Muon Decay with Parity Nonconserving Interactions and Radiative Corrections in the Two-Component Theory*

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The decay of a polarized muon is studied in the case of the general four-component neutrino theory with the most general parity-nonconserving interaction. A three-parameter formula for the decay-electron distribution is obtained as a generalization of the Michel formula for an unpolarized muon. This general formula is examined to determine to what extent the observed spectrum enables one to decide whether any particular theory is correct or not. It is seen, among other things, that by the observation of the muon decay spectrum alone one cannot test the validity of the two-component neutrino theory. To facilitate a possible accurate experimental test of the two-component neutrino theory, the radiative correction for the decay of a polarized muon is worked out to the lowest order in α. Although the correction is rather complicated, it can still be expressed approximately by the three-parameter formula mentioned above. It is found, in particular, that the corrected Michel parameter for the two-component theory is 0.70_6 when a neutrino and an antineutrino are emitted in the final state, which is 6% smaller than the value 0.75 predicted by the simple theory.

1. INTRODUCTION

THE suggestion of Lee and Yang[1] on the possible nonconservation of parity in weak interactions has been verified beyond doubt by the observation of strong left-right asymmetry of secondary particles in the processes such as the β decay[2] and the π-μ-e decay.[3] Lee and Yang,[4] Salam,[5] and Landau[6] have proposed independently that the strong violation of parity conservation in these reactions in which the neutrino participates could be explained if one assumes that the neutrino always violates the parity conservation because of its intrinsic nature. That such a theory of the neutrino, called the two-component theory, is possible within the frame work of relativity, has been known for a long time[7] but has not attracted any attention because of its failure to conserve parity. It is quite interesting that the new experimental evidence seems to imply the existence of just such a particle.

Predictions of the two-component theory are in general much more specific than those of the ordinary theory of the neutrino. Thus many of its consequences are subject to direct experimental confirmation. Although evidence available so far is strongly in favor of this theory, more detailed work will be required before a definite conclusion is obtainable about the validity of the two-component theory.

As an attempt in this direction, two separate considerations are presented in this paper concerning the predictions of two-component theory on the muon decay process. The first is how much information one can derive from the study of the experimental spectrum of muon decay. For this purpose, the muon decay spectrum is discussed in Sec. 2 in the case of the general four-component neutrino theory with the most general parity-nonconserving interaction. The decay spectrum of a completely polarized muon can be described by a set of three parameters when the mass of the electron is neglected, in close analogy with the one-parameter formula of Michel[8] for an unpolarized muon. Usually there are infinitely many possible choices of decay coupling constants for given values of these parameters. Thus one can find various four-component interactions which give exactly the same decay spectrum as that of the two-component theory. This argument is independent of whether the two neutrinos in the final state are identical or not. One cannot therefore tell more than whether the two-component theory is consistent or not by looking at the shape of the decay spectrum alone.

Secondly the accuracy of the predictions of the two-component theory of muon decay is improved by taking the effect of the radiative correction into account. This correction would be necessary for the precise comparison of theory and experiment since the radiative correction is not at all insignificant in this particular case, being of the order of

$$\alpha[\ln(\mu/m)]^2 \approx (137)^{-1}\times 28.4, \tag{1.1}$$

rather than α itself, where μ and m are the masses of muon and electron, respectively. Detailed calculation shows that its effect on the shape of the decay spectrum is, to the lowest order in α, about $+10\%$ for the lower momenta of electron and -4% at the upper end. With the inclusion of the radiative correction, the accuracy of the corrected theoretical spectrum will be better than 1% over the whole range of electron momentum. Qualitatively speaking, the radiative correction has a tendency to shift the electron spectrum to the low-

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[2] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
[3] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).
[4] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[5] A. Salam, Nuovo cimento **5**, 299 (1957).
[6] L. Landau, Nuclear Phys. **3**, 127 (1957).
[7] W. Pauli, *Handbuch der Physik* (Verlag Julius Springer, Berlin, 1933), Vol. 24, pp. 226–227.
[8] L. Michel, Proc. Phys. Soc. (London) **A63**, 514 (1950).

momentum side. Although the corrected spectrum is a rather complicated function of the electron momentum, it can still be approximated by a three-parameter formula given in Sec. 2. It is found, in particular, that the corrected Michel parameter for the two-component theory is 0.70_6 instead of 0.75 predicted by the simple theory.

2. MUON DECAY SPECTRUM IN THE GENERAL CASE

Let us first derive the spectrum of muon decay for the case of the four-component neutrino theory and examine the predictions of the two-component theory in the light of the former.

In the general case of the four-component theory, the muon decay process,

$$\mu \rightarrow e+\nu+\bar{\nu}, \tag{2.1}$$

is described by the Fermi interaction

$$H'=\sum_i[g_i(\bar{\psi}_e\Gamma_i\psi_\mu)(\bar{\psi}_\nu\Gamma_i\psi_\nu) +g_i'(\bar{\psi}_e\Gamma_i\psi_\mu)(\bar{\psi}_\nu\Gamma_i\gamma_5\psi_\nu)]+\text{H.c.}, \tag{2.2}$$

where Γ_i stands for the five Dirac matrices[9]

$$\begin{aligned} &\Gamma_S=1, && \Gamma_P=-i\gamma_5,\\ &(\Gamma_V)_\rho=\gamma_\rho, && (\Gamma_A)_\rho=-i\gamma_\rho\gamma_5,\\ &(\Gamma_T)_{\mu\nu}=(i/2\sqrt{2})(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu), \end{aligned} \tag{2.3}$$

and "H.c." means "Hermitian conjugate." For a muon at rest with its spin completely polarized, the electron distribution is given by the three-parameter formula[10]:

$$dN(x,\theta)=A\{3(1-x)+2\rho(\tfrac{4}{3}x-1) -\xi\cos\theta[(1-x)+2\delta(\tfrac{4}{3}x-1)]\}x^2dxd\Omega, \tag{2.4}$$

the electron mass being neglected compared with its momentum.[11] The parameter x is the electron momentum measured in terms of the maximum electron momentum, θ is the angle between the electron momentum and the spin direction of muon, and

$$A=\left(\frac{\mu^5}{3\times2^9\times\pi^4}\right)(a+4b+6c), \tag{2.5}$$

where

$$\begin{aligned} a&=|g_S|^2+|g_S'|^2+|g_P|^2+|g_P'|^2,\\ b&=|g_V|^2+|g_V'|^2+|g_A|^2+|g_A'|^2,\\ c&=|g_T|^2+|g_T'|^2. \end{aligned} \tag{2.6}$$

The quantity ρ is the Michel parameter[8] defined by

$$\rho=(3b+6c)/(a+4b+6c); \tag{2.7}$$

The parameter δ determines the shape of the $\cos\theta$ dependent part of the spectrum and plays a role similar to that of the Michel parameter ρ. It is related to the coupling constants by

$$\delta=(3b'+6c')/(-3a'+4b'+14c'), \tag{2.8}$$

where

$$\begin{aligned} a'&=g_Sg_P'^*+g_P'g_S^*+g_Pg_S'^*+g_S'g_P^*,\\ b'&=g_Vg_A'^*+g_A'g_V^*+g_Ag_V'^*+g_V'g_A^*,\\ c'&=g_Tg_T'^*+g_T'g_T^*. \end{aligned} \tag{2.9}$$

Finally, ξ is defined by

$$\xi=(3a'-4b'-14c')/(a+4b+6c), \tag{2.10}$$

and characterizes the magnitude of the $\cos\theta$ dependence of the electron distribution.

In many cases, it will be the integrated spectrum,

$$N(x,\theta)d\Omega=\int_x^1\frac{dN}{dx}dx, \tag{2.11}$$

that is conveniently compared with experiments. One finds from (2.4) that

$$N(x,\theta)=\tfrac{1}{4}A[1-4x^3+3x^4+(8/3)\rho(x^3-x^4)] \times[1-\xi Q(x)\cos\theta], \tag{2.12}$$

where

$$Q(x)=\frac{1+x+x^2-3x^3+8\delta x^3}{3(1+x+x^2-3x^3)+8\rho x^3}. \tag{2.13}$$

The three parameters in the formula (2.4) are functions of ten complex numbers. However, it is not difficult to find some of their characteristic features. We note that the inequalities

$$|\xi\delta|\leqq\rho, \tag{2.14}$$

$$0\leqq\rho\leqq1, \tag{2.15}$$

and

$$0\leqq|\xi|\leqq3-\tfrac{2}{3}\rho, \tag{2.16}$$

always hold because of $|a'|\leqq a$, $|b'|\leqq b$, $|c'|\leqq c$.

All relations before (2.15) are also valid in the case of emission of two identical neutrinos:

$$\mu\rightarrow e+\nu+\nu. \tag{2.17}$$

In this case, we have in addition the following simplification:

$$g_V=g_T=g_T'=g_A'=0. \tag{2.18}$$

This implies that

$$c=0,\quad c'=0, \tag{2.19}$$

and thus (2.15) and (2.16) are superseded by the stronger relations

$$0\leqq\rho\leqq\tfrac{3}{4},\quad\text{and}\quad 0\leqq|\xi|\leqq3-(8/3)\rho. \tag{2.20}$$

If one decomposes ψ_ν into two parts as follows:

$$\psi_\nu=\varphi+\chi, \tag{2.21}$$

[9] For the definition of the γ_ρ's used in this paper, see R. P. Feynman, Phys. Rev. **76**, 769 (1949).

[10] All formulas in this paper are written in the form appropriate for the description of the μ^--e^- decay. Formulas for the μ^+-e^+ decay are obtained by changing the signs of the $\cos\theta$ terms in (2.4), (2.12), (3.3), etc.

[11] After this work was completed, a preprint of a paper by C. Bouchiat and L. Michel [Phys. Rev. **106**, 170 (1957)] became available to us, in which a formula equivalent to our Eq. (2.4) has been derived.

where

$$\varphi = \tfrac{1}{2}(1+\gamma_5)\psi_\nu, \quad \chi = \tfrac{1}{2}(1-\gamma_5)\psi_\nu, \tag{2.22}$$

it is easy to see that, for different neutrinos, the S, T, and P interactions contain only cross terms of φ and χ, while the V and A interactions have no cross term. Since the two-component neutrino theory can be regarded as a special case of the general theory where ψ_ν satisfies

$$\gamma_5\psi_\nu = -\psi_\nu, \quad \text{i.e.,} \quad \varphi = 0, \tag{2.23}$$

only the V and A interactions are possible for nonidentical neutrinos.[4] Equation (2.23) further requires that

$$g_V' = -g_V, \quad g_A' = -g_A. \tag{2.24}$$

Thus, in the two-component theory, the electron distribution (2.4) is reduced to

$$dN(x,\theta) = \tfrac{1}{2}A[3-2x+\xi\cos\theta(1-2x)]x^2dxd\Omega, \tag{2.25}$$

when one uses (2.24) and puts $g_S = g_T = g_P = g_S' = g_T' = g_P' = 0$. It corresponds therefore to a special case of the general theory where

$$\rho = \delta = \tfrac{3}{4}. \tag{2.26}$$

The parameter ξ is then reduced to

$$\xi = (g_Vg_A^* + g_Ag_V^*)/(|g_V|^2 + |g_A|^2). \tag{2.27}$$

It obviously satisfies the relation

$$0 \leqq |\xi| \leqq 1. \tag{2.28}$$

The equalities and inequalities derived above lead us to various conclusions, some of which are given in the following:

(a) If the observed spectrum does not satisfy some of the relations (2.14), (2.15), and (2.16), it would indicate that our Hamiltonian (2.2) is inadequate for the description of muon decay. If the condition (2.20) is not satisfied, the emission of two identical neutrinos (2.17) is ruled out.

(b) If $|\xi| > 1$, the two-component neutrino theory must be abandoned.

(c) As is seen from (2.12), the asymmetry of the integrated spectrum is described by $-\xi Q(x)$. Using (2.14) and (2.15), one easily finds that

$$|\xi Q(x)| = \left|\frac{\xi(1+x+x^2-3x^3)+8\xi\delta x^3}{3(1+x+x^2-3x^3)+8\rho x^3}\right| \leqq 1, \tag{2.29}$$

for any x in the range (0,1). Thus the magnitude of the asymmetry cannot exceed 1, which is trivial since the decay spectrum must be non-negative for all energies and angles of the emitted electrons.

(d) Equation (2.29) shows that the integrated asymmetry at the upper end ($x=1$) of the spectrum is $|\xi\delta|/\rho$ whose maximum value is 1. The asymmetry of the complete integral spectrum is given by $\frac{1}{3}|\xi|$. Thus, if the two-component theory is correct, this asymmetry cannot exceed the value $\frac{1}{3}$. It is noteworthy that a much larger asymmetry of the integrated electron distribution is possible in the general case than is permitted in the two-component theory. If $\rho = 3\delta$, the integrated spectrum shows no momentum dependence as is obvious from (2.29). In particular, the asymmetry is ± 1 for all x if $\xi = \pm 3$. However, $\rho = \delta = 0$ in this case.

(e) The two-component theory predicts $\rho = \delta = \frac{3}{4}$, $0 \leqq |\xi| \leqq 1$. However, exactly the same prediction can be made in the general theory if one chooses a suitable set of coupling constants. In fact, it is immediately seen from (2.7) and (2.8) that

$$a = 2c, \quad \text{and} \quad a' = 2c' \tag{2.30}$$

are the necessary and sufficient conditions for this purpose. Since the muon decay spectrum is completely characterized by the three parameters ρ, δ, and ξ, this implies that it is impossible to distinguish the two-component theory from the four-component theory by looking at the muon decay spectrum if the latter satisfies (2.30). Thus the experimental verification of the spectrum (2.25) by itself does not unambiguously select the two-component neutrino theory.

(f) The above argument is based on the possibility of an arbitrary choice of a and a' only if (2.30) is satisfied. It is interesting to notice that the theory is not completely determined even if one assumes $\rho = \delta = \frac{3}{4}$ and $a = a' = 0$. In this case we obtain

$$\xi = -\frac{g_Vg_A'^* + g_A'g_V^* + g_Ag_V'^* + g_V'g_A^*}{|g_V|^2 + |g_V'|^2 + |g_A|^2 + |g_A'|^2}. \tag{2.31}$$

If one assumes (2.24), this ξ of course reduces to (2.27) of the two-component theory. Of particular interest is the case

$$g_V = g_A' = 0, \tag{2.32}$$

which is required when the two neutrinos emitted are identical. The other requirements of (2.18) are already satisfied by the assumption $c=0$. In this case we obtain

$$\xi = -(g_V'g_A^* + g_Ag_V'^*)/(|g_V'|^2 + |g_A|^2). \tag{2.33}$$

Thus, exactly the same spectrum as that of the two-component neutrino theory of Lee and Yang is obtained if one assumes identical neutrinos in the muon decay and $a = c = 0$. The maximum polarization $\xi = \pm 1$ is attained by the choice

$$g_V' = \mp g_A. \tag{2.34}$$

As was shown by Lee and Yang,[4] such a picture is not possible if there is only one spin state for the neutrino field ($\rho = \delta = 0$ in this case). Insofar as the two spin states are accessible for the neutrino, however, there is nothing to prevent the existence of such a case. Obviously it does not matter whether the neutrino is described by the Dirac theory or the Majorana theory.

(g) In the general case of two identical neutrinos, the inequality

$$3(3-4\rho)\geqq\xi(3-4\delta)\geqq-3(3-4\rho) \qquad (2.35)$$

is obtained from (2.7), (2.8), (2.10), and (2.19). This relation may be useful for the test of consistency of the assumption (2.17). In particular, it is seen that $\rho=\frac{3}{4}$ means $\delta=\frac{3}{4}$. Thus, if the experiment finds $\rho=\frac{3}{4}$ but $\delta\neq\frac{3}{4}$, the theory of identical neutrinos (including the Majorana theory) must be ruled out together with the two-component theory.

(h) The above arguments of course do not reduce at all the strong possibility that the two-component theory is the correct description for the neutrino. It only stresses a logical difficulty in deriving such a conclusion from the study of muon decay alone. It is in fact necessary to study other phenomena to settle the question. To produce a highly polarized muon from the π-μ decay, for instance, a rather special kind of parity-nonconserving decay interaction is required and the two-component theory would be the simplest solution to such a problem.

3. RADIATIVE CORRECTION IN THE TWO-COMPONENT THEORY

In this section we shall consider the effect of the radiative correction in the two-component theory of muon decay. The interaction Hamiltonian will now be

$$H=H'+H'', \qquad (3.1)$$

where

$$H''=e\bar{\psi}_\mu\gamma_\rho\psi_\mu A_\rho+e\bar{\psi}_e\gamma_\rho\psi_e A_\rho. \qquad (3.2)$$

Since the radiative interaction does not alter the properties of neutrinos, we have to consider only the V and A terms of H'. Thus the decay spectrum including the radiative correction can be written as

$$dN_r(x,\theta)=\tfrac{1}{2}A\left\{\left[3-2x+\frac{\alpha}{2\pi}f(x)\right]+\xi\cos\theta\left[1-2x+\frac{\alpha}{2\pi}h(x)\right]+6\zeta\frac{m}{\mu}\frac{1-x}{x}\right\}x^2dxd\Omega, \qquad (3.3)$$

where one neglects terms of higher order than α and m/μ, and where

$$\zeta=(|g_A|^2-|g_V|^2)/(|g_A|^2+|g_V|^2). \qquad (3.4)$$

The last term, which was neglected before since $m\ll\mu$, is included here since $m/\mu\sim\alpha$. Its actual magnitude, however, is only about a quarter of the radiative correction even if one assumes that $\zeta=1$. Since ζ is restricted by

$$\zeta^2\leqq1-\xi^2, \qquad (3.5)$$

where ξ is given by (2.27), the effect of this term would be even smaller when $|\xi|$ is close to 1 as is observed. Because of this, we shall neglect this term completely in the following discussion.

The quantities $f(x)$ and $h(x)$ represent the radiative corrections to the isotropic and $\cos\theta$ terms of (2.25), respectively. The function $f(x)$ was obtained previously by Behrends, Finkelstein, and Sirlin,[12] and $h(x)$ has been evaluated by making use of the same method. Their explicit forms are given in the Appendix. Numerical values of $(\alpha/2\pi)x^2f(x)$ and $(\alpha/2\pi)x^2h(x)$ are listed in columns 1 and 3 of Table I.

Both $f(x)$ and $h(x)$ consist of radiative corrections due to the virtual photon emission and the inner bremsstrahlung. Since the Fermi-type interactions are unrenormalizable in general, it is expected that the virtual emission of photons gives divergent results even if the ordinary renormalization of the mass and charge is carried out. It is interesting to notice that in the cases of V and A interactions, which are the only cases of interest to us, all ultraviolet divergences to order α can be removed by the renormalization of charge and mass of the muon and electron. Thus the two-component neutrino theory is distinguished from the general case by its finite radiative correction to the muon decay.

The infrared divergence due to the emission of low-frequency virtual quanta is of course cancelled by that of the real emission. The correction terms $f(x)$ and $h(x)$ are therefore finite and unambiguous for $0<x<1$. The divergence of these functions at $x=0$ is not real. It occurs simply because we have neglected the electron mass, which is certainly justifiable for $x\gtrsim0.1$. The case $x=1$ corresponds to the emission of an electron with maximum momentum. This is a singular configuration in which the emission of real quanta is prohibited by the conservation law of energy. As a result, the infrared divergence of the virtual photon is not canceled at

TABLE I. Radiative corrections to the isotropic and $\cos\theta$ terms of the muon decay spectrum and related functions.

x	$\frac{\alpha}{2\pi}x^2f(x)$ $\times10^2$	$\frac{\alpha}{2\pi}F(x)$ $\times10^2$	$\frac{\alpha}{2\pi}x^2h(x)$ $\times10^2$	$\frac{\alpha}{2\pi}H(x)$ $\times10^2$	$\frac{Q_r(x)}{Q(x)}$
0	...	2.973	...	3.260	1.0028
0.1	0.718	2.886	−0.070	3.238	1.0033
0.2	1.178	2.697	−0.108	3.186	1.0040
0.3	1.660	2.414	−0.203	3.097	1.0047
0.4	2.109	2.036	−0.393	2.923	1.0050
0.5	2.457	1.577	−0.686	2.604	1.0049
0.6	2.627	1.065	−1.062	2.083	1.0047
0.7	2.535	0.543	−1.441	1.329	1.0040
0.8	2.050	0.077	−1.631	0.390	1.0029
0.9	0.829	−0.230	−1.083	−0.490	1.0016
0.95	−0.528	−0.253	0.115	−0.665	1.0011
0.99	−3.752	−0.116	3.532	−0.339	1.0002
1.0	...	0	...	0	...

[12] Behrends, Finkelstein, and Sirlin, Phys. Rev. **101**, 866 (1956). In Table I of this reference, there is a numerical error in the value of the radiative correction for the vector case at $x=0.95$. We are thankful to Dr. K. M. Crowe for kindly pointing this out to us.

$x=1$, and $f(x)$ and $h(x)$ diverge there logarithmically. This divergence can be easily removed if one remembers that the energy resolution in actual measurements cannot be infinitely sharp.[13] Here we shall simply leave it as it is since it does no harm to the total probability of muon decay.

The isotropic and $\cos\theta$ terms of the differential spectrum (3.3) are plotted in Figs. 1 and 2 together with the corresponding terms without radiative corrections. The radiative correction reduces the magnitude of the electron distribution in the neighborhood of the upper end of the spectrum by about 4% but enhances it in most other regions. This may be interpreted as the result of a shift of the electron distribution to the low-energy side, part of the electron energy being lost in the form of a radiation. In the lowest energy region, the increase is quite large percentagewise, but its absolute magnitude is small. Some aspects of the corrected spectrum (3.3) will be discussed in the following.

(*a*) *Michel parameter.*—We shall first discuss the effect of radiative correction to the decay spectrum of an unpolarized muon. This has been discussed before for the general case[12] but we shall repeat it here in a slightly different manner. For an unpolarized muon, the electron distribution without the radiative correction is given by

$$dN(x)=4\pi A[3(1-x)+2\rho(\tfrac{4}{3}x-1)]x^2dx. \qquad (3.6)$$

ρ is $\frac{3}{4}$ in the two component theory. When the radiative correction is included, the spectrum is no longer describable by such a simple formula. The shape of the corrected curve in Fig. 1 indicates, however, that it could still be approximated with sufficient accuracy by

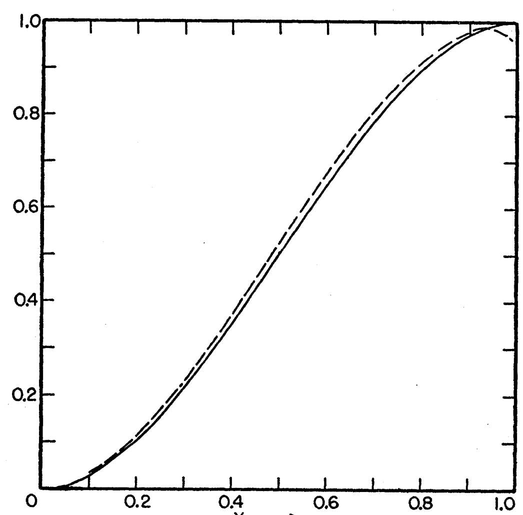

FIG. 1. The isotropic part of the muon decay spectrum in the two-component neutrino theory. The solid curve represents the uncorrected spectrum ($\rho=0.75$) for the emission of a neutrino and an antineutrino. The dashed curve is obtained by including the effect of the radiative correction to the solid curve.

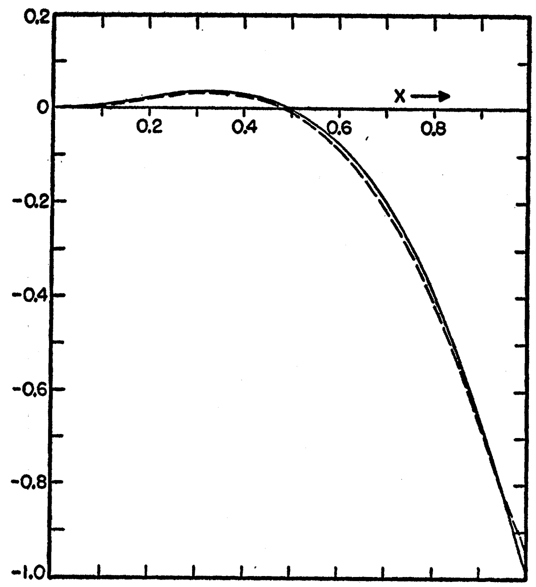

FIG. 2. The $\cos\theta$ part of the muon decay spectrum in the two-component neutrino theory. The solid curve represents the uncorrected spectrum ($\delta=0.75$) for the emission of a neutrino and an antineutrino. The dashed curve is obtained by including the effect of the radiative correction to the solid curve.

a Michel formula (3.6). To see whether this is the case or not, it is more convenient to study the quantity

$$3-2x+(\alpha/2\pi)f(x) \qquad (3.7)$$

rather than the spectrum (3.3) itself. If one neglects the last term, this is a linear function of x. Our concern is how well one can approximate (3.7) by a linear function. The function (3.7) is plotted against x in Fig. 3. As is seen immediately, (3.7) is close to a straight line for $0.3\lesssim x\lesssim 0.95$. When one fits this curve to the Michel formula in this range, the value of ρ is found to be

$$\rho=0.70_6, \qquad (3.8)$$

which is about 6% smaller than the uncorrected value.[14] It must be noted that this procedure is not unambiguous and the result depends slightly on the momentum range chosen.[15] The result (3.8) seems to be in fair agreement with the observed result.[16]

[13] See reference 12 for more detailed discussion about this.

[14] The value 0.727 for ρ given in reference 12 was based on a slightly different definition [Eq. (28a) of reference 12]. There, the experimental data was to be compared with a modified spectrum [Eq. (28c) of reference 12]. Here we suggest that the data be fitted with the uncorrected Michel formula (3.6) and compare the value of ρ thus determined with (3.8). The definition of ρ in the two cases is not identical and thus leads to different numerical values. But of course both methods of comparing theory and experiment are equivalent.

[15] If one uses the data up to the upper end of the spectrum, the resulting ρ would be even smaller than the value (3.8). But the parameter ρ would then lose its good physical meaning. If it is not convenient to neglect the data very close to the end of the spectrum for experimental reasons, it would be necessary to return to the original formula (3.3). It must be noted, however, that the procedure of Sec. 3 is restricted to the case where only the V and A interactions are present in the muon decay. For the more general

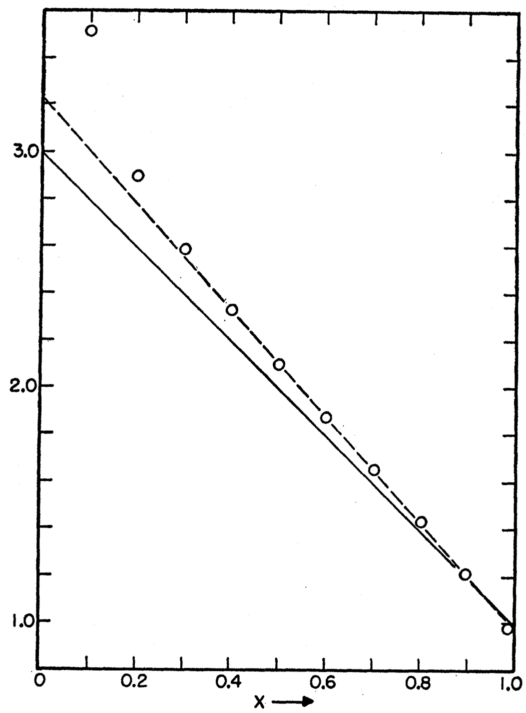

FIG. 3. The function $3-2x+(\alpha/2\pi)f(x)$ is evaluated for several values of x (open circles). The solid straight line represents the function $3-2x$ and the dashed straight line is a linear approximation to $3-2x+(\alpha/2\pi)f(x)$. The value at $x=0.95$ lies precisely on the dotted line. Because of an oversight, it has been omitted from the figure.

(*b*) *Parameter* δ.—The same considerations as above may be applied to the $\cos\theta$-dependent term of (3.3). It is found that the curve for the function

$$1-2x+(\alpha/2\pi)h(x) \tag{3.9}$$

is very close to a straight line for $0.2\lesssim x\lesssim 0.9$ but deviates appreciably from it for $0.9<x<1$. The parameter δ determined from the straight part is

$$\delta=0.74_6, \tag{3.10}$$

which differs from the uncorrected value by only 0.5%. Since the one-parameter approximation of the $\cos\theta$ term is not very good at the upper end of the spectrum, it may not be as useful as that for the isotropic term.[15]

(*c*) *Corrections to* ξ *and* $|g_V|^2+|g_A|^2$.—As a by-product of these considerations, we find immediately the radiative correction to the asymmetry parameter ξ and the coupling constant $|g_V|^2+|g_A|^2$. It is found that ξ is increased by 0.3% while the correction to the coupling constant is $+3\%$. We can therefore neglect the correction to the asymmetry parameter completely in our discussion. The correction to the coupling constant has no observable effect as far as we are concerned with the muon decay only. The results of this paragraph can also be derived from the consideration of $N_r(0,\theta)$ of (3.13).

(*d*) *Forward-backward asymmetry*.—In the experiment of Garwin, Lederman, and Weinrich,[3] it is the forward-backward asymmetry of the integrated spectrum that is directly measured. In the case of no radiative correction, one finds from (2.12) that

$$N(x,\theta)=\tfrac{1}{4}A(1-2x^3+x^4)[1-\xi Q(x)\cos\theta], \tag{3.11}$$

where

$$Q(x)=\frac{1}{3}\left(\frac{1+x+x^2+3x^3}{1+x+x^2-x^3}\right) \tag{3.12}$$

is a function which increases from $\frac{1}{3}$ to 1 monotonically as x increases.

The integrated distribution including the radiative correction is obtained from (3.3):

$$N_r(x,\theta)=\tfrac{1}{4}A[1-2x^3+x^4+(\alpha/2\pi)F(x)]\times[1-\xi Q_r(x)\cos\theta], \tag{3.13}$$

with

$$Q_r(x)=\frac{1}{3}\left(\frac{1+2x^3-3x^4+(\alpha/2\pi)H(x)}{1-2x^3+x^4+(\alpha/2\pi)F(x)}\right). \tag{3.14}$$

$F(x)$ and $H(x)$ are the functions derived from $f(x)$ and $h(x)$ by integration. Their explicit forms are given in the Appendix. Numerical values of $(\alpha/2\pi)F(x)$ and $(\alpha/2\pi)H(x)$ are listed in columns 2 and 4 of Table I.

The forward-backward asymmetry of the integrated spectrum, with and without the radiative correction, is given by

$$R_r(x)=\frac{N_r(x,\pi)}{N_r(x,0)}=\frac{1+\xi Q_r(x)}{1-\xi Q_r(x)}, \tag{3.15}$$

and

$$R(x)=\frac{N(x,\pi)}{N(x,0)}=\frac{1+\xi Q(x)}{1-\xi Q(x)}, \tag{3.16}$$

respectively. As is seen from Table I, the ratio $Q_r(x)/Q(x)$ is almost constant taking values between 1 and 1.005 for all x. Accordingly, $R_r(x)/R(x)$ varies in the range between 1 and 1.01. Thus as far as the forward-backward asymmetry of the integrated spectrum is concerned, it would be good enough for practical purposes to neglect the effect of the radiative correction completely. This is because the integrated radiative corrections $F(x)$ and $H(x)$ behave in a similar manner as functions of x, and thus the major part of the radiative effect is canceled out when taking the ratio (3.15).

discussion, it is necessary to compute radiative corrections to the cases of the S, T, and P interactions, too. These corrections have so far been calculated only for the isotropic terms of the decay spectrum. (See reference 12.)

[15] Sargent, Rinehart, Lederman, and Rogers, Phys. Rev. **99**, 885 (1955). *Note added in proof*.—More recent ρ values are 0.67 $\pm$0.05 obtained by L. Rosenson (to be published) and 0.68$\pm$0.02 by K. M. Crowe *et al.* (Bull. Am. Phys. Soc. Ser. II, 2, 206 (1957)). These values already include the effect of radiative corrections and are to be compared with the uncorrected value $\rho=0.75$ to test the validity of the two-component theory. W. F. Dudziak and R. Sagane are also measuring the ρ value (private communication). If the ρ value were smaller than 0.75 as is suggested by these measurements, it would be a serious difficulty for the two-component theory, at least in its simplest form.

4. DISCUSSION

The two-component neutrino theory gives much more definite predictions than the general four-component theory concerning the reactions in which the neutrino participates. Thus it gives a decay spectrum which is completely determined in the case of an unpolarized muon and depends only on one parameter ξ when it is polarized. If a detailed measurement reveals that the observed spectrum does not agree with the theoretical curve, or the shape parameters ρ and δ, within experimental accuracy, there is no doubt that the two-component theory, at least in its simplest form, has to be rejected. If the experiment reproduces the theoretical spectrum of this model, however, it is not an unambiguous proof of the two-component theory as was mentioned in Sec. 2.

So far we have discussed the spectrum of muon decay assuming that the muon spin is completely polarized. Since the polarization will not necessarily be complete in the actual circumstances, the measured asymmetry parameter P of the integrated $1+P\cos\theta$ distribution will have to be compared with

$$r\xi Q(x), \tag{4.1}$$

where $Q(x)$ is defined by (2.13) and r represents the effective percent polarization of a muon at the moment of decay. In the experiment of Garwin *et al.*,[3] the asymmetry parameter $P=-\frac{1}{3}$ is observed when positrons of range>8 g/cm^2 are detected. Assuming that this corresponds to $x=0.5$, one obtains from (2.13)

$$r|\xi|=(33+8\rho)/|33+24\delta|. \tag{4.2}$$

Since $r\leqq 1$ in any case, (4.2) gives the lower bound to $|\xi|$:

$$|\xi|\geqq(33+8\rho)/|33+24\delta|. \tag{4.3}$$

Making use of the relations (2.14), (2.15), and (2.16), one finds that

$$3\geqq|\xi|\geqq 17/33=0.52 \tag{4.4}$$

must hold whatever the values of ρ and δ are. In particular, if $\rho=\delta=\frac{3}{4}$, it follows from (4.3) that

$$1\geqq|\xi|\geqq 13/17=0.76. \tag{4.5}$$

Obviously our information about ξ is restricted essentially by the lack of information on r. It is quite important to determine accurately the degree of polarization of the muon at the instant of its decay for the further development of the muon decay theory.

APPENDIX

The explicit forms of the functions $f(x)$ and $h(x)$ which are encountered in (3.3) are as follows:

$$\begin{aligned} f(x)&=2(3-2x)u(x)+(6-6x)\ln x\\ &\quad+\frac{(1-x)}{3x^2}[(5+17x-34x^2)(\omega+\ln x)-22x+34x^2], \end{aligned} \tag{A.1}$$

$$\begin{aligned} h(x)&=2(1-2x)u(x)+(2-6x)\ln x\\ &\quad+\frac{(1-x)}{3x^2}\bigg[(-1-x-34x^2)(\omega+\ln x)\\ &\quad-3+7x+32x^2-\frac{4(1-x)^2}{x}\ln(1-x)\bigg], \end{aligned} \tag{A.2}$$

where

$$\begin{aligned} u(x)&=\omega^2+\omega(\tfrac{1}{2}-2\ln 2)+2\ln 2-3\\ &\quad+\ln x[3\ln(1-x)-\ln x-2\ln 2]\\ &\quad+(2\omega-1-1/x)\ln(1-x)+L(1)-2L(x), \end{aligned} \tag{A.3}$$

$$L(x)=\int_0^x dt\frac{\ln(1-t)}{t},\quad L(1)=-\pi^2/6, \tag{A.4}$$

and

$$\omega=\ln(\mu/m)=5.332. \tag{A.5}$$

$F(x)$ and $H(x)$ are functions obtained by integrating $2x^2f(x)$ and $-6x^2h(x)$ over an interval $(x,1)$. They read as follows:

$$\begin{aligned} F(x)&=(1-2x^3+x^4)v(x)+2[L(1)-L(x)]\\ &\quad+2x(2-x)(x\ln x)^2+(1-x)F_1(x)\\ &\quad+x\ln xF_2(x)+(1-x)\ln(1-x)F_3(x), \end{aligned} \tag{A.6}$$

$$\begin{aligned} H(x)&=(1+2x^3-3x^4)v(x)+10[L(1)-L(x)]\\ &\quad+2x(-2+3x)(x\ln x)^2+(1-x)H_1(x)\\ &\quad+x\ln xH_2(x)+(1-x)\ln(1-x)H_3(x), \end{aligned} \tag{A.7}$$

where

$$\begin{aligned} v(x)&=2[\omega^2+\omega(\tfrac{1}{2}-2\ln 2)+2\ln 2-3]\\ &\quad+6\ln x\ln(1-x)+2L(1)-4L(x),\\ F_1(x)&=\tfrac{1}{3}\ln 2(5+5x+5x^2-3x^3)\\ &\quad-\tfrac{1}{3}\omega(14+12x+18x^2-20x^3)\\ &\quad+(1/36)(422+104x+275x^2-255x^3),\\ F_2(x)&=\tfrac{1}{6}(16-6x+40x^2-19x^3)+4\ln 2(2x^2-x^3),\\ F_3(x)&=(4\omega-2)(1+x+x^2-x^3)\\ &\quad-\tfrac{1}{6}(25+25x-11x^2-3x^3),\\ H_1(x)&=\tfrac{1}{3}\ln 2(1+x+x^2+9x^3)\\ &\quad-\tfrac{1}{3}\omega(10+4x-2x^2+60x^3)\\ &\quad+\tfrac{1}{12}(254-92x+41x^2+243x^3),\\ H_2(x)&=\tfrac{1}{6}(24+18x-80x^2+57x^3)\\ &\quad+4\ln 2(-2x^2+3x^3),\\ H_3(x)&=(4\omega-2)(1+x+x^2+3x^3)\\ &\quad-\tfrac{1}{6}(101-43x+65x^2+9x^3). \end{aligned} \tag{A.8}$$