Nuclear Reactions in Stars. Buildup from Helium*

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The rates are calculated for thermonuclear reactions involving alpha particles, at temperature T of the order of 10^8 °K. In particular, the conversion of three alpha particles into a C^{12} nucleus and (α,γ) reactions on C^{12}, O^{16}, and Ne^{20} are considered, as well as $C^{13}(\alpha,n)O^{16}$. Rough estimates are also given for alpha reactions on the other isotopes of N, O, F, and Ne.

The reaction rates are used to re-evaluate some characteristic central temperatures T_c for Hoyle and Schwarzschild's model for the evolution of a population II red giant: for stars "at the tip of the red-giant branch," where helium burning first becomes important, $T_c \approx 1.0\times10^8$ °K; for stars at the beginning of the "horizontal branch," where helium burning contributes appreciably to the energy production, $T_c \approx 1.3\times10^8$ °K. At these temperatures the rates of formation of C^{12}, O^{16}, and Ne^{20} are roughly comparable (if the spin and parity of a certain resonance level in Ne^{20} is suitable), but the production rate of Mg^{24} and heavier nuclei is negligible. Of the alpha reactions on the rarer isotopes, by far the fastest is $C^{13}(\alpha,n)$ and most of the C^{13} is used up near $T_c = 0.8\times10^8$ °K.

1. INTRODUCTION

IN two previous papers[1] those thermonuclear reactions were discussed which might take place in the interior of the most common types of stars—those which consist mainly of hydrogen. As is well known, the main effect of these reactions is the conversion of hydrogen into He^4. Since hydrogen made up the bulk of the star originally and since chemical mixing is unimportant in most stars, a core consisting mainly of helium will build up slowly in these stars. After a certain fraction of the stellar mass is contained in this helium core, the core begins to contract.[2] The density and temperature of this core continue to increase with time, accompanied by an expansion of the outer layers of the star, until further thermonuclear reactions begin to occur inside the core. It was suggested by Öpik and by Salpeter[3] that the first and most important of such reactions is the conversion of three alpha particles into one C^{12} nucleus. Once C^{12} has been formed, O^{16}, Ne^{20}, etc., can be built up by successive (α,γ) reactions.

The $3\alpha\rightarrow C$ reaction, and related reactions, in stars is of interest from two different points of view: first of all, as a source of energy production in stars; secondly, in connection with theories on the formation of all the nuclear species by means of nuclear reactions in the interior of stars. From both points of view the $3\alpha\rightarrow C$ conversion is the key reaction since it is the gateway to all reactions which build up heavier elements and which produce energy, starting from helium. The present paper therefore concentrates mainly on the reaction rate for the $3\alpha\rightarrow C$ conversion. However, the relative importance of other reactions is rather different for the two different purposes. For purposes of nucleogenesis one is also interested in building up very rare nuclear species. Thus, even some kinds of reactions which involve only a small fraction of the matter of the star are of importance. Further, for predicting abundance ratios of various nuclear species we unfortunately need to know the ratios of various reaction rates with reasonable accuracy. For these purposes, then, our knowledge of the relevant reaction rates can still stand a lot of improving. What conclusions can be reached with our present knowledge has been discussed well and in detail elsewhere in the literature[4–6] and this paper will add little to the question of nucleogenesis.

From the point of view of sources of energy production, the reactions starting from the main constituent of the stellar core, helium, should be most important. Further, the rate of energy production is controlled mainly by the rate of the first, the $3\alpha\rightarrow C$, reaction: the energy release per C^{12} nucleus is 7.3 Mev, if this nucleus undergoes no further reactions. If the rates for the reactions $C^{12}(\alpha,\gamma)O^{16}$ and $O^{16}(\alpha,\gamma)Ne^{20}$, say, are very fast then an additional energy of 11.9 Mev per C^{12} nucleus is released by its conversion into Ne^{20}, increasing the rate of energy production only by a factor of 2.6. In stellar interior calculations one essentially uses the nuclear reaction rates only to determine the temperature at which energy is produced at a required rate. The rate p of the $3\alpha\rightarrow C$ reaction is highly temperature sensitive ($p\propto T^{40}$ for T near 10^8 °K) and a fairly large error in the reaction rate will lead to quite a small error in the calculated temperature. The energy produced by reactions involving minor constituents of the stellar core could be of some importance only at one particular stage of the evolution of the star. This will be discussed at the end of Sec. 6.

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[5] A. G. W. Cameron, Astrophys. J. **121**, 144 (1955); Fowler, Burbidge, and Burbidge, Astrophys. J. **122**, 271 (1955).

[6] Nakagawa, Ohmura, Takebe, and Obi, Progr. Theoret. Phys. (Japan) **16**, 389 (1956); Hayakawa, Hayashi, Imoto, and Kikuchi, Progr. Theoret. Phys. (Japan) **16**, 507 (1956).

The rate of the $3\alpha \rightarrow C^{12}$ reaction is enormously sensitive to the presence of any resonances in the suitable energy regions. If this reaction had to proceed via genuine three-body collisions, unassisted by any resonances, its rate would be negligibly small at the relevant temperatures of about 10^8 °K. The existence of the Be^8 ground-state level, at 94-kev relative kinetic energy between two alpha particles, already enhances the reaction rate enormously. A dynamic equilibrium is set up involving a minute but well-known concentration of Be^8 and our problem is reduced to finding the rate of the $Be^8(\alpha,\gamma)C^{12}$ reaction. Even without a suitable resonance in C^{12}, the reaction would proceed at a non-negligible rate, but would be much slower than the subsequent (α,γ) reactions on C^{12} and O^{16}. Hoyle[4] pointed out that a resonance level in C^{12} at 7.6- to 7.7-Mev excitation would enhance the $Be^8(\alpha,\gamma)C^{12}$ rate further to a sufficient extent, so that the production rate of C is comparable with the rate of its destruction. One such level was in fact found subsequently.[7] If this level has the right spin and parity to be accessible to a Be^8 nucleus plus alpha-particle, most of the $3\alpha \rightarrow C$ reaction proceeds via this resonance level in C^{12} and the rate depends critically only on some of the properties of this level (in particular, on its exact energy value and one of the partial widths for its decay). After the discovery of this resonance level, estimated rates for the $3\alpha \rightarrow C$ reaction had been calculated by Hoyle,[4] by Salpeter,[8] and, more recently, by the Kyoto[6] and Tokyo[6] groups. More definitive experimental work on this resonance level in C^{12} has been done recently and is reported on in the preceding paper.[9] We re-evaluate in this paper the reaction rates in the light of the new experimental work and discuss the various sources of errors.

2. 7.65-Mev STATE IN C^{12}

We discuss now the energy position and the partial widths of the second excited level in C^{12}. The relevant part of the energy-level diagram is given in Fig. 1.

We write $E_r = Q_1 + Q_2$, where Q_1 is the energy release in the decay of the second excited state of C^{12} into Be^8 plus an alpha particle and Q_2 the decay energy for $Be^8 \rightarrow 2\alpha$. We call the excitation energy of the second excited C^{12} state E_g. The energy we shall need to know most accurately is E_r, since it occurs in a Maxwell-Boltzmann factor in the final formula for the $3\alpha \rightarrow C^{12}$ reaction rate. The earlier experiments did not measure E_r directly, but measured the excitation energy E_g. From the measured Q-values of various nuclear-reaction cycles[10] we find, for the binding energy of the three alpha particles in the C^{12} ground state, $3\alpha - C^{12} = (7.282 \pm 0.008)$ Mev. Using this value, the Dunbar *et al.*[7] value for E_g gives $E_r = (398 \pm 32)$ kev, the Pauli[11] and Ahnlund[11] values give $E_r = (378 \pm 24)$ kev and $E_r = (376 \pm 29)$ kev, respectively. These values are quite consistent with the much more accurate and direct measurements of E_r reported in the preceding paper (CFLL). We shall adopt this new value of

$$Q_1 = (278 \pm 4) \text{ kev}, \quad E_r = (372 \pm 4) \text{ kev}. \tag{1}$$

This accuracy for E_r is quite sufficient for our present purposes; an error of 4 kev changes the reaction rate at 1.2×10^8 °K by a factor of less than 1.5. The corresponding excitation energy of the C^{12} level is

$$E_g = (7.654 \pm 0.009) \text{ Mev}.$$

We discuss next the experimental data on the spin and parity of the 7.65-Mev level in C^{12} and on the three partial widths Γ_α, Γ_γ, and Γ_g for its decay into $Be^8 + \alpha$, for a γ-ray cascade to the ground state via the 4.43-Mev $(2+)$ level and for a direct transition to the C^{12} ground state $(0+)$. γ-ray transitions directly to the ground state have never been observed and there is general agreement that Γ_g is very small. Much of the earlier experimental work indicated that Γ_α is much larger than Γ_γ (for instance, from the absence of C^{12} recoils in the excitation of the 7.65-Mev level[12]). The experiments described in the preceding paper (CFLL) establish this fact beyond doubt and demonstrate directly the existence and preponderance of the alpha-particle decay channel. These experiments further yield two inequalities for the branching ratios of the partial widths,

$$\Gamma_\gamma < 0.01 \Gamma_\alpha > \Gamma_g. \tag{2}$$

The preponderance of Γ_α establishes the important fact that the 7.65-Mev state in C^{12} can be reached from $Be^8 + \alpha$ and restricts its possible spin and parity to $0+$, $1-$, $2+$, $3-$, etc.

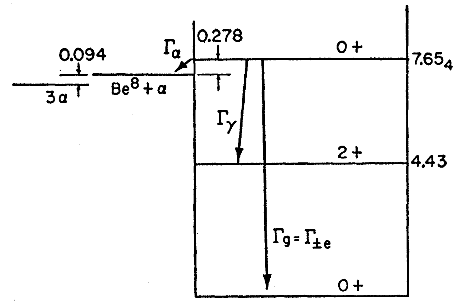

FIG. 1. The relevant energy levels in C^{12}. Energies are expressed in Mev.

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[7] Dunbar, Pixley, Wenzel, and Whaling, Phys. Rev. **92**, 649 (1953).

[8] E. E. Salpeter, in "Symposium on Astrophysics, University of Michigan, July, 1953" (unpublished).

[9] Cook, Fowler, Lauritsen, and Lauritsen, preceding paper [Phys. Rev. **107**, 508 (1957)]. Hereafter referred to as CFLL.

[10] D. M. Van Patter and W. Whaling, Revs. Modern Phys. **26**, 402 (1954); A. H. Wapstra, Physica **21**, 367 (1955).

[11] R. T. Pauli, Arkiv. Fysik **9**, 571 (1955); K. Ahnlund, Arkiv Fysik **10**, 369 (1956).

[12] Rasmussen, Miller, and Sampson, Phys. Rev. **100**, 181 (1955). Only one of the earlier investigations [R. G. Uebergang, Australian J. Phys. **7**, 279 (1954)] seems to be inconsistent with the result that $\Gamma_\alpha \gg \Gamma_\gamma$.

From the purely theoretical grounds of nuclear models, the assignments 0+ or 2+ are the most probable ones. All the experiments relating to the 7.65-Mev level are consistent with it being 0+ and many of them give at least very strong circumstantial evidence against any other spin and parity assignment (such as the absence or extreme weakness of γ-ray transitions directly to the ground state. For other arguments, see CFLL). Conclusive proof that transitions directly to the ground state proceed via electron-positron pair emission would clinch the 0+ assignment. The presence of such pairs has been reported, but possibly not yet conclusively. However, some quite definite conclusions can also be drawn from the Stanford experiments[13] on inelastic high-energy electron scattering from C^{12}: Both absolute scattering cross sections and angular distributions have been measured for inelastic electrons corresponding to excitation from the C^{12} ground state (0+) to the 4.43-Mev level (2+) and to the 7.65-Mev level in question. An analysis[14] of the angular distribution, if it were known completely, would determine the spin and parity of the level. The present data are compatible with 0+ and 2+, although 0+ gives the better fit. If we assume the 0+ assignment, then the absolute inelastic scattering cross section gives the electric-monopole matrix element.[15] This in turn is related to the partial width $\Gamma_{\pm e}=\Gamma_g$ for the electron-positron pair emission accompanying the direct transition from the 7.65-Mev level to the ground state and gives

$$\Gamma_{\pm e}=\Gamma_g=4\times10^{-5}\text{ ev}. \tag{3}$$

On the other hand, if we were to assume a 2+ assignment, the same electron-scattering data would yield quite a different value for Γ_g, which would now correspond to an electric-quadrupole γ-ray transition directly to the ground state. This value would be about $\Gamma_g\approx4\times10^{-2}$ ev, which we shall see is much too large to be compatible with the California Institute of Technology data embodied in Eq. (2).

We come now to theoretical estimates of the partial widths Γ_α and Γ_γ. An upper limit to the alpha-particle width Γ_α is given by the so-called Wigner limit[16] and a rough estimate for the radiation width Γ_γ is given by the Weisskopf formula,[16] once we have made a spin and parity assignment. The Wigner limit Γ_α depends on the l-value of the alpha particle (and hence on the spin assignment for the level) and on the nuclear radius of C^{12} through the barrier-penetration factor. Our uncertainty in nuclear radii introduces an error of about a factor of two or three into the Wigner limit; with a C^{12} radius of 5.2×10^{-13} cm we find $\Gamma_\alpha<7$ ev if the state is 0+ (s-wave alpha-particles) and $\Gamma_\alpha<0.2$ ev if the state is 2+ (d-wave alpha-particle). For the 2+ assignment one would get from the Wigner limit and from Eq. (2) the inequality $\Gamma_g<2\times10^{-3}$ ev, which would clearly be incompatible with the value of $\Gamma_g\approx4\times10^{-2}$ ev, which the electron-scattering data would give for 2+. We therefore discard the 2+ assignment, assume 0+ and hence Eq. (3) for $\Gamma_{\pm e}=\Gamma_g$. The first step in the cascade transition from the 7.65-Mev level to the ground state is then an electric-quadrupole transition to the 4.43-Mev state with the emission of a 3.32-Mev γ ray. The Weisskopf estimate for Γ_γ gives about 5×10^{-3} ev.

Instead of considering the Wigner limit for Γ_α and the Weisskopf estimate for Γ_γ, it may be of interest to use an extremely crude but explicit model for the C^{12} nucleus: We consider the nucleus as a two-particle system made up of one alpha-particle and one ("point-particle") Be^8 nucleus. For the potential $V(r)$ between the two particles we take a repulsive core of radius r_c, surrounded by an attractive square-well potential of depth V_0 out to a radius R. We fit the three parameters V_0, r_c, and R by requiring the excitation energies of the lowest D-state and the first excited S-state to have the correct experimental values of 4.43 Mev and 7.65 Mev, respectively, and for the binding energy Q_1 of the first excited S-state to be 278 kev, Eq. (1). We find $r_c=0.7\times10^{-13}$ cm and $R=5.2\times10^{-13}$ cm. Γ_α and Γ_γ can now be evaluated explicitly and the results are $\Gamma_\alpha=2.5$ ev and $\Gamma_\gamma=2.2\times10^{-2}$ ev. This model should give upper limits both for Γ_α and for Γ_γ, since the overlap integral between initial and final state of the substructure which we call the "Be^8-particle" has been replaced by unity in our model. In fact, according to the simplest "pure" alpha-particle model or shell model one would expect Γ_γ to be extremely small. However, it has been shown[15,17] that both the 4.43-Mev and the 7.65-Mev states must have fairly "impure" wave functions from the point of view of either model and we expect Γ_γ to be lower than our upper limit of 2×10^{-2} ev by not much more than a factor of 10. The quantity we shall need for the reaction rate is the sum of the two partial widths for transitions which lead to the ground state and we adopt

$$\Gamma_{\pm e}+\Gamma_\gamma=1\times10^{-3}\text{ ev}. \tag{4}$$

We have the value of Eq. (3) as a rigorous lower limit for $\Gamma_{\pm e}+\Gamma_\gamma$ and the upper limit of 2×10^{-2} ev given by the model described above. The value of Eq. (4) can thus be in error by a factor of at most twenty and in reality the error is probably considerably smaller than that.

3. RATE OF THE $3\alpha\rightarrow C^{12}$ REACTION

The individual steps in the process we are considering are

$$\begin{gathered}2\alpha+94\text{ kev}\rightarrow Be^8,\\ Be^8+\alpha+278\text{ kev}\rightarrow C^{12*},\\ C^{12*}\rightarrow C^{12}+2\gamma+7.654\text{ Mev},\end{gathered} \tag{5}$$

[13] J. H. Fregeau and R. Hofstadter, Phys. Rev. **99**, 1503 (1955); Phys. Rev. **104**, 225 (1956).
[14] L. I. Schiff, Phys. Rev. **96**, 765 (1954).
[15] B. F. Sherman and D. G. Ravenhall, Phys. Rev. **103**, 949 (1956); also private communication from Dr. Ravenhall.
[16] J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).
[17] R. A. Ferrell and M. Visscher, Phys. Rev. **104**, 475 (1956).

and the net result is

$$3\alpha \rightarrow C^{12} + 2\gamma + 7.282 \text{ Mev}. \quad (6)$$

We are interested in the rate of this reaction for temperatures of the order of 10^8 °K. Between about 0.8 and 10 ($\times 10^8$ °K) the overwhelming contribution to the rate of reaction (6) comes from the resonances of Eq. (5) and we can neglect the nonresonent contribution to the reaction. The reaction rate can be calculated by using the Breit-Wigner single-level formula for reaction cross sections for the steps in Eq. (5) and integrating the Maxwell energy distribution of the alpha particles over each of the two relevant resonances. However, we can obtain the same result more simply from statistical mechanics.

The partial width for the absorption of an alpha particle by a Be^8 nucleus is negligibly small compared with the partial width for the breakup of a Be^8 nucleus into two alpha particles. A dynamic equilibrium is thus set up between the concentrations of Be^8 and He^4. Similarly the inequality, Eq. (2), shows that the partial width for the decay of the second excited state C^{12*} to the C^{12} ground state is small compared with the width for its breakup into $Be^8+\alpha$. Thus the concentration of the C^{12*} state is also in equilibrium with those of Be^8 and He^4. The law of mass action[18] then gives the equilibrium concentration n_* of second excited states C^{12*} of C^{12} in terms of the concentration n_α of He^4 (just as for the equilibrium between a triatomic molecule and its constituent atoms). This relation is

$$n_* = n_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha kT} \right)^3 3^{\frac{3}{2}} e^{-E_r/kT}, \quad (7)$$

where M_α is the mass of the alpha particle and E_r is the energy difference between C^{12*} and three free alpha particles, Eq. (1). The number P of C^{12} nuclei in their ground state which are formed per cc per second is then simply $P = n_*(\Gamma_g + \Gamma_\gamma)/\hbar$, where $\hbar/(\Gamma_g+\Gamma_\gamma)$, Eq. (4), is the mean decay time for a transition from C^{12*} to the C^{12} ground state. The mean rate of destruction p_α of He^4 per alpha particle per second is $3P/n_\alpha$ and the rate of energy production ϵ per g per second is PQ/ρ, where $Q = 7.282$ Mev is the energy release per reaction, ρ is the density in g/cc and x_α is the fractional abundance (by mass) of He^4.

We find then

$$p_\alpha = 3^{\frac{5}{2}} n_\alpha^2 \left(\frac{2\pi\hbar^2}{M_\alpha kT} \right)^3 e^{-E_r/kT} \left(\frac{\Gamma_{\pm e} + \Gamma_\gamma}{\hbar} \right)$$

$$= 2.37 \times 10^3 \left(\frac{\rho x_\alpha}{10^5} \right)^2 \left(\frac{\Gamma_{\pm e} + \Gamma_\gamma}{10^{-3} \text{ ev}} \right) T^{-3} \times \exp\left(-\frac{43.2}{T} \right) \text{ sec}^{-1}, \quad (8)$$

TABLE I. The rate of energy production ϵ_0 (in erg/g/sec) for temperatures T_0 (in 10^8 °K). The quantity n is the exponent in Eq. (10).

T_0	0.85	1.0	1.1	1.2	1.35	1.5	2.0
ϵ_0	0.19	2.4×10^2	9.1×10^3	1.85×10^5	7.1×10^6	1.3×10^8	7.3×10^{10}
n	47.8	40.2	36.2	33.0	29.0	25.8	18.6

where T is the temperature in units of 10^8 °K. The rate of energy production is

$$\epsilon = 1.38 \times 10^{21} \left(\frac{\rho}{10^5} \right)^2 x_\alpha^3 \left(\frac{\Gamma_{\pm e} + \Gamma_\gamma}{10^{-3} \text{ ev}} \right) T^{-3} \times \exp\left(-\frac{43.2}{T} \right) \text{ erg/g sec}. \quad (9)$$

For temperatures T near T_0, ϵ can be written in the form

$$\epsilon = \epsilon_0 (T/T_0)^n. \quad (10)$$

For $\rho^2 x_\alpha^2 = 10^{10}$ and for $\Gamma_{\pm e} + \Gamma_\gamma = 10^{-3}$ ev, the parameters ϵ_0 and n are given in Table I for a number of temperatures T_0. The mean rate p_α of destruction of alpha particles, Eq. (8), has the same strong temperature dependence as the energy production in Table I. For $\rho x_\alpha = 10^5$ and $T = 1.0$, for instance, the mean life of helium, p_α^{-1}, is 7.7×10^7 years.

The possible errors inherent in Eq. (8) and (9), or the numerical values in Table I, can be summarized as follows: we have included in the reaction rate only the resonance contribution from the 7.65-Mev level in C^{12}. In stellar cores which are "burning helium," only a fairly narrow temperature range is of importance; for T less than about 0.9 ($\times 10^8$ °K) the rate of energy production is negligible; for T larger than 2 or 3 ($\times 10^8$ °K) the rate is so enormous that practically all the helium has been used up before such temperatures are reached. In this temperature range the error due to neglecting the nonresonant contribution is negligibly small. The error in the rate due to the uncertainty in the resonance energy E_r is only about 40%. The largest uncertainty lies in the partial width, $\Gamma_{\pm e} + \Gamma_\gamma \approx 10^{-3}$ ev, which could be in error by a factor of not more than 20 either way. The rates in Eq. (8) and (9) are simply proportional to the square of the density ρ. At high densities ($\rho \gtrsim 10^5$) the effects of electron screening can increase the rate by an appreciable amount. This is discussed in Sec. 5.

4. SUBSEQUENT (α,γ) REACTIONS

We calculate next the rates of the successive (α,γ) reactions, starting with the C^{12} produced by the $3\alpha \rightarrow C$ reaction. Such (α,γ) reactions are exoenergetic for the whole chain of nuclei with $Z = N =$ even, from C^{12} up to Ca^{40}. However, at temperatures less than 2×10^8 °K, (α,γ) reactions on Mg^{24} and heavier nuclei are negligibly slow and we only consider the three (α,γ) reactions

[18] J. E. Mayer and M. G. Mayer, *Statistical Mechanics* (John Wiley and Sons, Inc., New York, 1940), p. 206.

TABLE II. Some of the relevant quantities involved in the calculation of the rates of three (α,γ) reactions.

	$\tau T^{\frac{1}{3}}$	Q (Mev)	E_m (Mev)	R (10^{-13} cm)	y	ξ_l^2	γ_W^* (Mev)	θ_α^2
C→O	69.2	7.15	0.23	5.27	13.0	3.5×10^6	0.74	0.07
O→Ne	85.6	4.75	0.29	5.5	19.7	2.8×10^8	0.63	0.02
Ne→Mg	100.7	9.31	0.34	5.7	26	4.1×10^9	0.57	0.10

on C^{12}, O^{16}, and Ne^{20}. The energy release Q (in Mev) for these three reactions is given in the second column in Table II.

Let $A_1=2Z_1=4$, $A_2=2Z_2$ be the atomic weight of the alpha-particle and of the colliding nucleus (C^{12}, O^{16}, or Ne^{20}, spin zero, parity even) and E their relative kinetic energy (center-of-mass coordinates). We shall use the Breit-Wigner single-level formula for the contribution to the reaction rate from a level of spin l and parity $(-)^l$ of the compound nucleus A_1+A_2 (O^{16}, Ne^{20}, or Mg^{24}), whose energy (relative to a free α particle plus colliding nucleus A_1) is E_r. We need first of all theoretical estimates for the partial alpha-decay width Γ_α of the level at energy E (not E_r). We define a nuclear radius R for the collision by

$$R=[1.3(A_1+A_2)^{\frac{1}{3}}+2.0]\times10^{-13}\text{ cm}. \quad (11)$$

Our rather arbitrary recipe Eq. (11) for the fiction of a sharp nuclear boundary R gives values which are slightly smaller and vary slightly less strongly with A_2 than the more usual $1.45(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})\times10^{-13}$ cm. We assign a probable error of about 10% to our values of R (see column three, Table II).

We further define two dimensionless quantities

$$y=\frac{R2\mu e^2Z_1Z_2}{\hbar^2},\quad \eta=\frac{e^2}{\hbar v}=0.157Z_1Z_2\left[\frac{A_1A_2}{(A_1+A_2)E}\right]^{\frac{1}{2}}, \quad (12)$$

where μ is the reduced mass and E is the relative kinetic energy expressed in Mev. For our cases, $2\pi\eta$ is large compared with unity and the α width Γ_α can be written in the form

$$\Gamma_\alpha(E)=\xi_l^2e^{-2\pi\eta}\gamma^*\equiv\xi_l^2e^{-2\pi\eta}\theta_l^2\gamma_W^*,$$
$$\xi_l^2\equiv\frac{ye^{2\pi\eta}}{\eta G_l^2(\eta,y)},\quad \gamma_W^*\equiv\frac{\gamma_W^2}{R}=\frac{3\hbar^2}{2MR^2}. \quad (13)$$

In Eq. (13), $\gamma^*\equiv\gamma^2/R$ is the reduced alpha-particle width in energy units, γ_W^* is the Wigner-Teichmann upper limit to this quantity and θ_l^2 is a constant, smaller than unity, which depends on the internal structure of the resonance level. $G_l(\eta,y)$ is the irregular Coulomb wave function, normalized in the usual way.[19] The dimensionless quantity ξ_l^2 is a slowly varying function of η and hence of the energy E. The cross-section factor S [S in laboratory coordinates, see I, Eq. (7)], using the Breit-Wigner single-level formula, is then given by

$$\frac{A_2S}{A_1+A_2}\equiv e^{2\pi\eta}E\sigma(E)$$
$$=(2l+1)\frac{\pi\hbar^2}{2\mu}\frac{(\xi_l^2\gamma^*)\Gamma_\gamma}{[(E-E_r)^2+\Gamma^2/4]}, \quad (14)$$

where μ is the reduced mass, Γ_γ is the partial width for a γ-ray transition from the resonance level to the ground state of the compound nucleus, and $\Gamma=\Gamma_\alpha+\Gamma_\gamma$ is the total width.

The total reaction rate p is obtained by integrating the cross section $\sigma(E)$, obtained from Eq. (14), times the Boltzmann factor which is proportional to $e^{-E/kT}$, over all energies E. In the integrand the function ξ_l varies slowly with the energy E and the two factors which vary strongly with E are the energy denominator in Eq. (14) and the exponential factor $\exp(-2\pi\eta-E/kT)$. This exponential factor has a maximum value of $e^{-\tau}$ at an energy E_m and the behavior of the factor near its maximum is

$$\exp(-2\pi\eta-E/kT)\approx e^{-\tau}\exp\left[-\left(\frac{E-E_m}{2E_m/\sqrt{\tau}}\right)^2\right],$$
$$E_m=kT\tau/3,\quad \tau=42.48\left[\frac{Z_1^2Z_2^2A_1A_2}{(A_1+A_2)100T}\right]^{\frac{1}{3}}, \quad (15)$$

where T is the temperature in units of 10^8 °K. In the first and third columns of Table II the constants $T\tau^{\frac{1}{3}}$ and the values of E_m for a temperature of $T=1.2$ are given. In all our cases, $\tau\gg1$ and the energy width of the Gaussian in Eq. (15), $2E_m/\sqrt{\tau}$, is small compared with E_m but very large compared with the total width Γ of the resonance level. The integral over E which yields the reaction rate p can then be approximated by the sum of two terms. One of them, the nonresonant contribution p_n from energies near E_m, is obtained by substituting the expression (14) for S into Eq. (9) of reference I. It is

$$\frac{p_n}{x_1}=\frac{(2l+1)(A_1+A_2)^2}{A_1^3A_2^2Z_1Z_2}\tau^2e^{-\tau}\frac{\xi_l^2\gamma^*\Gamma_\gamma}{(E_m-E_r)^2+\Gamma^2/4}\times2.82\times10^8\text{ sec}^{-1}, \quad (16)$$

where ξ_l is evaluated at an energy of E_m, ρ is the density in g/cc and x_1 is the fractional abundance (by mass) of species 1 (the alpha particles). The quantity p^{-1} is the mean lifetime of a nucleus of species 2 in seconds. If E_r corresponds to a positive relative kinetic energy between the nuclei 1 and 2, then we get another contribution p_r to the reaction rate from energies in the vicinity of E_r. This contribution is

$$\frac{p_r}{x_1}=\frac{2l+1}{A_1}e^{-E_r/kT}N_0\left(\frac{2\pi\hbar^2}{\mu kT}\right)^{\frac{3}{2}}\frac{\Gamma_\alpha\Gamma_\gamma}{\hbar(\Gamma_\alpha+\Gamma_\gamma)}, \quad (17)$$

[19] I. Bloch *et al.*, Revs. Modern Phys. **23**, 147 (1951).

where N_0 is Avogadro's number, μ is the reduced mass, and Γ_α is the partial width, Eq. (13), evaluated at an energy of E_r. If $\Gamma_\alpha \ll \Gamma_\gamma$, then p_r does not depend on Γ_γ and Eq. (17) reduces to

$$\frac{p_r}{\rho x_1}=\frac{1}{A_1}\left(\frac{A_1+A_2}{A_1A_2}\right)^{\frac{3}{2}}T^{-\frac{3}{2}}\exp\left(-\frac{E_r}{kT}-2\pi\eta_r\right)\times\xi_l^2(2l+1)\gamma^*\times 4.87\times 10^{12}\ \mathrm{sec}^{-1}, \quad (18)$$

where the expressions η_r and ξ_l^2, Eqs. (12) and (13), are evaluated at an energy of E_r. T is the temperature in units of 10^8 °K and γ^* is the reduced alpha-particle width in Mev.

In the last three columns of Table II we give the values we shall adopt for the factors ξ_l^2 and θ_α^2 and the Wigner limit γ_W^* to γ^* for the three (α,γ) reactions considered. We now discuss the choice of these numbers and the reaction rates in more detail.

(i) $C^{12}(\alpha,\gamma)O^{16}$

The energy release[10] Q of this reaction is (7.148 $\pm$0.008) Mev. There are no resonance levels[20] in O^{16} for positive values of E_r less than at least 1.5 Mev (the 8.6-Mev level). We can therefore neglect any resonant contribution p_r to the reaction rate and consider only Eq. (16) for the nonresonant contribution p_n. This contribution comes overwhelmingly from the fourth excited (7.115$\pm$0.012)-Mev level, i.e., from $E_r=-(0.033 \pm 0.02)$ Mev. The contributions from the three lower excited states is negligible because their gamma-ray widths Γ_γ are much smaller than those from the higher excited states because the energy denominator in Eq. (16) is much larger for them.

From Eq. (15) the reaction mainly involves relative kinetic energies near $E_m=0.20T^{\frac{2}{3}}$ (with T in 10^8 °K) and the energy denominator in Eq. (16) reduces approximately to

$$(E_m-E_r)^2\approx(0.20T^{\frac{2}{3}}\ \mathrm{Mev})^2(1+0.3T^{-\frac{2}{3}}).$$

The 7.12-Mev resonance level has spin and parity 1$-$ and requires an alpha-particle with $l=1$. The Wigner limit of the reduced alpha width γ_W^* and the factor ξ_l^2 are given in Table II for our assumed nuclear radius of $R=5.27\times10^{-13}$ cm. An analysis of elastic alpha-particle scattering[21] from C^{12} indicates that (for the higher resonance levels) the value of R is uncertain by not much more than 10%, which introduces an uncertainty of a factor of about two into ξ_l^2. Elastic α-scattering also gives values for the reduced widths γ^* of the levels at higher energy. For most of these, the value of $\theta_\alpha^2=\gamma^*/\gamma_W^*$ lies between 0.01 and 1. For the 7.12-Mev level we adopt a value of $\gamma^*=0.05$ Mev, i.e., $\theta_\alpha^2=0.07$, which should be in error by a factor of less than fifteen either way.

The 7.12-Mev gamma-ray transition to the ground state is an electric-dipole ($E1$) one. For an allowed $E1$ transition the Weisskopf estimate for the gamma-ray width Γ_γ is about 200 ev. However, to a first approximation, the transition is forbidden since both initial and final states have isotopic spin zero. Theoretical estimates[22] of isotopic-spin mixing indicate a width of the order of 10^{-3} of the Weisskopf estimate. Nuclear-recoil experiments gave a limit of $\Gamma_\gamma \gtrsim 0.08$ ev and a recent resonance-fluorescence experiment[23] gives a value of $\Gamma_\gamma\approx0.13$ ev, to within a factor of about two.

Using this value of $\Gamma_\gamma=0.13$ ev, we thus find for the cross section factor S, defined in Eq. (14),

$$S=\frac{2.0\times10^4}{(0.20T^{\frac{2}{3}}+0.03)^2}\ \mathrm{ev\ barns}.$$

The reaction rate, given by Eq. (16), is

$$\frac{p}{\rho x_\alpha}=\frac{5.3\times10^9\times e^{-69.20/T^{\frac{1}{3}}}}{T^2(1+0.3T^{-\frac{2}{3}})}\ \mathrm{sec}^{-1}, \quad (19)$$

to within a factor of about twenty either way.

(ii) $O^{16}(\alpha,\gamma)Ne^{20}$

The energy release Q of this reaction[10] is (4.75$\pm$0.02) Mev. There are two resonance levels of Ne^{20} in the relevant energy region[24] at (4.95$\pm$0.02) Mev and at (5.62$\pm$0.02) Mev, i.e., with $E_r=(0.20\pm0.03)$ Mev and $E_r=(0.87\pm0.03)$ Mev. Unfortunately nothing is known about the spin and parity of these two levels and such a level can contribute to the reaction rate only if it is 0+, 1$-$, 2+, etc. We shall therefore estimate the resonance contribution p_r for each of the two levels separately and also the nonresonant contribution p_n.

For the lower level, Γ_α is about 10^{-15} ev and thus certainly less than Γ_γ and we can use Eq. (18) for the resonance contribution p_r to the reaction rate. The Wigner limit γ_W^* for the reduced width is given in Table II as well as ξ_l^2 for $l=0$ and $E_r=0.2$ Mev. The factor $(2l+1)\xi_l^2$ does not decrease very rapidly with increasing l-values until $l\sim4$ and is about twenty times smaller for $l=3$ than for $l=0$. We shall use the value of ξ_l for $l=0$, but use a rather small value, $\theta_\alpha^2=0.02$, for γ^*/γ_W^*. This gives for the reaction rate

$$\frac{p_l}{\rho x_\alpha}=T^{-\frac{3}{2}}e^{-23.21/T}\times2.4\times10^{-10}\ \mathrm{sec}^{-1}. \quad (20)$$

The uncertainty in the nuclear radius R, and in the resonance energy E_r, make p uncertain by a factor of

[20] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[21] R. W. Hill, Phys. Rev. **90**, 845 (1953); J. W. Bittner and R. D. Moffat, Phys. Rev. **96**, 374 (1954).

[22] L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 39 (1954); D. H. Wilkinson, Phil. Mag. **1**, 379 (1956).

[23] C. P. Swann and F. R. Metzger, Bull. Am. Phys. Soc. Ser. II, **1**, 211 (1956).

[24] R. G. Freemantle *et al.*, Phys. Rev. **96**, 1270 (1954).

about 3 and of 5, respectively. The biggest uncertainty comes from $(2l+1)\xi_l^2\theta_\alpha^2$—about a factor of 50 either way, if the level is 0+, 1−, 2+, or 3−. The contribution is of course zero if the level is 1+, 2−, etc.

For the resonance contribution p_u from the upper level at $E_r=0.87$ Mev we don't even know whether Γ_α or Γ_γ is the larger. Nevertheless the quantity $\Gamma_\alpha\Gamma_\gamma/(\Gamma_\alpha+\Gamma_\gamma)$, which we need to substitute into Eq. (17), can be estimated to lie within about 2×10^{-4} ev and 2 ev, if the state is 0+, 1−, 2+, 3−, or 4+: for instance, for $l=1$, $\theta_\alpha^2=1$ and a large $E1$ gamma-ray width we get 2 ev for this expression; for $l=3$, $\theta_\alpha^2=0.005$, we get 10^{-3} ev. We thus adopt 0.02 ev for this quantity, to within a factor of 100 either way, and get[25]

$$p_u/\rho x_\alpha=T^{-\frac{3}{2}}e^{-101.0/T}\times5\times10^3\ \mathrm{sec}^{-1}. \qquad (21)$$

To get an order of magnitude estimate for the nonresonant reaction rate p_n, we assume the following hypothetical level: $E_m-E_r\sim1$ Mev, $l=0$, $\theta_\alpha^2\sim0.03$, and $\Gamma_\gamma\sim0.05$ ev. These numbers and Eq. (16) give

$$p_n/\rho x_\alpha\sim10^9T^{-\frac{2}{3}}e^{-85.6/T^{\frac{1}{3}}}\ \mathrm{sec}^{-1}. \qquad (22)$$

In the temperature range $T=1$ to 2, both the contributions p_u and p_n, Eqs. (21) and (22), are negligibly small compared with the resonance contribution p_l from the lower level, Eq. (20).

(iii) $\mathrm{Ne}^{20}(\alpha,\gamma)\mathrm{Mg}^{24}$

The energy release of this reaction is 9.31 Mev. The resonance levels in Mg^{24} have been studied by elastic alpha-particle scattering[26] for α-energies of $E_r=2$ Mev and larger. The average level spacing between levels with 0+, 1−, 2+, or 3− is about 150 kev. The optimum energy E_m, defined in Eq. (15), varies from 290 kev at $T=1$ to 460 kev at $T=2$ and the width $2E_m/\sqrt{\tau}$ of the Gaussian factor in Eq. (15) is about 60 kev. We therefore overestimate the reaction rate by a factor not much bigger than unity if, for each temperature T separately, we assume there is just one level with energy $E_m=E_r$. At these energies, $\Gamma_\alpha\ll\Gamma_\gamma$ and we can use Eq. (18). If we assume $l=0$ and $\theta_\alpha^2=0.1$, we obtain

$$p/\rho x_\alpha=T^{-\frac{2}{3}}e^{-100.7/T^{\frac{1}{3}}}\times4.6\times10^{19}\ \mathrm{sec}^{-1}. \qquad (23)$$

This expression is probably an overestimate and the correct rate should lie between about 10^{-3} and 10 times the rate given in Eq. (23).

Even an upper limit for the rate of the $\mathrm{Mg}^{24}(\alpha,\gamma)\mathrm{Si}^{28}$ reaction is much smaller than the rate in Eq. (23) for $\mathrm{Ne}^{20}(\alpha,\gamma)\mathrm{Mg}^{24}$ and we shall not consider this or subsequent reactions.

[25] W. W. Buechner and A. Sperduto, Massachusetts Institute of Technology Annual Progress Report, May, 1955 (unpublished), give preliminary values of $E_r=0.22$ Mev and 1.06 Mev for the lower and upper level (instead of Freemantle's 0.20 and 0.87). These values, if confirmed, would increase p_l slightly and decrease p_u quite considerably. The contribution from the upper level would then be even more negligible.

[26] E. Goldberg *et al.*, Phys. Rev. **93**, 799 (1954).

5. OTHER REACTIONS AND RESULTS

We have so far discussed only the reaction chain which can take place, starting from pure He^4. The interior of the star may also contain small amounts of other nuclear species, in particular carbon and heavier elements. At temperatures in the range $T=1$ to 2 (in 10^8 °K) we can neglect alpha-particle reactions on nuclei heavier than neon. The alpha-particle reactions on all the isotopes from carbon to neon (besides those on C^{12}, O^{16}, Ne^{20} which we have already discussed) are listed in Table III.

The importance of the $\mathrm{C}^{13}(\alpha,n)$ reaction for the formation of heavy and neutron-rich nuclei in the interior of stars was first pointed out by Cameron.[5] The role played by the other two (α,n) reactions in Table III in nucleogenesis has been discussed by Fowler *et al.*,[5] and some of the reaction rates have also been calculated.[27] From the point of view of energy production (at temperatures below $T=2$), only those reactions in Table III could be of any importance which are much faster than the $3\alpha\rightarrow\mathrm{C}$ reaction, since the initial nuclei in Table III have low abundances. We shall see that only the $\mathrm{C}^{13}(\alpha,n)$ reaction is very fast at the lower temperatures.

We have calculated the rate of the $\mathrm{C}^{13}(\alpha,n)\mathrm{O}^{16}$ reaction as follows: The energies and neutron widths Γ_n of all resonance levels in O^{17} (near the relevant energy region) are known from elastic scattering of neutrons from O^{16} [see Table I (17) of reference 20]. The main contribution comes from the $J=\frac{1}{2}+$level at 6.30 or 6.34 Mev ($E_r=-0.04$ or 0 Mev) with neutron width $\Gamma_n=120$ kev. This state can be reached by an alpha particle with $l=1$ impinging on the ground state of C^{13}. We shall use the nonresonant single-level Breit-Wigner contribution, Eqs. (14) and (16), just as we had done for $\mathrm{C}^{12}(\alpha,\gamma)$ in the previous section, except for two modifications: Γ_γ is replaced by $\Gamma_n=120$ kev and the statistical weight factor $(2l+1)$ has to be generalized to $(2J_{17}+1)/(2J_{13}+1)=1$, where J_{17} is the spin of the resonance level in O^{17} and J_{13} that of C^{13}. For the Wigner limit to the alpha-particle width we use the

TABLE III. The reactants ("final") in alpha-particle reactions on various nuclei ("initial"). Q is the energy release in Mev and t is the mean life for the reaction (in years) for $T=1.2$ and $\rho x_\alpha=10^5$.

Initial	Final		Q	$\log_{10}t$
C^{13}	O^{16}	n	2.10	1.4
O^{17}	Ne^{20}	n	0.60	6
Ne^{21}	Mg^{24}	n	2.58	8
N^{14}	F^{18}	γ	4.41	8
N^{15}	F^{19}	γ	3.99	5
O^{18}	Ne^{22}	γ	9.66	5
Ne^{22}	Mg^{26}	γ	10.64	9
F^{19}	Ne^{22}	p	1.70	7

[27] J. B. Marion and W. A. Fowler, Astrophys. J. **125**, 221 (1957).

same numbers as for $C^{12}(\alpha,\gamma)$, for θ_α^2 we use 0.03, accurate to within a factor of 30 either way. Our cross-section factor is about 1.4×10^5 times that for $C^{12}(\alpha,\gamma)$ and our reaction rate[28] for $C^{13}(\alpha,n)O^{16}$ is

$$p=(\rho x_\alpha)7\times10^{14}\times T^{-2}\exp(-69.65/T^{\frac{1}{3}})\ \mathrm{sec}^{-1} \quad (24)$$

to within a factor of about 30 either way.

For the other reactions in Table III we have estimated the rates only crudely, using the methods described in Sec. 4, but making rougher approximations. In the last column in Table III we give our rough estimates for t, the mean reaction time (in years) at temperature $T=1.2$ and $\rho=10^5$. It will be seen that the other reactions are much slower than $C^{13}(\alpha,n)$: The reaction on N^{14} is slow since there is no resonance in F^{18} in the relevant energy region; for N^{15} the main contribution does come from a resonance but the resonance energy is rather high ($E_r=0.42$ Mev). For $O^{17}(\alpha,n)$ there is again no resonance in the relevant energy region. For the reactions on O^{18}, F^{19}, and the Ne isotopes we have overestimated the rates somewhat by assuming one resonance level just at the optimum energy E_m (the level densities in these cases are indeed large[29] and there should be levels fairly close to E_m).

The main numerical results of this paper are summarized in Fig. 2, in which the logarithm of the mean reaction time t ($t=p^{-1}$) is plotted against temperature T. The curves are for density $\rho=10^5$ g/cc and almost pure He ($x_\alpha=1$). For other densities and He-abundances note that t^{-1} is proportional to $(\rho x_\alpha)^2$ for $3\alpha\rightarrow$C and proportional to ρx_α for all the other reactions. The four solid curves refer to $3\alpha\rightarrow$C and to the three subsequent

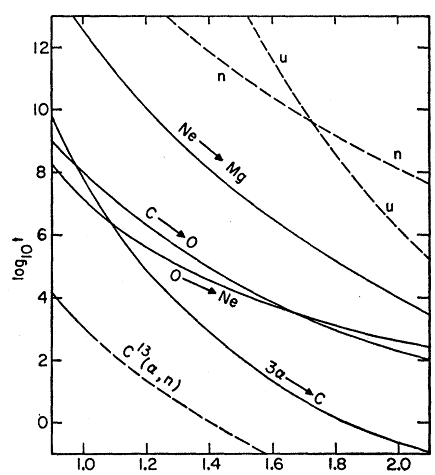

FIG. 2. The mean lifetime t (in years) plotted against temperature T (in 10^8 °K) for a helium density of 10^5 g/cc. The dotted curves marked u and n refer to the O→Ne reaction, assuming only the resonance contribution from the upper level and the nonresonant contribution, respectively.

[28] Our rate is somewhat faster than that calculated by Cameron[5] and somewhat slower than that of Marion and Fowler.[27]

[29] P. M. Endt and J. C. Kluyver, Revs. Modern Phys. **26**, 95 (1954).

TABLE IV. The logarithm of the mean lifetime t (in years) at $T=1.2$ of He, C^{12}, O^{16}, Ne^{20}, and C^{13}, for densities such that $\rho x_\alpha=10^5$ g/cc, together with estimates for the possible error in this logarithm. $(O-Ne)_l$ holds if the lower resonance state in Ne^{20} has suitable spin and parity, otherwise $(O-Ne)_n$ holds. $\Delta\log t$ is the effect on the lifetime of electron screening at density $\rho=10^5$ and temperature $T=1.2$.

	3α→C	C→O	(O→Ne)$_l$	(O→Ne)$_n$	Ne→Mg	$C^{13}(\alpha,n)O^{16}$
$\log_{10}t$	5.0	6.3	5.6	13.6	10.0	1.4
error	±1.2	±1.3	±2.0	±3.0	±2.0	±1.5
$\Delta\log_{10}t$	−0.26	−0.22	−0.26	−0.26	−0.30	−0.22

(α,γ) reactions; for O→Ne the solid curve assumes that the lower resonance level in Ne^{20} does have the required spin-parity relation. The dotted curves in the upper right-hand corner were calculated for O→Ne, assuming only the nonresonant contribution in one case, and only the contribution from the upper resonance level in the other case. The dotted curve in the lower left-hand corner refers to the $C^{13}(\alpha,n)O^{16}$ reaction.[30]

In Table IV we give again $\log t$ for the various reactions discussed at one temperature $T=1.2$, together with estimated limits of error for $\log t$. Note how remarkably similar the reaction times are for the first three reactions 3α→C, C→O, O→Ne. The Ne→Mg reaction is considerably slower. In the last row of Table IV we give rough estimates for the effect of electron screening[31] on the reaction times calculated again for density $\rho=10^5$. For much lower values of ρ the effect is negligible; for higher values up to about $\rho\sim10^7$, the $\Delta\log t$ given in Table IV can be re-estimated by using the rough relationship

$$\Delta\log t\propto(\rho T^{-3})^{0.4}. \quad (25)$$

6. ASTROPHYSICAL DISCUSSION

From the point of view of energy production, the 3α→C and subsequent reactions are now understood well enough. The uncertainty in the rate of the 3α→C reaction itself is now at most a factor of 30 either way and probably much smaller. Any uncertainty about the subsequent reactions has a small effect on the rate of energy production. If the subsequent reactions were slow compared with 3α→C, then the rate of energy production ϵ is given by Eq. (9) or Table I. If the reaction chain from C^{12} to Ne^{20} is much faster than 3α→C, then the energy release per original reaction is 19.2 Mev instead of 7.3 Mev. In this case the values for ϵ in Eq. (9) and Table I would have to be multiplied by a factor of 2.6. Since the conversion to Ne is probably not complete (and the production of Mg is quite negligible below $T\sim1.5$) the correct multiplying factor is probably nearer to 2.0.

In previous calculations on the evolution of Type II stars, Hoyle and Schwarzschild[2] did not use an explicit

[30] The values in Fig. 2 and in Table IV for Ne→Mg are for $\theta_\alpha^2=0.01$, instead of the 0.1 of Table II.

[31] E. E. Salpeter, Australian J. Sci. **7**, 373 (1954).

rate of energy production from $3\alpha \rightarrow C$, but assumed that the rate will become appreciable for temperatures near $T=1.2$ ($\times 10^8$ °K). This qualitative assumption is still justified in the light of our present numbers, but greater accuracy will now be possible in future calculations. In the evolution of Type II red-giant stars,[2] the $3\alpha \rightarrow C$ reaction is important at two different stages. In the first, the electrons are degenerate and the density slightly larger than 10^5 g/cc (and $T \sim 1.0$). The effect of electron screening [see Table IV and Eq. (25)] is to increase the reaction rate by a factor of about 2.0. In the next stage of evolution the density has dropped well below 10^4 g/cc, with a small increase in temperature, and the effect of electron screening is negligible.

As regards nucleogenesis,[4-6] the following can be said. Observationally, the cosmic abundances of C^{12}, O^{16}, and Ne^{20} are roughly comparable. As Table IV and Fig. 2 show, the reaction times for $3\alpha \rightarrow C$, for $C \rightarrow O$ and for $O \rightarrow Ne$ are also roughly the same, if the lower resonance in Ne^{20} has the right spin-parity (and $Ne \rightarrow Mg$ is much slower). For a given temperature, density, and life-history of a star, one can calculate predicted abundance ratios for the C, O, and Ne produced and these ratios will not differ very greatly from unity. This qualitative agreement with observation at present is at least reassuring, considering the extremely large numerical factors that enter in the calculation of these ratios. For instance, if the lower resonance in Ne^{20} did not contribute to the $O \rightarrow Ne$ reaction the Ne-production would be too low by a factor of 10^8; if the resonance level in C^{12} had been at an energy different (from the value which was predicted by Hoyle and later observed) by, say, 200 kev the $3\alpha \rightarrow C$ rate would be off by a similar factor, etc. On the other hand, quantitative comparison with the observational abundance ratios is still difficult. As the limits of error in Table IV show, any abundance ratio predicted at the moment could be in error by a factor of more than 100 either way. Using the numerical rates given in Fig. 2 and simple assumptions about stellar evolution, the predicted Ne/O abundance ratio is larger than the observed one by a factor of more than 10. In view of the present uncertainties, this discrepancy should not be taken too seriously.

We finally discuss the significance of the $C^{13}(\alpha,n)O^{16}$ reaction. If any C^{13} was originally present in the core of the red-giant star, it will certainly be burned up at temperatures well below those ($T \sim 1.2$) necessary for the $3\alpha \rightarrow C^{12}$ reaction. At a density of helium of 10^5 g/cc, the mean reaction time for $C^{13}(\alpha,n)$ is about 4×10^6 years at $T=0.70$ and about 5×10^4 years at $T=0.85$ (in 10^8 °K). From the calculations of Hoyle and Schwarzschild,[2] one can find the central temperature T_c, density ρ_c and total luminosity L as a function of time, for a Type II red-giant star. At one stage of its evolution, the electrons in the helium-core are degenerate and T_c and L are rising rather rapidly. At a particular time, for instance, ρ_c is about 10^5 g/cc, T_c is 0.80 and T_c is rising about 0.01 every 10^5 years. About 2×10^6 years later, T_c has risen from 0.8 to 1.2 and the luminosity L has increased by a factor of about 2.5. One can now find the temperature range in which most of the C^{13} will burn up, namely near $T_c \sim 0.75$ to 0.80. At lower temperatures the reaction rate is too low and before the star has had time to increase T_c much above 0.8 most of the C^{13} has already burned out. Thus the typical reaction time for $C^{13}(\alpha,n)$ in such stars will be about 5×10^5 years. We can now also estimate the peak rate of energy production from $C^{13}(\alpha,n)O^{16}$ and from the subsequent neutron absorption. From the core as a whole we find an energy production of slightly less than $10^4 x_{13} L_0$, where x_{13} is the abundance by mass of C^{13} and L_0 is the sun's luminosity. If this rate of energy production were to exceed the rate of gravitational energy release, about $5L_0$, the structure of the core could change drastically. This would require $x_{13} \gtrsim 10^{-3}$. C^{13} probably constitutes only about $\frac{1}{2}\%$ of the C, N isotopes which have previously gone through the C, N cycle and in most stars the original value of x_{13} is probably less than the required amount. Nevertheless, in view of the uncertainties in some of our numbers, the possible influence of the $C^{13}(\alpha,n)$ reaction on the development of the stellar core should perhaps be kept in mind.

Neglecting the effect of the $C^{13}(\alpha,n)$ reaction, we can now estimate somewhat more accurately the temperatures at which helium burning is important in Hoyle and Schwarzschild's[2] model for the evolution of a red-giant star of about 1.2 times the mass of the sun. Near $T_c \sim 1.0$ ($\times 10^8$ °K), the central density is about $\rho_c = 1.7\times 10^5$ g/cc and the gravitational energy release about $10L_0$. The structure of the star's core will begin to change drastically when the total energy production from helium burning roughly equals this amount. If one assumes that the rate of energy production from helium burning averaged over the whole mass of the star is about 5% of the rate at the center of the star, the required central rate of energy production is about 500 erg/g sec. Using the above value of ρ_c, Table I, a multiplying factor of two for the extra energy released in $C \rightarrow Ne$ and another multiplying factor of 2.5 for the effect of electron screening on the reaction rate, we find a required central temperature of $T_c \approx 0.99$. Stars near the "tip of the red-giant branch" should have central temperatures near this value (or slightly higher), but this numerical value is uncertain by about 10%.

After helium burning first takes place, the density of the core decreases, the structure of the star changes and the star is now on the "horizontal branch" of the Hertzsprung-Russell diagram, according to Hoyle and Schwarzschild.[2] About 5% of the total luminosity of the star is now supplied by helium burning in the core (the rest by hydrogen burning in an outer shell) and the central density is about 4×10^3 g/cc. Assuming a central energy-production rate of a few thousand ger/g sec, a central temperature of about $T_c \approx 1.3$ is required (the effect of electron screening is negligible in this case).

I am indebted to Dr. G. Ravenhall for some interesting discussions and to Dr. C. Cook, Dr. W. Fowler, Dr. C. Lauritsen, and Dr. T. Lauritsen both for helpful communications and also for holding up the publication of their paper to coincide with this one. I also wish to thank M. Nauenberg and A. Sirlin for help with some of the calculations.

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Neutron Total Cross Section of Np^{237} from 0.02 to 2.8 ev*†

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The total neutron cross section of Np^{237} has been measured in the energy region 0.02 to 2.8 ev with the Materials Testing Reactor crystal spectrometer. Measurements were made on a sample containing 152 mg of Np^{237} in oxide form dissolved in deuterated nitric acid. A value for the absorption cross section at 0.025 ev of 170±22 barns was obtained. Resonances were observed at energies 0.489±0.002, 1.337±0.015, and 1.488±0.018 ev, with the respective values of $\sigma_0\Gamma$ being 84.2 ev barns, approximately 29 ev barns, and approximately 140 ev barns. Values for σ_0 of 2600±100 barns, Γ of 0.032±0.003 ev, and $g\Gamma_n$ of 0.016 mv were determined for the 0.489-ev resonance.

I. INTRODUCTION

THE thermal-activation cross section of Np^{237} was determined by Jaffey and Magnusson[1] from radiometric measurements of the Np^{238} and Pu^{238} activities resulting from neutron capture in Np^{237}. A recent re-evaluation[2] of this experiment, in which a later value of the half-life of Pu^{238} was used, gave 170±20 barns for the activation cross section of Np^{237}. Since Np^{237} is used as a fast-neutron flux monitor in the presence of slow neutrons, a knowledge of the thermal neutron cross section is necessary to correct for the fission in the Pu^{238} formed. It is of interest to nuclear theory to know the level spacing and other parameters for this nucleus, which is one of the heaviest odd proton-even neutron nuclei available for such measurements.

The problems associated with transmission measurements on limited quantities of material have been treated by Shull and Wollan,[3] and by Bernstein *et al.*[4] An important consideration in such measurements is the arrangement of the sample in the thickest manner consistant with a beam sufficiently large to provide satisfactory counting statistics. The theoretical aspects of optimizing sample geometry have been considered by Rose and Shapiro.[5] With the high neutron flux provided by the Materials Testing Reactor the requirements for sample sizes are less critical, and transmission measurements may be extended to higher energies.

II. EXPERIMENTAL DETAILS

The spectrometer system consists of a collimator located in the reactor shielding, the spectrometer proper, a monochromating crystal, an automatic sample changer, slits to define the Bragg beam, a neutron detector, and the associated control and counting electronics. The neutron beam from the reactor was defined by a steel collimator of inside dimensions $\frac{1}{8}$ in.$\times 2\frac{3}{4}$ in.$\times$96 in. This system, when the (240) planes of NaCl crystal were used, had an overall resolution of 0.9 μsec/meter. The (220) planes of NaCl were used to obtain some of the lower energy data.

The cross-section data reported here were obtained by transmission measurements on a sample of Np^{237} having a total sample cross section of 0.06 cm^2 at 0.025 ev. Since Np^{237} has a half-life of 2.20×10^6 years, the problems of radiolysis and alpha heating are not troublesome, and specialized handling equipment is not necessary. Since only 152 mg of Np were available for the measurements, a special sample changer was developed to allow the placement of the optimum amount of sample in the Bragg beam. For these measurements the Bragg beam was reduced to a width of 0.24 cm and a height of 1.5 cm by a boron carbide and paraffin collimator 12.7 cm long placed adjacent to the sample between the sample and the detector.

The detector was a group of three $B^{10}F_3$ proportional counters located with the cylindrical axes parallel to

* Work carried out under contract with the U. S. Atomic Energy Commission.

† A preliminary report of these measurements was made at the 1955 Washington, D. C. meeting of the American Physical Society [Phys. Rev. **99**, 611(A) (1955)].

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