Electron Polarization in Beta Processes

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Recent experiments have confirmed the hypothesis of Lee and Yang that there exist terms in the beta interaction which destroy the invariance of the theory under space inversion and charge conjugation; the possibility of the lack of invariance under time reversal had not yet been tested conclusively. We explore further consequences of these additional interactions, limiting the discussion to experiments involving the detection of polarized electrons and gamma rays from unpolarized nuclei. In particular, formulas are derived for the longitudinal polarization of electrons in beta decay, and for the correlation of the transverse polarization of electrons with the direction of a gamma ray. Such experiments would lead to an independent verification of the violation of selection rules for parity and charge conjugation, and to a possible test of the invariance under time reversal.

I. INTRODUCTION

THE recent experiments of Wu, Ambler, Hayward, Hoppes, and Hudson[1,2] have proven conclusively that there are beta transitions in Co^{60} and Co^{58} which violate parity selection rules; that is, they observed a correlation between a pseudovector (the nuclear polarization) and a vector (the electron momentum). Measurement of the magnitude of the correlation also proved that charge-conjugation invariance was violated. The experiments in principle could have yielded information on the time-reversal invariance as well, but were not conclusive.

It is the purpose of this work to investigate further experiments with nuclear beta decay which can provide an independent verification of these results, and can also reveal violations of time-reversal invariance, if present. Clearly the generalization of the beta-decay interaction to include terms which violate these invariance requirements suggests many new experiments. In particular there will be extra terms in the transition rate for beta decay and in the beta-gamma correlation function. We shall restrict the discussion to only those experiments involving the detection of electrons and gamma rays emitted by unpolarized nuclei. To observe nonconservation of parity one must then detect the polarization vector of either the electron or the gamma ray; we shall (rather arbitrarily) select the measurement of electron polarizations as being more feasible. The problem then splits rather naturally into the discussion of the experiments arising from a measurement of the *longitudinal* electron polarization, and of the *transverse* polarization. Accordingly, we shall discuss in Sec. II the information which can be obtained by measuring the longitudinal polarization of electrons from unpolarized beta emitters. It will be shown that a situation very similar to that of the experiment of Wu *et al.* pertains; the detection of a polarization will prove nonconservation of parity, while the measurement of the magnitude and energy dependence of the polarization will provide possible proof of the violation of charge-conjugation invariance and of time-reversal invariance, respectively.

In Sec. III we discuss the information gained from the measurement of transverse electron polarizations in coincidence with gamma rays. By measuring the magnitude of the polarization alone, both *in* the plane of the electron and gamma ray, and *perpendicular* to the plane, it is possible to observe violations of charge-conjugation invariance and of time-reversal invariance, respectively.

II. LONGITUDINAL POLARIZATION

Suppose we measure only the momentum $\mathbf{p}$ and the spin direction $\mathbf{l}$ of an electron emitted in the decay of an unpolarized beta source. Since there are only two directions specified, the rotational invariance of the theory requires that the transition rate has the form

$$N\sim a+b(\mathbf{p}\cdot\mathbf{l}). \quad (1)$$

Because the second term is a pseudoscalar, it could not appear in a theory in which parity is conserved, and hence the electron spin would have no correlation with its momentum; i.e., the electron would be unpolarized. If conservation of parity is not required, then the expression (1) pertains, and the electrons can be polarized along their directions of motion. It seems surprising at first that this polarization could escape detection for so long, until one recalls that a longitudinally polarized beam of particles does not give rise to an asymmetry on single scattering, but only on double scattering.[3] For example, the experiments[4] utilizing beta sources to check the validity of the Mott scattering formula, would not be influenced by the presence of a longitudinal polarization of the electrons. To analyze the longitudinal polarization by scattering,[5] one could

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[1] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

[2] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[3] See for example R. Oehme, Phys. Rev. **98**, 147 (1955).

[4] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Oxford University Press, New York, 1949), second edition, p. 83.

[5] See for example H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956).

first rotate the polarization into a transverse direction by passing the electrons through an electrostatic field, and then perform a single Coulomb scattering. Alternatively, one could perform a double Coulomb scattering, and search for an asymmetry in the plane perpendicular to the plane of the first scattering. It is interesting to note that this experiment was attempted many years ago,[6] and a positive result obtained, indicating the existence of longitudinally polarized beta rays. Subsequent experiments failed to reproduce the effect;[7] in the light of present knowledge about the magnitude of such asymmetries, it is unlikely that the effect was due to the polarization of the beta rays.

Using the beta-decay interaction proposed by Lee and Yang, one can calculate the transition rate for an allowed beta decay into a state in which the electron has momentum **p** and spin in the direction of the unit vector **l**, which is chosen either parallel or antiparallel to **p**. The result is[8]

$$N(\mathbf{p},\mathbf{l})dWd\Omega_p=\frac{1}{8\pi^4}F(Z,W)pW(W_0-W)^2dWd\Omega_p \times\tfrac{1}{2}\xi\left[1+b\frac{m}{W}+d\frac{\mathbf{p}\cdot\mathbf{l}}{W}\right]. \quad (2)$$

We are using a notation identical to that of reference 2, with the additional definition

$$\xi d=\{|M_{\mathrm{F}}|^2[C_SC_S'^*-C_VC_V'^*+\text{c.c.}] +|M_{\mathrm{GT}}|^2[C_TC_T'^*-C_AC_A'^*+\text{c.c.}]\} -\left\{\frac{i\alpha Z}{p}|M_{\mathrm{F}}|^2[C_SC_V'^*+C_S'C_V^*-\text{c.c.} +\frac{i\alpha Z}{p}|M_{\mathrm{GT}}|^2[C_TC_A'^*+C_T'C_A^*-\text{c.c.}]\right\}, \quad (3)$$

where "c.c." means "complex conjugate." Defining the polarization as

$$P=\frac{N(\mathbf{p},\mathbf{p}/p)-N(\mathbf{p},-\mathbf{p}/p)}{N(\mathbf{p},\mathbf{p}/p)+N(\mathbf{p},-\mathbf{p}/p)},$$

we find that

$$P=\frac{v_e}{c}\left(\frac{d}{1+bm/W}\right). \quad (4)$$

Let us note here that under the special conditions assumed in the Lee-Yang neutrino theory[9] ($C=-C'$), and with a single term in the interaction Hamiltonian, the polarization is just $\pm v/c$,[10] and therefore *complete* for a fast electron. Since this is the assignment of coupling constants suggested by the experiment of Wu *et al.*, let us make clear the origin of this result. Under the above conditions, the interaction Hamiltonian contains the light-particle wave functions in the form $\psi^\dagger\mathcal{O}(1-\gamma_5)\phi$, where $\mathcal{O}$ is any of the operators appearing in the various interactions. For S, T, and P, we find that γ_5 commutes with $\mathcal{O}$, while for V and A, it anticommutes. Therefore we can write it as $\psi^\dagger(1\pm\gamma_5)\mathcal{O}\phi$. But it has been emphasized that if the mass of a Dirac particle is zero (or negligible, as for a fast electron), the operators $\frac{1}{2}(1\pm\gamma_5)$ acting on positive energy states, are projection operators for states with spin parallel or antiparallel to the momentum.[11] Therefore fast electrons, as well as the neutrino, are emitted in only one of the two possible spin states, and are completely polarized. Notice that this should be true for the basic differential process, before averaging over the neutrino directions, or making the expansion into degrees of forbiddenness, etc.

The polarization arising from a pure interaction will therefore approach a constant for high electron energies; the same result is true for the polarization arising from the interference between S, T, and P, and between V and A. The polarizations arising from S-V and T-A interference, however, will vanish like $1/W$ for fast electrons. This difference in the energy dependence of direct and interference terms is characteristic of beta decay, and is very important in deciding the form of the beta interaction from experiment.

Examination of the formula (3) for the polarization shows that it provides exactly the same information as is obtained by the experiment of Wu *et al.* [see Eq. (A.6) in reference 2]. The detection of a polarization proves the violation of parity conservation; a nonvanishing value for the first (energy-independent) term in d violates charge-conjugation invariance, while a nonvanishing value for the second (energy-dependent) term violates time-reversal invariance.[12] There is the further possibility of studying pure Fermi transitions ($0\rightarrow0$, no), for which there is no analogous experiment with polarized nuclei.

As is well known, the only interference terms for an allowed beta transition of an unpolarized source are between S and V and between T and A (the Fierz interferences). Since it is generally concluded that V and A are both absent in nuclear beta decay, this would prevent the observation of terms violating time-reversal invariance in an allowed transition.[13] For this

[6] Cox, McIlwraith, and Kurrelmeyer, Proc. Natl. Acad. Sci. U. S. **14**, 544 (1928).

[7] C. T. Chase, Phys. Rev. **34**, 1069 (1929).

[8] This same result has been derived in references 14 and 16, as well as by M. E. Ebel and G. Feldman (unpublished preprint).

[9] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957). See also L. Landau, Nuclear Phys. **3**, 127 (1957); A. Salam, Nuovo cimento **5**, 299 (1957).

[10] Perhaps we should point out here that, if one assumes the Lee-Yang neutrino theory, time-reversal invariance implies complete polarization, but the converse is not true. The polarization in an allowed transition would be complete even if there were no time-reversal invariance, but if $|C_S||C_V|=0$ held.

[11] We are neglecting the Coulomb field here.

[12] Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).

[13] We should note that the Fierz condition, which now reads $[C_SC_V^*+C_S'C_V'^*+\text{c.c.}]=0$ does not necessarily imply the vanishing of the S-V and T-A interference terms in the polarization, which contain $[C_SC_V'^*+C_S'C_V^*-\text{c.c.}]$.

reason, it may be important to examine a forbidden transition, where S-T interference can occur; the case of RaE may be a good example.

In this case (first-forbidden, 1→0, yes) we obtain for the polarization[14]

$$P=B/A,$$

where

$$\begin{aligned}A=&\alpha_{SS}|(\|\int\mathbf{r}\|)|^2[L_1+\tfrac{1}{6}q^2L_0+\tfrac{1}{2}M_0+\tfrac{1}{3}qN_0]\\&+\tfrac{1}{4}\alpha_{TT}|(\|\int\boldsymbol{\sigma}\times\mathbf{r}\|)|^2[L_1+\tfrac{1}{3}q^2L_0+2M_0-\tfrac{4}{3}qN_0]\\&+\tfrac{1}{2}\alpha_{TT}|(\|\int\beta\boldsymbol{\alpha}\|)|^2[L_0]\\&-\tfrac{1}{2}\{i\alpha_{ST}(\|\int\mathbf{r}\|)(\|\int\boldsymbol{\sigma}\times\mathbf{r}\|)^*+\text{c.c.}\}[L_1-M_0]\\&+\tfrac{1}{2}\{i\alpha_{ST}(\|\int\mathbf{r}\|)(\|\int\beta\boldsymbol{\alpha}\|)^*+\text{c.c.}\}[\tfrac{1}{3}qL_0+N_0]\\&-\tfrac{1}{2}\{\alpha_{TT}(\|\int\boldsymbol{\sigma}\times\mathbf{r}\|)(\|\int\beta\boldsymbol{\alpha}\|)^*+\text{c.c.}\}[\tfrac{1}{3}qL_0-N_0],\end{aligned}\tag{5}$$

and

$$\begin{aligned}B=&\beta_{SS}|(\|\int\mathbf{r}\|)|^2[L_1'+\tfrac{1}{6}q^2L_0'+\tfrac{1}{2}M_0'+\tfrac{1}{3}qN_0']\\&+\tfrac{1}{4}\beta_{TT}|(\|\int\boldsymbol{\sigma}\times\mathbf{r}\|)|^2[L_1'+\tfrac{1}{3}q^2L_0'+2M_0'-\tfrac{4}{3}qN_0']\\&+\tfrac{1}{2}\beta_{TT}|(\|\int\beta\boldsymbol{\alpha}\|)|^2[L_0']\\&-\tfrac{1}{2}\{i\beta_{ST}(\|\int\mathbf{r}\|)(\|\int\boldsymbol{\sigma}\times\mathbf{r}\|)^*+\text{c.c.}\}[L_1'-M_0']\\&+\tfrac{1}{2}\{i\beta_{ST}(\|\int\mathbf{r}\|)(\|\int\beta\boldsymbol{\alpha}\|)^*+\text{c.c.}\}[\tfrac{1}{3}qL_0'+N_0']\\&-\tfrac{1}{2}\{\beta_{TT}(\|\int\boldsymbol{\sigma}\times\mathbf{r}\|)(\|\int\beta\boldsymbol{\alpha}\|)^*+\text{c.c.}\}[\tfrac{1}{3}qL_0'-N_0']\\&+\{\beta_{ST}(\|\int\mathbf{r}\|)(\|\int\boldsymbol{\sigma}\times\mathbf{r}\|)^*+\text{c c }\}[\tfrac{1}{3}qR_0']\\&-\{\beta_{ST}(\|\int\mathbf{r}\|)(\|\int\beta\boldsymbol{\alpha}\|)^*+\text{c.c.}\}[\tfrac{1}{2}R_0']\\&+\{i\beta_{TT}(\|\int\boldsymbol{\sigma}\times\mathbf{r}\|)(\|\int\beta\boldsymbol{\alpha}\|)^*+\text{c.c.}\}[\tfrac{1}{2}R_0'].\end{aligned}\tag{6}$$

We have made the useful abbreviations $\alpha_{ij}=C_iC_j^*+C_i'C_j'^*$ and $\beta_{ij}=C_iC_j'^*+C_i'C_j^*$. The functions L_1, L_0, M_0, N_0, L_1', L_0', M_0', N_0', R_0' are defined in the appendix. The symbol $(\|\int\mathcal{O}\|)$ is an abbreviation for the reduced matrix element of a vector operator for the nuclear transition being considered, and the assumption has been made that only the scalar and tensor forces contribute.

If time reversal is valid for the nuclear forces, then $(\|\int\boldsymbol{\sigma}\times\mathbf{r}\|)(\|\int\beta\boldsymbol{\alpha}\|)^*$ is real and the last term in the expression for B is zero. The preceding two terms will be nonvanishing only if time-reversal invariance is violated in β decay. These terms are proportional to $\alpha Z/p$ and thus are a Coulomb effect.

For a heavy element, the functions M_0 and M_0' are very large and approximately energy-independent. In general one would therefore expect the β spectrum to have an allowed shape, and the polarization to be independent of the terms violating time-reversal invariance. However, RaE does not have an allowed shape, owing to cancellation of the terms containing M_0. If, as seems likely, $C_i=\pm C_i'$, then a similar cancellation occurs among the terms containing M_0'. In this event, the terms violating time-reversal invariance might be observable. It seems unlikely that this experiment will prove useful in checking the invariance under time reversal in the theory; however, polarization experiments on RaE can provide new information on the S, T coupling constants.

[14] This result has also been obtained by Alder, Stech, and Winther, Phys. Rev. (to be published).

III. TRANSVERSE POLARIZATION

If another direction is introduced into the problem by measuring another particle, the electron can have components of its polarization transverse to its momentum, as well as longitudinal.[15] For example, Jackson, Treiman, and Wyld[16] have suggested experiments in which nuclear recoils or nuclear polarizations are measured in addition to electron polarizations. We shall consider the case in which a gamma ray is detected; that is, the measurement of β polarization-gamma correlation functions. One easily discovers that no such correlations exist for allowed beta transitions; we therefore consider first-forbidden transitions. For simplicity we shall also neglect Coulomb effects, and consider only those beta transitions involving the scalar nuclear matrix element $\int\mathbf{r}$ and the tensor matrix elements $\int\boldsymbol{\sigma}\times\mathbf{r}$. It is our main purpose to present illustrative examples, rather than an extensive tabulation of correlation functions.

A similar argument to that of Sec. II, based on rotational invariance, leads us to expect the following terms in the transition rate:

$$N\sim a+b[3(\mathbf{p}\cdot\mathbf{k})^2-p^2k^2]+c\mathbf{l}\cdot\mathbf{k}\mathbf{p}\cdot\mathbf{k}+d\mathbf{l}\cdot\mathbf{p}\times\mathbf{k}\mathbf{p}\cdot\mathbf{k}.\tag{7}$$

Here $\mathbf{p}$ is the electron momentum, $\mathbf{l}$ is a unit vector orthogonal to $\mathbf{p}$ in the direction of the electron spin,[17] and $\mathbf{k}$ is a unit vector in the direction of the photon. The first two terms are the ordinary β-γ correlation function; the third term leads to a polarization *in* the plane of $\mathbf{p}$ and $\mathbf{k}$, and the last term leads to a polarization *perpendicular* to the plane. A calculation of the transition rate for the emission of an electron $\mathbf{p}$, $\mathbf{l}$ and a photon $\mathbf{k}$ gives

$$\begin{aligned}N(\mathbf{p},\mathbf{l},\mathbf{k})dWd\Omega_pd\Omega_k\sim&\,pW(W_0^2-W^2)d\Omega_pd\Omega_k\\&\times\{A+f(J_1,J_2,J_3,L)[B(3(\mathbf{p}\cdot\mathbf{k})^2-p^2)\\&+C\mathbf{p}\cdot\mathbf{k}\mathbf{l}\cdot\mathbf{k}+D\mathbf{l}\cdot\mathbf{p}\times\mathbf{k}\mathbf{p}\cdot\mathbf{k}]\}dW,\end{aligned}\tag{8}$$

[15] We shall drop longitudinal polarization terms in this section; we take $\mathbf{p}\cdot\mathbf{l}=0$.

[16] Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957).

[17] The transverse polarization of a beam of relativistic particles is usually defined as the ensemble average $\langle\beta\sigma\rangle$ in the rest frame of the particles. Defining the polarization to transform like a four-(pseudo) vector, it is given in an arbitrary frame by $\langle i\gamma_5\gamma_\mu\rangle$. An electron polarized in its rest system in the direction $\mathbf{l}$, is thus in an eigenstate of $\beta\boldsymbol{\sigma}\cdot\mathbf{l}$, and in an arbitrary frame is in an eigenstate of $i\gamma_5\gamma_\mu l_\mu$. Formula (8) gives the transition rate into such a state. Here l_μ is defined to transform like a four-(pseudo) vector, and is $(\mathbf{l},0)$ in the rest frame, and satisfies $l_\mu l_\mu=1$, $p_\mu l_\mu=0$. Note that a unit vector transverse to $\mathbf{p}$ is unchanged by a Lorentz transformation along $\mathbf{p}$.

where $f(J_1J_2J_3,L)$ is a numerical coefficient and

$$A=\alpha_{SS}|(J_2\|\int\mathbf{r}\|J_1)|^2\{\tfrac{1}{2}(p^2+q^2)-\tfrac{1}{3}qp^2/W\}+\alpha_{TT}|(J_2\|\int\boldsymbol{\sigma}\times\mathbf{r}\|J_1)|^2\{\tfrac{1}{4}(p^2+q^2)+\tfrac{1}{3}qp^2/W\},$$

$$B=\alpha_{SS}|(J_2\|\int\mathbf{r}\|J_1)|^2\{(2q-3W)/8W\}+\alpha_{TT}|(J_2\|\int\boldsymbol{\sigma}\times\mathbf{r}\|J_1)|^2\{(4q+3W)/32W\}-i\alpha_{ST}(J_2\|\int\mathbf{r}\|J_1)(J_2\|\int\boldsymbol{\sigma}\times\mathbf{r}\|J_1)^*\{3q/16W\}+\text{c.c.},\quad(9)$$

$$C=\beta_{SS}|(J_2\|\int\mathbf{r}\|J_1)|^2\{3q/4W\}+\beta_{TT}|(J_2\|\int\boldsymbol{\sigma}\times\mathbf{r}\|J_1)|^2\{3q/8W\}-i\beta_{ST}(J_2\|\int\mathbf{r}\|J_1)(J_2\|\int\boldsymbol{\sigma}\times\mathbf{r}\|J_1)^*\{9q/16W\}+\text{c.c.},$$

$$D=-\alpha_{ST}(J_2\|\int\mathbf{r}\|J_1)(J_2\|\int\boldsymbol{\sigma}\times\mathbf{r}\|J_1)^*\{9/16W\}+\text{c.c.}$$

The transition is $J_1-{}^{\beta}\!\!\rightarrow J_2-{}^{\gamma}\!\!\rightarrow J_3$, where the β transition is through the matrix elements given above and the γ-transition is through 2^L-pole radiation. For $L=1$, 2 we have

$$f(J_1J_21)=-\frac{2}{3}\frac{W(J_2J_211;2J_1)W(J_2J_211;2J_3)}{W(J_2J_211;0J_1)W(J_2J_211;0J_3)},$$
$$f(J_1J_2J_32)=-\frac{2}{3}\left(\frac{5}{7}\right)^{\frac{1}{2}}\frac{W(J_2J_211;2J_1)W(J_2J_222;2J_3)}{W(J_2J_211;0J_1)W(J_2J_222;0J_3)}.\quad(10)$$

Here $W(abcd;ef)$ is the Racah coefficient.

We define the polarization in the direction $\mathbf{l}$ as before. If we introduce a coordinate system such that $\mathbf{p}$ lies along the z axis, $\mathbf{k}$ in the xz plane, and the y axis along $\mathbf{p}\times\mathbf{k}$, i.e., $\mathbf{p}=p(0,0,1)$, $\mathbf{k}=(\sin\theta,0,\cos\theta)$ then the polarization in the x direction is

$$P_x=\frac{fpC\sin\theta\cos\theta}{A+f(J_1J_2J_3L)Bp^2(3\cos^2\theta-1)},\quad(11)$$

and in the y direction is

$$P_y=\frac{fp^2D\sin\theta\cos\theta}{A+f(J_1J_2J_3L)Bp^2(3\cos^2\theta-1)}.\quad(12)$$

A detection of P_x proves that parity is not conserved and, if the Coulomb effects are negligible,[18] that charge-conjugation invariance is also violated. If the Coulomb effects are negligible, P_y will be present only if time-reversal invariance is violated. Of course P_y will vanish unless there are both S and T matrix elements present and only such beta emitters should be chosen. If Coulomb effects are not negligible, one cannot necessarily conclude that the existence of such a polarization component will violate time-reversal invariance, and the analysis of the experiment is more difficult. It might then again be necessary to study the energy dependence of the polarization to identify the terms present.

[18] The conditions for the validity of neglecting Coulomb effects are that $\alpha Z\ll 1$ and $(\alpha Z/pR)\ll 1$, where R is the nuclear radius.

The magnitude and energy dependence of the transverse polarization cannot be estimated as easily, since they depend on the assignment of nuclear matrix elements, spin values, etc. However, we can say in general that polarizations are rather small. This comes about mainly because some of the electrons are emitted in $j=\frac{1}{2}$ states, which show no correlation with the gamma ray. The energy dependence for fast electrons shows an inverse behavior to that of the longitudinal polarization: the terms arising from pure interactions, and from S-T-P interference and V-A interference, are proportional to m/W, and the terms arising from S-V interference and T-A interference, etc., are proportional to 1. This is clearly associated with the fact that, if the *longitudinal* polarization is large, the *transverse* must be small (and vice versa).

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APPENDIX

Certain combinations of Coulomb radial wave functions continually reoccur in the discussion of β decay. Before considerations of parity nonconservation were made, the functions defined by Konopinski and Uhlenbeck[19] and tabulated by Rose and Perry[20] were sufficient. These are the unprimed functions in Eq. (5). However, there is now a need for new combinations, denoted by the primed functions in Eq. (6). For convenience we tabulate the definitions of these functions and their limiting value for $\alpha Z\ll 1$, $pR\ll 1$, $\alpha Z/pR\sim 1$, where R is the nuclear radius. The functions f_κ and g_κ are the radial wave functions given by Rose[21] normalized to one particle per unit sphere, and $F_0(Z,W)$ is the Fermi function. Here $k=|\kappa|$, $\Lambda=\alpha Z/p$, $\xi=\alpha Z/2R$, and $S(p,k)=[2(2p)^{k-1}k!/(2k)!]^2$.

$$L_{k-1}=(2p^2F_0)^{-1}(f_k^2+g_{-k}^2)R^{-2k+2}\rightarrow S(p,k),$$

$$M_{k-1}=(2p^2F_0)^{-1}(f_{-k}^2+g_k^2)R^{-2k}\rightarrow S(p,k)\left[\frac{1}{k^2}\xi^2+\frac{2}{k(2k+1)}\left(\frac{p^2}{W}\right)\xi+\frac{p^2}{(2k+1)^2}\right],$$

$$N_{k-1}=(2p^2F_0)^{-1}(f_{-k}g_{-k}-f_kg_k)R^{-2k+1}\rightarrow -S(p,k)\left[\frac{1}{k}\xi+\frac{1}{2k+1}\left(\frac{p^2}{W}\right)\right],$$

[19] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).

[20] M. E. Rose and C. L. Perry, Phys. Rev. **90**, 479 (1953).

[21] M. E. Rose, Phys. Rev. **51**, 484 (1937). Dr. Rose has pointed out that the bilinear combinations of radial functions which occur are always independent of the (arbitrary) choice of signs of the radial functions f_k, g_k. This ambiguity of signs arises from the possibility of adding an arbitrary multiple of π to each phase shift. In this appendix we have chosen the phase shifts to satisfy

$$\cos(\Delta_k-\Delta_{-k})=\frac{k}{(k^2+\Lambda^2)^{\frac{1}{2}}}$$

$$\sin(\Delta_k-\Delta_{-k})=-\frac{\Lambda}{(k^2+\Lambda^2)^{\frac{1}{2}}}.$$

$$L_{k-1}' = (2p^2F_0)^{-1}(2f_k g_{-k})\frac{k}{(k^2+\Lambda^2)^{\frac{1}{2}}}R^{-2k+2} \to S(p,k)\frac{p}{W}$$

$$M_{k-1}' = (2p^2F_0)^{-1}(-2f_{-k}g_k)\frac{k}{(k^2+\Lambda^2)^{\frac{1}{2}}}R^{-2k} \to S(p,k)\frac{p}{W}\left[\frac{1}{k^2}\xi^2+\frac{2W}{k(2k+1)}\xi+\frac{p^2}{(2k+1)^2}\right],$$

$$N_{k-1}' = (2p^2F_0)^{-1}(f_kf_{-k}-g_kg_{-k})\frac{k}{(k^2+\Lambda^2)^{\frac{1}{2}}}R^{-2k+1} \to -S(p,k)\frac{p}{W}\left[\frac{1}{k}\xi+\frac{W}{2k+1}\right],$$

$$R_{k-1}' = (2p^2F_0)^{-1}(f_kf_{-k}+g_kg_{-k})\frac{\Lambda}{(k^2+\Lambda^2)^{\frac{1}{2}}}R^{-2k+1} \to S(p,k)\frac{p}{W}\frac{\Lambda}{2k+1}.$$

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Measurements of the Proton Strength Function*

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The thick-target yield of gamma rays from the $(p,p'\gamma)$ reaction has been used to calculate values of the strength function $\langle\gamma^2\rangle_{Av}/\bar{D}$, of states in the compound nucleus for the channel of *s*-wave inelastic protons. Measurements of inelastic scattering to the 2+ first excited states of nine even-even nuclides, ranging from Ca^{44} to Zn^{64}, indicate a constant value for $\langle\gamma^2\rangle_{Av}/\bar{D}$ of 2.7×10^{-14} cm, within experimental error. This may be compared with the resonance in neutron strength function near atomic weight 55 as found in neutron elastic scattering.

INTRODUCTION

IN comparing theories for various nuclear models, certain average properties of nuclear levels are of considerable interest. One such property is the average strength of levels in a nucleus as measured by the strength function, $\langle\gamma^2\rangle_{Av}/\bar{D}$.[1–3] Here $\langle\gamma^2\rangle_{Av}$ is the average reduced width, for a particular reaction channel, of levels with the same quantum numbers π and J, and $\bar{D}$ is the average spacing of such levels.

Previous measurements of the strength function have come primarily from experiments utilizing neutrons from nuclear reactors[4] or neutrons in the kev region obtained from the $Li^7(p,n)Be^7$ reaction by use of electrostatic accelerators.[5] The centrifugal barrier limits most of the resonances in such experiments to those formed by *s*-wave neutrons. It has become customary to define a modified reduced width, $\Gamma_n{}^0=\Gamma_n/E_n{}^{\frac{1}{2}}$, where both E_n and Γ_n are measured in ev. This $\Gamma_n{}^0$, defined only for *s*-wave neutrons, differs from the customary reduced width γ^2 only by a constant factor.

Feshbach, Porter, and Weisskopf[2] have discussed two models for the variation of the neutron strength function with atomic weight. Assuming very strong absorption of the neutron in nuclear matter, they expect the strength function to stay constant with atomic weight and to have the value of $1/(\pi K)=2.3\times10^{-14}$ cm.[6] This is sometimes called the "black nucleus" model. Here K is the wave number of the neutron inside the nuclear potential well and is taken to be about 1.4×10^{13} cm^{-1}. If less absorption is assumed for the neutron, they find that maxima in the strength function would be expected at values of the atomic weight for which the nuclear radius is $(n+\frac{1}{2})\pi/K$, where n is an integer. The data from neutron experiments were found to be consistent with this latter assumption. A maximum at $A\approx55$ was found both by Bollinger,[4] using slow neutrons on nuclei of odd Z, and by Newson,[5] using neutron energies in the kev region. Even better evidence has been obtained for a second maximum at $A\approx150$.[4] Lane, Thomas, and Wigner[3] also have explained these results by a more detailed treatment where they consider the maxima in the strength function to be due to independent particle states whose strength has been spread out to neighboring levels.

Recently an experimental technique has been used in which a large number of resonances in the compound nucleus are analyzed, and their reduced widths deter-

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