renormalized coupling constant and it is still equal and opposite for positive and negative scattering amplitudes.

Furthermore, poles at $\omega=E_\Lambda$ and $\omega=E_\Sigma$ (where $E_H$ represents the mass difference of hyperons and nucleons) do not contribute because the interaction Hamiltonian conserves strangeness, and therefore matrix elements of the current between nucleon and hyperon physical states vanish identically.

We conclude that the $\Lambda$ and $\Sigma$ fields, interacting directly with pions, and through the $K$ field with nucleons, do not alter the structure of the dispersion relations. The validity of this result is not limited to the very simple Hamiltonian assumed above; in fact it turns out that the same conclusions are reached by following the procedure of Goldberger which leads to the conventional dispersion relations.

It seems therefore that, in the light of our present knowledge about strong interactions, the results of the Bologna group cannot be explained. Should they be confirmed, very drastic changes to the present theory would be necessary, in our opinion, such as dropping the charge symmetry of some strong interactions or the principle of microscopic causality.

Details of the calculations reported above will be published elsewhere.

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## Spin-Orbit Coupling and Tensor Forces*

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We have investigated, in the framework of Bruckner's theory,[1] if the spin-orbit coupling which is responsible for the shell model can be accounted for by second-order effects of the tensor forces. The spin-orbit coupling which is computed in this way is found, with some restrictions, to be an order of magnitude too small. This result is in disagreement with the conclusions of Kisslinger.[2]

The nucleus is treated in the Thomas-Fermi approximation. First, we define a modified reaction amplitude for the collisions of 2 particles $i$ and $j$ inside nuclear matter of uniform density, pictured as a Fermi gas. This amplitude has a part $\theta_{ij}$ which is linear in the spins of these particles

$$\theta_{ij}=[\alpha+\beta(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)](\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\mathbf{A}_{ij}. \tag{1}$$

$\theta_{ij}$ depends on the density of the nuclear matter. Then we go to nuclear matter of varying density. A matrix element of the spin-dependent part $\boldsymbol{\sigma}_i\cdot\mathbf{V}$ of the optical potential for the $i$th nucleon is, between states of momenta $\mathbf{k}_i$ and $\mathbf{k}_i'$,

$$(\mathbf{k}_i'|\mathbf{V}|k_i)=4\sum_{\psi_j}(2\pi)^{-6}\int d^3k_j'\int d^3k_j\psi_j^*(\mathbf{k}_j')\times[\alpha(\mathbf{k}_i'\mathbf{k}_j'|\mathbf{A}|\mathbf{k}_i\mathbf{k}_j)-\tfrac{1}{2}(\alpha+3\beta)\times(\mathbf{k}_j'\mathbf{k}_i'|\mathbf{A}|\mathbf{k}_i\mathbf{k}_j)]\psi_j(\mathbf{k}_j), \tag{2}$$

where the $\psi_j(\mathbf{k}_j)$ are the Fourier transforms of the one-particle orbital states $\psi_j(\mathbf{r}_j)$. Equation (2) can be expanded in a power series involving successive derivatives of the density

$$\rho(\mathbf{r}_j)=4\sum\psi_j^*(\mathbf{r}_j)\psi_j(\mathbf{r}_j). \tag{3}$$

Keeping only the term containing the first derivative of (3), one obtains a spin-orbit potential[3,4]

$$\boldsymbol{\sigma}_i\cdot\mathbf{V}=a(1/r_i)(\partial\rho/\partial r_i)(\mathbf{l}_i\cdot\boldsymbol{\sigma}_i), \tag{4}$$

where $a$ is a function of momentum $k_i$ and, through $\mathbf{A}$, of the local Fermi momentum $f$ at the point $r_i$. For the last bound nucleon, we set $k_i=f(r_i)$. The quantity $a$ depends only on the values of (1) for small-angle scattering and scattering through angles near $\pi$

$$a=a_0+a_1=i[\alpha E_0(k_i,f)-\tfrac{1}{2}(\alpha+3\beta)E_1(k_i,f)], \tag{5}$$

where $E_0$ and $E_1$ are defined by

$$\lim_{\mathbf{k}_i'\to\mathbf{k}_i}\left(\frac{1}{8\pi^3}\right)\left(\frac{3}{4\pi f^3}\right)\int_{k_j<f}d^3k_j(\mathbf{k}_i'\mathbf{k}_j'|\mathbf{A}|\mathbf{k}_i\mathbf{k}_j)=\delta(\mathbf{k}_i'+\mathbf{k}_j'-\mathbf{k}_i-\mathbf{k}_j)(\mathbf{k}_j'-\mathbf{k}_j)\times\mathbf{k}_iE_0(k_i,f), \tag{6}$$

$$\lim_{\mathbf{k}_i'\to\mathbf{k}_i}\left(\frac{1}{8\pi^3}\right)\left(\frac{3}{4\pi f^3}\right)\int_{k_j<f}d^3k_j(\mathbf{k}_j'\mathbf{k}_i'|\mathbf{A}|\mathbf{k}_i\mathbf{k}_j)=\delta(\mathbf{k}_i'+\mathbf{k}_j'-\mathbf{k}_i-\mathbf{k}_j)(\mathbf{k}_j'-\mathbf{k}_j)\times\mathbf{k}_iE_1(k_i,f). \tag{7}$$

The relevant matrix elements of (1) were computed from a tensor force, using the second Born approximation and taking into account the exclusion principle for intermediate states which are already occupied by other nucleons, in the model of a uniform Fermi gas. The computation involves complicated integrals which can, however, be transformed to simple numerical one-variable integrals.

The experimental value of $a$ is about $+70$ Mev$\times(10^{-13}$ cm)[5,4,6] The values computed for the direct part $a_0$ of (5) are very sensitive to the range $(1/\mu)$ of the tensor force; $a_0$ increases with this range. For $f=1.27\times10^{-13}$ cm (this is the value far from the nuclear surface), $a_0=-4$ for an exponential tensor force[6] and $a_0<1$ for a Yukawa tensor force.[7] Actually, $f$ ought to be smaller in the nuclear surface, in the Thomas-Fermi approximation, and this would lead to even algebraically smaller values of $a_0$. In the case of a potential $r^2\exp(-\mu^2r^2)$, adjusted to look like the Gartenhaus potential,[8] $a_1=-6$.

The calculations have also been done, neglecting now the exclusion principle in intermediate states, in order to estimate the importance of this effect. It is found that the Pauli principle had diminished $a$ by a factor $\sim 5$ (although this factor is also very sensitive to the details of the tensor force). Anyhow, with all reasonable values for the parameters, $a$ remains negative or, if positive, too small.[9]

There is a possibility that spin-orbit forces in shell structure would involve more than two particles at a time.[10] Such effects would appear in a Brueckner formulation in the momentum-nonconserving terms[1] which have not been considered here. If our approximations are justified, and if more than two-body terms were inoperative, we should admit the existence of two-body elementary spin-orbit forces.

A more detailed account will be published later.

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## Magnetohydrodynamic Waves in the Atomic Nucleus

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HYDRODYNAMIC models are important for the understanding of some properties of the atomic nucleus. Since the nucleus has a magnetic moment, hydrodynamic phenomena should be affected by a magnetic field. Hence, the classical analogy should not be ordinary hydrodynamics but instead magnetohydrodynamics. It is of interest to calculate the order of magnitude of possible magnetohydrodynamic resonance frequencies. The electric conductivity is assumed to be infinite.

The magnetohydrodynamic velocity $V_{\mathrm{MH}}$ is

$$V_{\mathrm{MH}}=H(4\pi\rho)^{-\frac{1}{2}},$$

where $H$ is the magnetic field and $\rho$ the density. Suppose that a nucleus with radius $r$ has a uniform magnetization giving a magnetic moment of $B$ nuclear magnetons. The magnetic field inside the nucleus is

$$H=2B\mu_0r^{-3},$$

where $\mu_0=eh(4\pi M_pc)^{-1}=5\times10^{-24}$ gauss cm$^3$, and $r=r_0A^{\frac{1}{3}}$, with $r_0=1.5\times10^{-13}$ cm, $A$ = atomic weight, and $M_p$ = mass of a nucleon. Since the density is

$$\rho=AM_p(\tfrac{4}{3}\pi r^3)^{-1}=\tfrac{3}{4}\pi M_pr_0^{-3}=10^{14}\text{g cm}^{-3},$$

we get

$$V_{\mathrm{MH}}=\frac{B}{A\sqrt{3}}\left(\frac{eh}{2\pi cM_p^{\frac{1}{2}}r_0^{\frac{3}{2}}}\right)=\frac{B}{A}\times8\times10^7\text{ cm sec}^{-1}.$$

One of the lowest modes of oscillation is a torsional standing wave along the magnetic axis. The wavelength is somewhat smaller than $4r$, which means a frequency

$$\nu=V_{\mathrm{MH}}(4r)^{-1}.$$

This corresponds to an energy

$$W=h\nu=CW_{\mathrm{MH}},$$

where

$$W_{\mathrm{MH}}=\frac{e\hbar^2}{CM_p^{\frac{1}{2}}r_0^{\frac{5}{2}}}=10^{-6}\text{ erg}=0.5\text{ Mev}$$

and

$$C=\frac{\pi}{2\sqrt{3}}\left(\frac{B}{A^{\frac{4}{3}}}\right).$$

For reasonable values of $A$ and $B$, the value of $W$ is of the order of a few kev. If this classical phenomenon has a quantum-mechanical correspondence, we may expect that a final theory of the nucleus should contain fine-structure terms of this type.

## Charges and Parities of Elementary Particles

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IN this letter we outline a theory of elementary particles which has the following consequences:

1. Every particle (elementary or compound) has charge $\pm ne_0$, where $n$ is an integer or zero and $e_0$ is a constant.

2. The direct interaction between a particle and the electromagnetic field is invariant under either a space or time reflection (bosons) or a space-time reflection (fermions).

3. All singly charged bosons are pseudoscalars (or vectors, etc.).