and D-wave protons, but neglecting higher *l*-values. It is not quite clear whether the slight indications of peaks at $A \approx 55$ and \approx 110 and an increase at $A \approx$ 130 could be interpreted as maxima in the strength functions for protons with higher angular momenta.

It is estimated that the strength functions are accurate to better than 50% and that the relative accuracy for most of them is better than 25%. After allowing for these uncertainties, the evidence for a maximum at $A \approx 75$ is good. The agreement with the independent predictions based on the complex-potential model seems excellent. It would appear that, within the accuracy of the assumptions on which the calculations are based, the proton and the neutron potentials are equal in depth provided the same radii are taken for both.

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⁴ B. Margolis and V. F. Weisskopf, Phys. Rev. 107, 641 (1957), following letter.

Proton-Width Strength Function*

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HE ratio $\bar{\Gamma}_n^0/\bar{D}$ of the reduced neutron width divided by the level distances, when averaged over low-lying resonance levels (usually referred to as neutron-strength function), shows a characteristic behavior as a function of A with maxima near $A_n \simeq 55$ and $A_n \simeq 170$. These maxima come from the establishment of standing waves within the nucleus when

$$K_n R = (n + \frac{1}{2})\pi, \quad K_n = (2mV_n/\hbar^2)^{\frac{1}{2}},$$
 (1)

where K_n is the wave number of the incoming particle within nuclear matter for particles with zero incident energy; V_n is the depth of the nuclear potential well for neutrons which, in first order, is considered constant over the nucleus.

Similar maxima should occur for the proton strength function Γ_{p}/\bar{D} , as was pointed out by Schiffer and Lee.¹ In this case Γ_p^0 is the reduced proton width, which is the actual width corrected for Coulomb penetration and reduced to a fixed energy. However, the potential well V_p for protons is different from V_n on two accounts. It is less deep than the well for neutrons, and the part which comes from the electrostatic force has a characteristic variation with the radial coordinate r. We have, in fact, $V_p = V_p^0 - [\frac{3}{2} - \frac{1}{2}(r^2/R^2)](Ze^2/R)$, where V_{p^0} is the depth without Coulomb force. The average value of V_p over the nucleus is $\overline{V}_p = V_p^0 - (6/5)(Ze^2/R)$. Maxima in $\bar{\Gamma}_{p}^{0}/\bar{D}$ should occur whenever

$$\hbar^{-1} \int_{0}^{\pi} (2mV_{p})^{\frac{1}{2}} dr \\ \approx \left(\frac{2m}{\hbar^{2}}\right)^{\frac{1}{2}} \left(V_{p}^{0} - \frac{4}{3} \frac{Ze^{2}}{R}\right)^{\frac{1}{2}} R = \left(n + \frac{1}{2}\right) \pi.$$

p

We therefore get for protons an equation similar to (1) with

$$K_p = \left[\frac{2m}{\hbar^2} \left(\bar{V}_p - \frac{2}{15} \frac{Ze^2}{R}\right)\right]^{\frac{1}{2}}.$$

We obtain a simple estimate of \overline{V}_p and V_n from the separation energies S_n and S_p of neutrons or protons.

The following relation must hold: $T_F + S = \overline{V}$, where T_F is the kinetic energy on top of the Fermi distribution: $T_F = [(9/2)\pi]^{\frac{3}{2}} (\hbar^2/2mr_0^2) (n/2A)^{\frac{3}{2}} = C(n/2A)^{\frac{3}{2}}$. Here *m* is the mass of a nucleon, r_0 is given by $R = r_0 A^{\frac{1}{2}}$, and n is the number of protons or neutrons, respectively. Hence, we get

$$\overline{V}_p/V_n = [(T_F)_p + S_p]/[T_F]_n + S_n],$$

and from this formula

$$\frac{K_p}{K_n} = \left[\frac{C(n_p/2A)^{\frac{2}{3}} + S_p - (2/15)(Ze^2/R)}{C(n_n/2A)^{\frac{2}{3}} + S_n}\right]^{\frac{1}{3}}.$$
 (2)

The ratio of the values of A at which maxima occur is then given by $(A_p/A_n) = (K_n/K_p)^3$.

Expression (2) can be roughly evaluated for nuclei 55 < A < 70 by putting $r_0 = 1.2$ (C=83.3 Mev), $S_{p} \approx S_{n} \approx 8$ Mev, $(n_{p}/2A) = 0.225$ and $(n_{n}/2A) = 0.275$. We then get $(A_p/A_n) = 1.24$, and, with the well-known value of $A_n \approx 55$, we expect $A_p \approx 68$. According to these rough estimates the next maximum can be expected near $A_{p} \approx 230$. Recent measurements by Schiffer and Lee¹ seem to bear out the prediction of a maximum near A = 68.

* This work was supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission. ¹ J. P. Schiffer and L. L. Lee [Phys. Rev. 107, 640 (1957)],

preceding letter.

Electron-Neutrino Angular Correlation in the Positron Decay of Argon 35*

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HE positron-neutrino angular correlation coeffi-I cient has been measured for the decay of A³⁵. Since this decay occurs mainly through the Fermi matrix element,¹ the correlation should be sensitive to the ratio of the two Fermi coupling constants $|g_V^2|/|g_S^2|$. Two different experiments have been carried out, and in each case, the results have shown that the vector interaction is dominant.

A³⁵ has a half-life of 1.8 sec and decays by the emission of positrons with a maximum kinetic energy of 4.9 Mev into its mirror nucleus Cl³⁵. Although most of the beta transitions go to the ground state of Cl³⁵, about 7% of the decays go through beta transitions to the first two excited levels of Cl³⁵ which then decay by emission of gamma rays.1

The A³⁵ was produced by the reaction $S^{32}(\alpha, n)A^{35}$ in a target of gaseous SF₆. A steady stream of SF₆ carried the radioactive A³⁵ through a pipe line to the apparatus outside the shielding walls of the cyclotron.