reported that experiments with low-energy electrons (less than 0.6 Mev) result in defects stable at room temperature (i.e., vacancies and interstitials separated by more than the capture radius) at electron energies which are not likely to displace the germanium atoms by more than a few lattice spacings.

E. Confirmation of the Theory

The theory of diffusion-limited chemical reactions developed here and in the preceding paper provides a satisfactory description of a large portion of the annealing data discussed here. The data which are not fitted adequately appear to scatter considerably owing to experimental error, but may indicate that some of the assumptions outlined in Sec. VI are not satisfied. The present annealing problem does not provide a complete test of some aspects of the theory because the competition of many A's for each of the B's and *vice versa* does not appear to be important and, therefore, a less general theory might suffice. However, it does appear safe to conclude on the basis of the present work that the application of diffusion theory to the kinetics of certain reactions in the solid state leads to satisfactory results.

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High-Frequency Relaxation Processes in the Field-Effect Experiment

C. G. B. GARRETT
Bell Telephone Laboratories, Murray Hill, New Jersey
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A theoretical treatment is given of two dispersion phenomena in the field-effect experiment: (1) dispersion arising from the finite time required to generate minority carriers, and (2) relaxation of the fast surface states. It is shown that the in-phase part of the field-effect mobility is given (for an n-type semiconductor) by $\mu_{FE}=-\mu_n+A/(1+\omega^2\tau_1^2)-B/(1+\omega^2\tau_2^2)$, where $(\omega/2\pi)$ is the frequency of the applied field, A and B are constants, and τ_1 and τ_2 are characteristic times, all four quantities being functions of the body resistivity, surface potential, and of the densities, energy levels, and capture cross sections of the fast states. Under certain conditions, τ_1 is equal to the fundamental decay-mode lifetime of the sample, while τ_2 is expected to be much shorter, and depends primarily on the cross sections and the position of the state level in the gap. A comparison of the theory with recent experimental results of Montgomery shows (1) that reasonable agreement can be obtained, and (2) that the presence of any significant number of states in the region close to the center of the gap is unlikely.

1. INTRODUCTION

THE field-effect experiment is the observation of a change in the conductance of a thin slice of material, caused by the application of an external electric field normal to its surface. Such an effect was looked for in metals around the turn of the century, and the failure to detect it was discussed by Thomson.[1] Since the volume density of electric carriers may be many orders of magnitude less in a semiconductor than in a metal, a fresh attempt at the field-effect experiment was made by Shockley and Pearson[2] some ten years ago, using the high-purity silicon then available for the first time. A modulation of conductance was indeed found, but the magnitude of the effect was less than expected. One possible reason for this was suggested by Bardeen[3]: the semiconductor surface has on it electronic trapping levels, which tie up most of the induced charge in localized sites ("surface states"). Since then, many experiments have confirmed the truth of Bardeen's suggestion. Montgomery and Brown[4] have studied the field-effect experiment in germanium, as a function of the height of the surface space-charge barrier. Brattain and Garrett[5,6] have brought the field-effect experiment into reconciliation with a parallel line of work—the study of surface photovoltage and surface recombination.[7] It is now fairly well established (1) that there are two classes of surface states on germanium and silicon, distinguished by having relaxation times[8] of the order of a second or greater (the "slow states"), or a microsecond or less (the "fast

[1] J. J. Thomson, *The Corpuscular Theory of Matter* (Constable and Company, London, 1907), p. 80.
[2] W. Shockley and G. L. Pearson, Phys. Rev. **74**, 232 (1948).
[3] J. Bardeen, Phys. Rev. **71**, 717 (1947).
[4] H. C. Montgomery and W. L. Brown, Phys. Rev. **98**, 1165 (1955); **103**, 865 (1956).
[5] W. H. Brattain and C. G. B. Garrett, Bell System Tech. J. **35**, 1019 (1956).
[6] C. G. B. Garrett and W. H. Brattain, Bell System Tech. J. **35**, 1041 (1956).
[7] W. H. Brattain and J. Bardeen, Bell System Tech. J. **32**, 1 (1953).
[8] I.e., times for readjustment of state population, by interchange of charge with whichever band is the more accessible.

states"), and (2) that the fast states are also the states responsible for surface recombination.

For some time, the idea has been current that, if one could carry out the field-effect experiment at a high enough frequency, one could "outwit" the fast states, and obtain the full effect, in which every induced charge adds to or subtracts from the conductance of the slice by one electronic carrier. Recently Montgomery[9,10] has reported measurements of the field effect at frequencies up to 50 Mc/sec, using the Aigrain method,[11] which determines the real (in-phase) part of the conductivity modulation. His experiments do indeed show dispersion in the last decade of frequency, which is very reasonably associated with relaxation of the fast states. The experiments also show dispersion in the range where the frequency is comparable with the reciprocal of the minority-carrier filament lifetime. The interpretation of this effect, first given by Brown,[4,12] is this. Let us, for the moment, assume that there are no surface states. Now, if the body minority carrier is present in excess at the field-free surface, a small charge induced by the applied field must appear as a change in the surface excess of this carrier. This change must arise from a flow of minority carriers from the bulk to the surface or *vice versa*. At the frequencies in question, however, there is not time for these minority carriers to be supplied or taken up by thermal generation or recombination. Therefore, to preserve space-charge neutrality in the bulk, an equal number of *majority* carriers must leave or enter the sample via its ohmic contacts. Similar language may be used to describe the situation in which fast states are present.

The present paper offers a theoretical treatment of both dispersion phenomena. The object of the calculations is to present general expressions for the in-phase part of the *field-effect mobility*,[4,5] which is defined as the ratio of the change in surface conductivity to the induced charge density. The method will be to write down expressions for the rates of capture of carriers from the conduction and valence bands by the surface states, and match this to the rates at which carriers diffuse towards the surface from the interior, allowing for the accumulation of holes and electrons in the space-charge region. In this way we implicitly allow for the creation and annihilation of carrier-pairs through the surface states (surface recombination).

The calculations will be carried out for a single surface trapping level, having the properties of the traps considered in the Shockley-Read theory.[13] Of course, since it is not usually possible to account even for the low-frequency field-effect data in terms of a single surface level,[4,6] we must not expect the agreement between the present theory and Montgomery's results to be good over the entire range of surface potential. Properly, we ought to carry out the calculations taking into account at least a pair of levels, one high and one low in the gap[7]; possibly we should even generalize still further, and consider a continuous spectrum of levels.[6] However, it will turn out that the algebra involved is already complicated enough with a single trapping level. Should future developments demand a consideration of the more general case, it should be possible to proceed by a straightforward extension of the arguments to be given below, with no additional difficulties in principle.

2. FUNDAMENTAL EQUATIONS

We shall use the notation of Garrett and Brattain.[6,14] We suppose that there is, on the surface, a trapping level characterized by Shockley-Read parameters[13] C_p, C_n, p_1, and n_1, and let p_s, n_s, and f_t stand for the (time-dependent) concentrations of holes and electrons at the surface, and the (time-dependent) trap-filling factor. Then the rate U_{cn} of capture of electrons by the traps is

$$U_{cn}=C_n[(1-f_t)n_s-f_tn_1], \tag{1}$$

while the rate U_{cp} of capture of holes by the traps is

$$U_{cp}=C_p[f_tp_s-(1-f_t)p_1]. \tag{2}$$

The difference between these is the rate of filling of the traps:

$$U_{cn}-U_{cp}=N_tdf_t/dt, \tag{3}$$

where N_t is the density of traps on the surface.

Another condition to be satisfied is that of overall charge balance. Let Σ be the total charge density (expressed in electrons/cm^2) outside the semiconductor surface, consisting of the sum of the charge associated with slow states (assumed independent of time in what follows) and the field term ($\epsilon\epsilon_0E/4\pi e$). Also let Γ_p and Γ_n stand for the surface excesses[14] of holes and electrons. Then

$$(\Gamma_p-\Gamma_n)-N_tf_t=\Sigma. \tag{4}$$

Now we must bring in the transport of carriers across the space-charge region, and from the space-charge region into the interior. Taking the second topic first, it should be noted that it would be perfectly possible to set the equations up in ambipolar form, but that it is algebraically simpler to work the problem out for a moderately extrinsic semiconductor, so that the transport of added carriers through the bulk may be considered by evaluating the rate of diffusion of minority carriers. We shall therefore choose the semiconductor

[9] H. C. Montgomery (to be published).
[10] H. C. Montgomery and B. A. McLeod, Bull. Am. Phys. Soc. Ser. II, 1, 53 (1956).
[11] Aigrain, Lagrenaudi, and Liandrat, J. phys. radium **13**, 587 (1952).
[12] Garrett, Brattain, Brown, and Montgomery, *Proceedings of the 1956 Philadelphia Meeting on the Physics of Semiconductor Surfaces* (University of Pennsylvania Press, Philadelphia, 1956).
[13] W. Shockley and W. T. Read, Phys. Rev. **87**, 835 (1952).
[14] C. G. B. Garrett and W. H. Brattain, Phys. Rev. **99**, 376 (1955).

to be extrinsic n-type, so that holes are the minority carriers. We further introduce a quantity δ, defined as the geometric mean shift of the quasi-Fermi levels for holes and electrons near the surface, measured in units (kT/e). The minority-carrier current I_p away from the surface, at a point just below the space-charge region, may now be related to δ by solving the diffusion equation for minority-carrier transport below the surface, taking into account whatever boundary conditions are appropriate to the problem (see Sec. 4). In general we shall find that, for small-signal ac conditions,

$$I_p=Z\delta, \tag{5}$$

where Z will usually be a complex function of the frequency, lifetime, etc.:

$$Z=Z'+iZ''. \tag{6}$$

We turn lastly to the transport of carriers across the space-charge region. Firstly, it should be said that we are not primarily interested in transit-time effects in this paper, largely because their consideration presents severe analytic problems. It is also convenient to neglect recombination in the space-charge region. With these two assumptions, the hole and electron currents across the space-charge region may be treated as solenoidal. Their magnitudes are also sufficiently small for one to be able to assume that the change in quasi-Fermi levels for holes and electrons across the space-charge region may be neglected. Thus δ can stand indifferently for the geometric mean shift of the Fermi levels, either at the surface itself, or at a point just below the space-charge region. One may also assume, as in the dc case, that Γ_p and Γ_n are functions of the two independent variables δ and Y, the difference in electrostatic potential across the space-charge region, measured in units (kT/e). Finally we may write down the last fundamental equation:

$$U_{cp}+I_p+d\Gamma_p/dt=0. \tag{7}$$

The next stage of the analysis will be to look for sinusoidally-varying solutions of Eqs. (1) through (7). It will be convenient to evaluate the time-dependent solution for the two variables δ and Y. The amplitude of variation of Σ will be supposed sufficiently small for the problem to be linear. Thus, if we write

$$\delta=\tilde{\delta}e^{i\omega t},\quad \tilde{\delta}=\tilde{\delta}'+i\tilde{\delta}'',\quad Y=Y_0+\tilde{Y}e^{i\omega t},\quad \tilde{Y}=\tilde{Y}'+i\tilde{Y}'', \tag{8}$$

and

$$\Sigma=\Sigma_0+\sigma e^{i\omega t}, \tag{9}$$

the object of the next stage in the analysis will be to determine $\tilde{\delta}'$, $\tilde{\delta}''$, $\tilde{Y}'$, and $\tilde{Y}''$ as real multiples of σ.

3. DEVELOPMENT OF THE ac SOLUTION FOR THE VARIABLES δ AND Y

Equations (1) through (7) suffice to determine the small-signal ac solution for the geometric mean shift of Fermi levels $\tilde{\delta}$ and the variation in surface potential $\tilde{Y}$. The "induced charge" (the ac part of Σ) is supposed to be as shown in Eq. (9).

Subtracting (2) from (1), using (3), and then substituting for f_t from (4), we have

$$(C_n/N_t)[(N_t-\Gamma_p+\Gamma_n+\Sigma)n_s-(\Gamma_p-\Gamma_n-\Sigma)n_1]$$
$$-(C_p/N_t)[(\Gamma_p-\Gamma_n-\Sigma)p_s-(N_t-\Gamma_p+\Gamma_n+\Sigma)p_1]$$
$$=-\frac{d}{dt}(\Gamma_p-\Gamma_n)-\frac{d}{dt}\Sigma. \tag{10}$$

Again, from (2) and (7), using (4), we find:

$$(C_p/N_t)[(\Gamma_p-\Gamma_n-\Sigma)p_s-(N_t-\Gamma_p+\Gamma_n+\Sigma)p_1]$$
$$=-\frac{d}{dt}\Gamma_p-I_p. \tag{11}$$

We next write

$$\begin{aligned}
p_s&=p_{s0}+\left(\frac{\partial p_s}{\partial\delta}\right)_Y\tilde{\delta}+\left(\frac{\partial p_s}{\partial Y}\right)_\delta\tilde{Y},\\
n_s&=n_{s0}+\left(\frac{\partial n_s}{\partial\delta}\right)_Y\tilde{\delta}+\left(\frac{\partial n_s}{\partial Y}\right)_\delta\tilde{Y},\\
\Gamma_p&=\Gamma_{p0}+\left(\frac{\partial \Gamma_p}{\partial\delta}\right)_Y\tilde{\delta}+\left(\frac{\partial \Gamma_p}{\partial Y}\right)_\delta\tilde{Y},\\
\Gamma_n&=\Gamma_{n0}+\left(\frac{\partial \Gamma_n}{\partial\delta}\right)_Y\tilde{\delta}+\left(\frac{\partial \Gamma_n}{\partial Y}\right)_\delta\tilde{Y},
\end{aligned} \tag{12}$$

where the partial differentials are functions of the surface potential Y and of the bulk resistivity, and may be written down by use of the standard theory of the surface space-charge region.[14] Inserting the above expressions, and approximating for small-signal conditions at a frequency $(\omega/2\pi)$, we get, from (10),

$$\left\{C_pf_0\left(\frac{\partial p_s}{\partial\delta}\right)_Y-C_n(1-f_0)\left(\frac{\partial n_s}{\partial\delta}\right)_Y\right.$$
$$+\frac{1}{N_t}\left(\frac{C_pp_1}{f_0}+\frac{C_nn_1}{1-f_0}\right)\left[\left(\frac{\partial\Gamma_p}{\partial\delta}\right)_Y-\left(\frac{\partial\Gamma_n}{\partial\delta}\right)_Y\right]$$
$$\left.+i\omega\left[\left(\frac{\partial\Gamma_p}{\partial\delta}\right)_Y-\left(\frac{\partial\Gamma_n}{\partial\delta}\right)_Y\right]\right\}\tilde{\delta}$$
$$+\left\{C_pf_0\left(\frac{\partial p_s}{\partial Y}\right)_\delta-C_n(1-f_0)\left(\frac{\partial n_s}{\partial Y}\right)_\delta\right.$$
$$+\frac{1}{N_t}\left(\frac{C_pp_1}{f_0}+\frac{C_nn_1}{1-f_0}\right)\left[\left(\frac{\partial\Gamma_p}{\partial Y}\right)_\delta-\left(\frac{\partial\Gamma_n}{\partial Y}\right)_\delta\right]$$
$$\left.+i\omega\left[\left(\frac{\partial\Gamma_p}{\partial Y}\right)_\delta-\left(\frac{\partial\Gamma_n}{\partial Y}\right)_\delta\right]\right\}\tilde{Y}$$
$$=\left[\frac{1}{N_t}\left(\frac{C_pp_1}{f_0}+\frac{C_nn_1}{1-f_0}\right)+i\omega\right]\sigma, \tag{13}$$

and, from (11),

$$\left\{C_p f_0\left(\frac{\partial p_s}{\partial \delta}\right)_Y+\frac{C_p p_1}{N_t f_0}\left[\left(\frac{\partial \Gamma_p}{\partial \delta}\right)_Y-\left(\frac{\partial \Gamma_n}{\partial \delta}\right)_Y\right]\right.$$

$$\left.+Z'+i\left[\omega\left(\frac{\partial \Gamma_p}{\partial \delta}\right)_Y+Z''\right]\right\}\tilde{\delta}$$

$$+\left\{C_p f_0\left(\frac{\partial p_s}{\partial Y}\right)_\delta+\frac{C_p p_1}{N_t f_0}\left[\left(\frac{\partial \Gamma_p}{\partial Y}\right)_\delta-\left(\frac{\partial \Gamma_n}{\partial Y}\right)_\delta\right]\right.$$

$$\left.+i\omega\left(\frac{\partial \Gamma_p}{\partial Y}\right)_\delta\right\}\tilde{Y}=-\frac{C_p p_1}{N_t f_0}\sigma, \quad (14)$$

where f_0 is the mean (zero-field) value of the filling factor f_t.

The coefficients in Eqs. (13) and (14) may be simplified by making use of certain relations. One knows, for example, that the equilibrium filling factor f_0 is equal to $n_{s0}/(n_{s0}+n_1)$, and that $p_{s0}n_{s0}=p_1n_1=n_i^2$. One also knows that, to the first order in δ, $n_s=\lambda^{-1}n_ie^Y \times(1+\lambda\delta)$ and $p_s=\lambda n_ie^{-Y}(1+\lambda^{-1}\delta)$, where $\lambda=(n_0/n_i)$, so that the differentials $(\partial p_s/\partial Y)$, $(\partial p_s/\partial\delta)$, $(\partial n_s/\partial Y)$ and $(\partial n_s/\partial\delta)$ may be written down. Proceeding in this way, and separating into real and imaginary parts by (8) and (9), we may arrive at the solutions of (13) and (14):

$$\begin{aligned}
\frac{\tilde{\delta}'}{\sigma}&=\frac{\mathfrak{A}\mathfrak{C}+\mathfrak{B}\mathfrak{D}}{\mathfrak{A}^2+\mathfrak{B}^2},\\
\frac{\tilde{\delta}''}{\sigma}&=\frac{\mathfrak{A}\mathfrak{D}-\mathfrak{B}\mathfrak{C}}{\mathfrak{A}^2+\mathfrak{B}^2},\\
\frac{\tilde{Y}'}{\sigma}&=\frac{\mathfrak{A}\mathcal{E}+\mathfrak{B}\mathcal{F}}{\mathfrak{A}^2+\mathfrak{B}^2},\\
\frac{\tilde{Y}''}{\sigma}&=\frac{\mathfrak{A}\mathcal{F}-\mathfrak{B}\mathcal{E}}{\mathfrak{A}^2+\mathfrak{B}^2}.
\end{aligned} \quad (15)$$

Here the script letters $\mathfrak{A}$, $\mathfrak{B}$, . . . denote the following expressions:

$$\mathfrak{A}\equiv-N_t\Omega\left[f_0(1-f_0)-\frac{1}{N_t}\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)\right](sn_i+Z')$$

$$-\omega Z''\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)+\omega^2\left[\frac{\partial \Gamma_p}{\partial\delta}\frac{\partial \Gamma_n}{\partial Y}-\frac{\partial \Gamma_p}{\partial Y}\frac{\partial \Gamma_n}{\partial\delta}\right], \quad (16)$$

$$\mathfrak{B}\equiv-N_t\Omega Z''\left[f_0(1-f_0)-N_t^{-1}\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)\right]$$

$$+\omega Z'\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)$$

$$+\omega C_n(1-f_0)n_s\left[\lambda\frac{\partial \Gamma_p}{\partial Y}-\frac{\partial \Gamma_p}{\partial\delta}\right]$$

$$-\omega C_p f_0 p_s\left[\lambda^{-1}\frac{\partial \Gamma_n}{\partial Y}+\frac{\partial \Gamma_n}{\partial\delta}\right]$$

$$+\omega\Omega\left[\frac{\partial \Gamma_p}{\partial Y}\frac{\partial \Gamma_n}{\partial\delta}-\frac{\partial \Gamma_p}{\partial\delta}\frac{\partial \Gamma_n}{\partial Y}\right], \quad (17)$$

$$\mathfrak{C}\equiv\omega^2(\partial\Gamma_p/\partial Y), \quad (18)$$

$$\mathfrak{D}\equiv+\omega C_p f_0 p_s-\omega\Omega(\partial\Gamma_p/\partial Y), \quad (19)$$

$$\mathcal{E}\equiv\Omega(sn_i+Z')-\omega Z''-\omega^2(\partial\Gamma_p/\partial\delta), \quad (20)$$

$$\mathcal{F}\equiv\Omega[\omega(\partial\Gamma_p/\partial\delta)+Z'']+\omega Z'+\omega C_p f_0\lambda p_s, \quad (21)$$

in which it has been convenient to introduce s, the surface recombination velocity, given by:

$$s=\frac{(p_0+n_0)C_pC_n}{C_p(p_s+p_1)+C_n(n_s+n_1)}, \quad (22)$$

and also a frequency Ω, which, it will turn out, is closely connected with the higher-frequency relaxation process:

$$\Omega=[C_n(n_s+n_1)+C_p(p_s+p_1)]/N_t. \quad (23)$$

This concludes the working of this section. The complexity of the above results may look a little forbidding. There is no simple approximation to Eqs. (15) to (23) that is of general usefulness throughout the frequency range of interest. However, it will usually be true that the frequencies characterizing the two dispersion effects ("lifetime" and the relaxation of the fast states) are sufficiently different for one to be able to treat them separately, using a different approximation to (15) to (23) for the discussion of each. This will be done in Secs. 6 and 7.

Let us briefly review the progress so far. Starting with the fundamental equations for the rates of trapping of holes and electrons, charge balance, etc., and making use of the fact that the carrier concentrations at the surface and surface-carrier excesses are functions of the two system parameters δ and Y, it has been possible to deduce the complex amplitudes $\tilde{\delta}$ and $\tilde{Y}$ in response to the application of an ac field to the surface. The expressions derived in this way have involved a quantity Z, which relates, through Eq. (5), the ac minority-carrier current at a point just below the space-charge region to $\tilde{\delta}$, which describes the amplitude of the ac departures from equilibrium there. In the next section we relate Z to the bulk lifetime, geometry of the

sample, etc. Then, in Sec. 5, we shall proceed to take the ac solutions for $\tilde{\delta}$ and $\tilde{Y}$ and use them to deduce the in-phase ac conductivity modulation for a slice of given properties.

4. Z

It will be noticed, from Eq. (5) and the definition of δ given in the paragraph preceding that equation, that Z has the form of a generalized "input admittance" of the bulk, as seen from the surface. It therefore depends on the geometry of the sample, and may be calculated by an argument essentially identical to that used by Shockley[15] to determine the ac properties of a pn junction. Only the outlines of the argument will be presented here.

The differential equation to be satisfied at all points below the space-charge region is

$$D\frac{\partial^2\Delta p}{\partial x^2}=\frac{\partial\Delta p}{\partial t}+\frac{\Delta p}{\tau}, \tag{24}$$

where x is the perpendicular distance below the surface, Δp the added carrier concentration at that point, D the minority-carrier diffusion constant, and τ the bulk lifetime. The boundary condition at $x=0$ is

$$(\Delta p)_{x=0}=n_i\tilde{\delta}, \tag{25}$$

from the definition of δ.

The situation of most general interest concerns a slice of uniform thickness X, having, on the surface farthest from that to which the field is applied, a surface recombination velocity s^*, which may or may not be equal to s. This gives the other boundary condition,

$$(D\nabla\Delta p)_{x=X}=-(s^*\Delta p)_{x=X}. \tag{26}$$

The solution of (24) appropriate to a periodic δ of frequency $(\omega/2\pi)$ is of the form

$$\Delta p=A\exp[-xM/(D\tau)^{\frac{1}{2}}]+B\exp[xM/(D\tau)^{\frac{1}{2}}], \tag{27}$$

where A and B, determined by (25) and (26), are

$$A=\tfrac{1}{2}n_i\tilde{\delta}\frac{(v_DM+s^*)e^{\xi M}}{v_DM\cosh(\xi M)+s^*\sinh(\xi M)}, \tag{28}$$

$$B=\tfrac{1}{2}n_i\tilde{\delta}\frac{(v_DM-s^*)e^{-\xi M}}{v_DM\cosh(\xi M)+s^*\sinh(\xi M)}, \tag{29}$$

where $\xi=X/(D\tau)^{\frac{1}{2}}$, $v_D=(D/\tau)^{\frac{1}{2}}$, and $M=(1+i\omega\tau)^{\frac{1}{2}}$.

The quantity needed for substitution in the work of the preceding section is

$$\begin{aligned}Z&=(\tilde{\delta}^{-1}I_p)_{x=0}=-(\delta^{-1}D\nabla\Delta p)_{x=0}\\&=v_DM\tilde{\delta}^{-1}(A-B)\\&=n_iv_DM\frac{v_DM\sinh(\xi M)+s^*\cosh(\xi M)}{v_DM\cosh(\xi M)+s^*\sinh(\xi M)}.\end{aligned} \tag{30}$$

[15] W. Shockley, Bell System Tech. J. 28, 435 (1949).

It will be noticed that Eqs. (28), (29), and (30) include the body lifetime (through ξ and M) and the surface recombination velocity s^* at the back surface, but not the surface recombination velocity s at the front surface. This is because we have implicitly included recombination effects at this surface in the discussion of the rates of capture of carriers by traps in Sec. 2.

We have now deduced both the minority-carrier current at a point just below the space-charge region, and the actual distribution of carriers across the width of the slice, in terms of $\tilde{\delta}$. In the next section we shall complete the formal working by showing how this information may be used, together with (15) to (23), to deduce the conductivity modulation of the sample.

5. CONDUCTIVITY MODULATION

At low enough frequencies, at which there is always equilibrium inside the semiconductor ($\delta=0$), the only way in which the conductivity of the sample may change is by variations of the surface excesses Γ_p and Γ_n. At higher frequencies, however, one must consider in addition the existence of pairs of excess (or deficit) carriers in the space-charge-free region, fairly deep into the semiconductor. The observed change of sheet conductance of the sample will be the sum of the two, having regard to sign. Because the variation of δ and Y with time will not, in general, be in phase with that of Σ, there will be both an in-phase and an out-of-phase conductivity modulation. The experiments so far reported by Montgomery describe only the inphase component (except at the highest frequencies, where there is some uncertainty, owing to circuit phase shifts); presumably the out-of-phase component could also be measured by a straightforward modification of the measuring circuit.

We now write down the surface and volume contributions to the conductivity modulation of the sample in terms of $\tilde{\delta}$ and Y.

(i) Contribution from the Surface Excesses

The additional conductivity ΔG_1 due to the surface excesses is

$$\begin{aligned}\Delta G_1=e\mu_p&\left[\left(\frac{\partial\Gamma_p}{\partial\delta}\right)_Y+b\left(\frac{\partial\Gamma_n}{\partial\delta}\right)_Y\right]\tilde{\delta}\\&+\left[\left(\frac{\partial\Gamma_p}{\partial Y}\right)_\delta+b\left(\frac{\partial\Gamma_n}{\partial Y}\right)_\delta\right]\tilde{Y},\end{aligned} \tag{31}$$

where b is the ratio of the electron to the hole mobility.

(ii) Contribution from Excess Carriers in the Body

The distribution of added carriers in the body of the sample is known, for the case of a parallel-faced slice, from the work of the preceding section [Eqs. (27), (28), and (29)]. The additional conductivity ΔG_2

associated with these carriers is

$$\Delta G_2=(1+b)e\mu_p\int_0^X \Delta p dx. \tag{32}$$

Using (27)–(29), we obtain:

$$\int_0^X \Delta p dx$$

$$=n_i\tilde{\delta}\frac{(D\tau)^{\frac{1}{2}}}{M}\frac{v_D M\sinh(\xi M)+s^*[\cosh(\xi M)-1]}{v_D M\cosh(\xi M)+s^*\sinh(\xi M)}. \tag{33}$$

This completes the formal working. If, now, we wish to express the conductivity modulation in terms of a "field-effect mobility" μ_{FE}, which will in general be complex, we must sum the contributions from (i) and (ii):

$$\mu_{FE}=(\Delta G_1+\Delta G_2)/e\sigma. \tag{34}$$

To calculate the effective mobility, we need to start with (15) to (23), substitute Eq. (30) for Z, substitute the resulting expressions in (31) and (33), and then use (34). There would be little point in writing down the general expressions so obtained. It is convenient from this point on to take advantage of the actual magnitudes of the parameters occurring in the practical case of germanium, and to discuss separately the low- and high-frequency dispersion phenomena. This will be done in the next two sections.

6. THE LOW-FREQUENCY DISPERSION PHENOMENON IN A THIN SLICE

In the practical case of a thin slice of germanium having etched surfaces, the frequency Ω is of the order of 10^8 sec^{-1}. Thus, at frequencies small in comparison with this figure, one may drop the second term from each of (16), (17), (20), and (21). Certain other terms may also be neglected; and substitution of typical values for the various parameters suggests the following set of approximations:

$$\mathfrak{A}\approx -N_t\Omega(sn_i+Z')\left[f_0(1-f_0)-N_t^{-1}\frac{\partial}{\partial y}(\Gamma_p-\Gamma_n)\right], \tag{35}$$

$$\mathfrak{B}\approx -N_t\Omega Z''\left[f_0(1-f_0)-N_t^{-1}\frac{\partial}{\partial y}(\Gamma_p-\Gamma_n)\right], \tag{36}$$

$$\mathfrak{C}\approx 0, \tag{37}$$

$$\mathfrak{D}\approx\omega[C_p f_0 p_s-\Omega(\partial\Gamma_p/\partial y)], \tag{38}$$

$$\mathcal{E}\approx\Omega(sn_i+Z'), \tag{39}$$

$$\mathcal{F}\approx\Omega Z''. \tag{40}$$

We first note that, since $\mathfrak{D}$ is considerably smaller in magnitude than $\mathcal{E}$ and $\mathcal{F}$, $(\tilde{\delta}/\sigma)$ is quite small in comparison with $(\tilde{Y}/\sigma)$. Therefore, when we come to substitute into (31), it is reasonable to drop the term in $\tilde{\delta}$ and consider only that in $\tilde{Y}$.

We also see that, since $\mathfrak{AF}=\mathfrak{BE}$, the imaginary part of $(\tilde{Y}/\sigma)$ is zero, that is, the variations of surface potential are in phase with the applied field. Throughout the range of frequency under consideration, the real part of $(\tilde{Y}/\sigma)$ is seen to be given by:

$$\frac{\tilde{Y}'}{\sigma}=\left[\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)-N_t f_0(1-f_0)\right]. \tag{41}$$

This is the common-sense answer for the dc case, since the second term is just $N_t(\partial f_t/\partial Y)$. Since we have shown it to hold also in the ac case, one may say that the true field effect—the conductivity modulation arising from modulation of the surface excesses—can show no dispersion up to a frequency of the order of Ω. The same fraction of the induced charge goes into the fast states at any frequency in this range.

We turn now to the departures from equilibrium in the interior. Here it will be convenient to restrict the calculation to the case that the sample is thin in comparison with a diffusion length, and to make, for the moment, the additional limitation that $\omega\ll D/X^2$. From (30) and (33), we obtain

$$Z\to n_i\frac{s^*+v_D M^2\xi}{1+(s^*/v_D)\xi}, \tag{42}$$

$$\int_0^X \Delta p dx\to n_i\tilde{\delta}(D\tau)^{\frac{1}{2}}\xi\left(\frac{1+\frac{1}{2}(s^*/v_D)\xi}{1+(s^*/v_D)\xi}\right). \tag{43}$$

To get $\tilde{\delta}$, we use (15) with (35)–(38). Substituting, we find

$$\int_0^X \Delta p dx=-\Phi\left(1+\frac{s^*\xi}{2v_D}\right)\frac{\omega\tau_1(\omega\tau_1+i)}{1+\omega^2\tau_1^2}, \tag{44}$$

where we have written

$$\Phi=\frac{\Omega^{-1}C_p f_0 p_s-\partial\Gamma_p/\partial Y}{N_t f_0(1-f_0)-\partial(\Gamma_p-\Gamma_n)/\partial Y}, \tag{45}$$

a quantity independent of frequency, and depending only on the mean surface potential and the properties of the surface traps, and

$$\tau_1=\tau\Big/\left[1+\frac{s^*}{v_D\xi}+\frac{s}{v_D\xi}\left(1+\frac{s^*}{v_D}\xi\right)\right]. \tag{46}$$

The solution given in Eq. (44) has a number of interesting properties.

(i) Relation to the "Fundamental-Mode Lifetime"

The fundamental-mode lifetime, for the case s, $s^*\ll(D/X)$, may be written as follows:

$$\tau_{\text{fund}}=\tau/[1+(s^*+s)/v_D\xi]. \tag{47}$$

Comparing this expression with (46), one may make the following assertion: in the case that $(s^*\xi/v_D)\ll 1$, the time constant governing the low-frequency dispersion phenomenon in the field-effect experiment is the fundamental-mode lifetime. This is consonant with the experimental findings of Montgomery. Furthermore, the frequency factor in (44) is the Fourier transform of (e^{-t/τ_1}). Montgomery has pointed out that such a relation ought in certain cases to exist, since, if a pulse of carriers disappears uniformly across the thickness of the slice according to a certain law, the frequency behavior ought to be given by the Fourier transform of that law. However, we may see from (46) and (47) that this will not be true in general. If $s^*>(v_D/\xi)$ (which is equal to D/X), the two characteristic times are not the same. The reason for this is obvious: if the surface recombination velocity at the back surface is greater than (v_D/ξ), the decay will not be well represented by a simple exponential, since higher decay modes will become important. Montgomery's law should therefore hold only when the decay of photoconductivity after exposure to a short flash of carriers follows a simple exponential law.

(ii) Asymptotic Behavior at Frequencies such that $\omega\tau_1\gg 1$

First it should be remembered that we have temporarily required that $\omega\gg D/X^2$. Thus, Eq. (44) will hold out to $\omega\tau_1\gg 1$ only if $\tau_1\gg X^2/D$. This implies that we may explore this frequency range with the help of (44) only if $s^*\ll(v_D/\xi)$. Then, from (44) we see that the limiting value of the right-hand side, at $\omega\tau_1\gg 1$, is $-\Phi$. Looking at (45), we may see that it is of a form which is susceptible of a simple physical interpretation, at any rate in the condition that the mean surface potential Y has an extreme value.

If we consider the limit $Y_0\rightarrow-\infty$, the expression $(\partial\Gamma_n/\partial Y)$ in the denominator may be dropped, and, remembering that, under these conditions $\Omega\rightarrow C_p(p_s+p_1)/N_t$ [see (23)], we get:

$$d\left[\int\Delta p dx\right]/d\sigma\rightarrow-1.$$

The meaning of this is as follows. When $Y_0\rightarrow-\infty$, the surface is rich in holes and poor in electrons. Further, at $\omega\gg s^2/D$, all changes in surface charge have to be supplied by a flow of holes (minority carriers) from the interior. So, for each charge of unity that appears at the surface in response to a change of the external field, there must be a depletion of one hole-electron pair from the space-charge-free interior. This is in agreement with the qualitative argument given in Sec. 1.

At the other limit, $Y_0\rightarrow\infty$, the term $(\partial\Gamma_n/\partial Y)$ in the denominator becomes large without limit, so that $d[\int\Delta p dx]/d\sigma\rightarrow 0$. This too is common sense: the surface charge consists mainly of electrons, which can be supplied rapidly from the ohmic contacts to the sample, without upsetting the hole-electron equilibrium significantly.

Summing up the surface and volume contributions to the conductivity modulation, and expressing the result in terms of the field-effect mobility by means of (34), we get

$$\mu_{FE}'=\mu_p\left[\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)-N_t f_0(1-f_0)\right]^{-1}\left\{\frac{\partial\Gamma_p}{\partial Y}+b\frac{\partial\Gamma_n}{\partial Y}-\frac{\omega^2\tau_1^2}{1+\omega^2\tau_1^2}(1+b)\left[\frac{\partial\Gamma_p}{\partial Y}-\Omega^{-1}C_p f_0 p_s\right]\right\}. \quad (48)$$

7. HIGH-FREQUENCY DISPERSION PHENOMENON

At frequencies comparable to Ω, a different approximation procedure becomes appropriate. We begin by writing down asymptotic forms for Z [from (30)] and for $\int_0^X\Delta p dx$ [from (33)]:

$$Z\rightarrow n_i(D\omega/2)^{\frac{1}{2}}(1+i), \quad (49)$$

$$\int_0^X\Delta p dx\rightarrow n_i\bar{\delta}(D/2\omega)^{\frac{1}{2}}(1-i). \quad (50)$$

It will be noted that these expressions are independent of the geometry of the sample, bulk lifetime, etc.: they correspond simply to the state of affairs in which the diffusion length into the sample is $\sim(D/\omega)^{\frac{1}{2}}$.

A careful inspection of (16) to (21) suggests the following approximations as suitable for our present purpose:

$$\mathfrak{A}\approx-Z'[N_t\Omega f_0(1-f_0)-(\Omega-\omega)(\partial/\partial Y)(\Gamma_p-\Gamma_n)], \quad (51)$$

$$\mathfrak{B}\approx-Z'[N_t\Omega f_0(1-f_0)-(\Omega+\omega)(\partial/\partial Y)(\Gamma_p-\Gamma_n)], \quad (52)$$

$$\mathfrak{C}\approx\omega^2(\partial\Gamma_p/\partial Y), \quad (53)$$

$$\mathfrak{D}\approx\omega[C_p f_0 p_s-\Omega(\partial\Gamma_p/\partial Y)], \quad (54)$$

$$\mathfrak{E}\approx(\Omega-\omega)Z', \quad (55)$$

$$\mathfrak{F}\approx(\Omega+\omega)Z', \quad (56)$$

where $Z'\equiv n_i(D\omega/2)^{\frac{1}{2}}$ throughout.

Here we have dropped (i) terms in ω^2 from $\mathfrak{A}$ and $\mathfrak{E}$, on the grounds that these do not become important until ω becomes comparable with $(D/\mathfrak{L}^2)$ (where $\mathfrak{L}$ is a Debye length), at which point the analysis fails in any case, and (ii) terms linear in ω from $\mathfrak{B}$ and $\mathfrak{F}$, on the grounds that substitution of typical values for the various parameters suggests that they may generally be neglected in comparison with the terms shown.

The subsequent working proceeds as in the previous section. The real part of $(\tilde{Y}/\sigma)$ is found to be given by

$$\frac{\tilde{Y}'}{\sigma}=\left[\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)-N_t f_0(1-f_0)\right]^{-1}\times\left[1-\frac{\omega^2\tau_2^2}{1+\omega^2\tau_2^2}\frac{N_t f_0(1-f_0)}{\partial(\Gamma_p-\Gamma_n)/\partial Y}\right], \quad (57)$$

where

$$\tau_2=\Omega^{-1}\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)\Big/ \times\left[\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)-N_t f_0(1-f_0)\right]. \quad (58)$$

The ac variation in surface potential is seen to follow a simple dispersion law, the characteristic time τ_2 being of the order of, but rather less than, Ω^{-1}. On the low-frequency side of the dispersion,

$$(\tilde{Y}'/\sigma)\rightarrow[\partial(\Gamma_p-\Gamma_n)/\partial Y-N_t f_0(1-f_0)]^{-1},$$

which is the value given in Eq. (41); on the high-frequency side, $(\tilde{Y}'/\sigma)\rightarrow[\partial(\Gamma_p-\Gamma_n)/\partial Y]^{-1}$, which is the value one would calculate in the absence of surface states. This shows quite vividly that τ_2 does indeed characterize the relaxation of the surface states.

Turning to the added carriers in the body, we find, for the real part,

$$\mathrm{Re}\left[\frac{\int\Delta p dx}{\sigma}\right]=-\left[\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)-N_t f_0(1-f_0)\right]^{-1} \times\left\{\frac{\partial\Gamma_p}{\partial Y}-\Omega^{-1}C_p f_0 p_s-\frac{\omega^2\tau_2^2}{1+\omega^2\tau_2^2} \times\left[\frac{\partial\Gamma_p/\partial Y}{\partial(\Gamma_p-\Gamma_n)/\partial Y}N_t f_0(1-f_0)-\Omega^{-1}C_p f_0 p_s\right]\right\}. \quad (59)$$

This again follows a simple dispersion law, with the value on the low-frequency side agreeing with Eq. (45), as it must, and that on the high-frequency side being

$$-(\partial\Gamma_p/\partial Y)/[\partial(\Gamma_p-\Gamma_n)/\partial Y].$$

This latter limit has a very obvious meaning, when compared with the limiting value of $(\tilde{Y}'/\sigma)$, discussed above: it says that, for $\omega\tau_2\gg1$, every minority carrier (hole) appearing in the surface excess must have come from the bulk and *vice versa*. Since the surface states are no longer operative, this is just what we should expect.

We write down finally, the expression giving the real part of the field-effect mobility in this frequency range, using (31), (32), and (34):

$$\mu_{FE}'=\mu_p\left[\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)-N_t f_0(1-f_0)\right]^{-1} \times\left\{-b\frac{\partial}{\partial Y}(\Gamma_p-\Gamma_n)+(1+b)\Omega^{-1}C_p f_0 p_s+\frac{\omega^2\tau_2^2}{1+\omega^2\tau_2^2} \times[bN_t f_0(1-f_0)-(1+b)\Omega^{-1}C_p f_0 p_s]\right\}. \quad (60)$$

In deriving this expression, we have taken advantage of the fact that, since $\mathfrak{C}$ and $\mathfrak{D}$ are generally small in magnitude in comparison with $\mathcal{E}$ and $\mathfrak{F}$, $|\tilde{\delta}|\ll|\tilde{Y}|$; this has allowed us to neglect the term in $\tilde{\delta}$ in (31).

8. COMPARISON WITH EXPERIMENT

The purpose of this section is to show how closely it is possible to fit the experimental measurements reported by Montgomery[9] by means of the theory developed in the preceding sections. The measurements, which covered the frequency range 10^2 to 10^8 cps, were made on a 20 ohm-cm sample of p-type germanium, kept in three different ambients, which, one would estimate from previous work,[4,5] cover the range of surface potential $|Y-\ln\lambda|<5$. The measured filament lifetime was 25 μsec, which, for the slice thickness given (0.5 mm), corresponds to a surface recombination velocity of 1000 cm/sec on each face, if the body lifetime is supposed to be infinite.

For the purpose of calculation, it will be convenient to combine Eqs. (48) and (60) to give a single expression for the effective mobility over the entire frequency range. It is true that we have not investigated the validity of (48) for frequencies $\omega\gg D/X^2$. However, so long as $\tau_1\gg X^2/D$ the entire low-frequency dispersion phenomenon lies at frequencies lower than D/X^2, and, since both real and imaginary parts match on to the low-frequency tail of (60), we may be assured that there is no other dispersion phenomenon at any frequency in between. In the case of Montgomery's sample, the condition $\tau_1\gg D/X^2$ is not too well satisfied, but we shall hope to get at any rate a qualitative description of his results by ignoring this fact.

Combining (48) and (60), we have, for an n-type semiconductor,

$$\frac{\mu_{FE}'}{\mu_p}=-b+\frac{A_n}{1+\omega^2\tau_1^2}+\frac{B_n}{1+\omega^2\tau_2^2}, \quad (61)$$

where

$$A_n=(1+b)\left[\frac{\partial\Gamma_p/\partial Y-\Omega^{-1}C_p f_0 p_s}{\partial(\Gamma_p-\Gamma_n)/\partial Y-N_t f_0(1-f_0)}\right], \quad (62)$$

$$B_n=\frac{(1+b)\Omega^{-1}C_p f_0 p_s-bN_t f_0(1-f_0)}{\partial(\Gamma_p-\Gamma_n)/\partial Y-N_t f_0(1-f_0)} \quad (63)$$

and τ_1 and τ_2 are given by (46) and (58), respectively. By analogy, we may write down the equivalent results holding for extrinsic p-type semiconductor:

$$\frac{\mu_{FE}'}{\mu_p}=1-\frac{A_p}{1+\omega^2\tau_1^2}+\frac{B_p}{1+\omega^2\tau_2^2}, \quad (64)$$

where:

$$A_p=(1+b)\frac{-\partial\Gamma_n/\partial Y-\Omega^{-1}C_n(1-f_0)n_s}{\partial(\Gamma_p-\Gamma_n)/\partial Y-N_t f_0(1-f_0)}, \quad (65)$$

$$B_p=\frac{(1+b)\Omega^{-1}C_n(1-f_0)n_s-bN_t f_0(1-f_0)}{\partial(\Gamma_p-\Gamma_n)/\partial Y-N_t f_0(1-f_0)}. \quad (66)$$

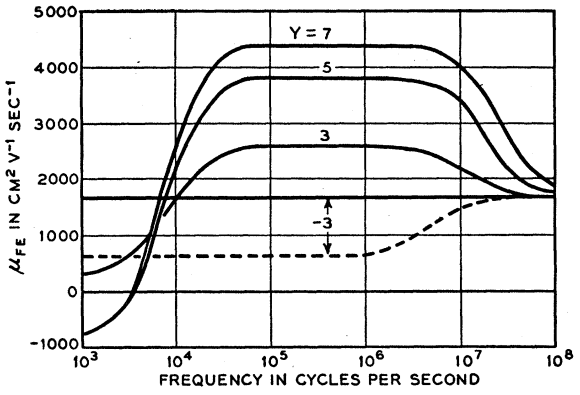

FIG. 1. Field-effect mobility as a function of frequency, for 20-ohm-cm p-type germanium, as a function of surface potential Y. Surface-state scheme as described in text.

Figure 1 shows the results of using equations (64), (65), and (66) to calculate the dependence of μ_{eff} on frequency for a p-type sample of the resistivity (20 ohm-cm) used by Montgomery. The parameters have been chosen to give the best fit: C_p has been set equal to 1.7×10^5 cm/sec, C_n to 1.1×10^7 cm/sec, and n_1 to 2.5×10^{16} cm^{-3}. This means that we have postulated a level of density 7×10^{11} cm^{-2}, situated 0.18 volt above the center of the gap. The position of this level agrees with that found by Statz *et al.*[16]; the density supposed, however, is larger than that reported by them. The C_p and C_n imply capture cross sections of 2.4×10^{-14} cm^2 for holes, and 1.6×10^{-16} cm^2 for electrons. The ratio of these cross sections agree with that reported by Garrett and Brattain,[6] but the magnitudes of each are here taken as four times as large as the values reported by those authors. Since we have assumed only one trapping level in the work of this paper, we should not expect the results of the calculations to give a good description when the surface potential is negative; and, indeed, the calculations suggest that the field-effect mobility should be equal to the bulk value for holes, and independent of frequency, in this range of Y. To patch up the solution in this region, we would have to work out the theory afresh, with two trapping levels instead of one. There would seem to be little merit to this procedure at the present stage. Qualitatively, we may argue as follows. If we introduce a new trapping level somewhere in the lower half of the gap, the low-frequency mobility will be reduced to some value below μ_p, as observed. The "lifetime" dispersion phenomenon will not now appear, since the expression (65) for A_p tends rapidly to zero with increasing negative Y, whatever the exact position of the trapping level. The high-frequency dispersion phenomenon, during the course of which the field-effect mobility must rise to μ_p, will still be found, and will be characterized by a new value of τ_2, given by inserting the parameters for the new trapping level into (58). It will be seen that, so long as the density and cross sections for the new set of traps do not differ too radically from those postulated above, and so long as they are located about as far *below* the center of the gap as the traps previously discussed lie above it, the value of τ_2 will not be too different. Because of this fact, we have indicated on Fig. 1, as a dashed line, the way in which we would expect the field-effect mobility to go at $Y=-3$, if these conditions are satisfied. (It must be emphasized that this curve is purely schematic.)

Comparing Fig. 1 with Montgomery's results, we may say that a reasonable qualitative description has been given by the present theory. Unfortunately, we are ignorant of the values of Y to be assigned to the three curves in Montgomery's paper, but one might guess $Y\sim7$ for ozone, $Y\sim2$ for dry oxygen, and $Y\sim-3$ for wet nitrogen. A simultaneous measurement of Y by the large-signal field-effect technique[4] would be informative here, as would measurements on an n-type sample.

Since the parameters of the trapping level or levels come into Eqs. (61) through (66) in a rather complicated way, it is difficult to decide what latitude in the assumed values of these parameters would be permissible. One definite statement can, however, be made. It is not possible to account for the smallness of the observed value of τ_2 without invoking levels that are relatively distant from the center of the gap.[17] In the simple case of one trapping level so situated, $\tau_2\sim1/[v_T(\sigma_p\sigma_n)^{\frac{1}{2}}p_1]$ or $1/[v_T(\sigma_p\sigma_n)^{\frac{1}{2}}n_1]$, where v_T is the thermal speed and σ_p and σ_n the trapping cross sections for holes and electrons, respectively. Thus, so long as τ_2 is of the order of 10^{-8} sec, and $(\sigma_p\sigma_n)^{\frac{1}{2}}\sim10^{-15}$ cm^2, there must be a trapping level situated such that $n_1\sim10^{16}$ cm^{-3}, or one situated such that $p_1\sim10^{16}$ cm^{-3}, or both. Otherwise, there should be some range of Y close to zero in which τ_2 would be found to be very much longer than the value quoted. This raises the interesting question: could there be, in addition to discrete states situated either high or low or both, other states close to the center of the gap? The evidence provided by Montgomery's experiments suggests that the answer is no. If the density of such states were sufficient to produce a noticeable effect on the low-frequency field-effect mobility, then this effect should disappear at a frequency of the order of the τ_2 appropriate to these states. In the extreme case of a complete distribution of states near the center of the gap, there should be a gradual change of the field-effect mobility over several decades in frequency. There is no sign of this in the measurements reported by Montgomery. Thus the high-frequency field-effect measurements

[16] Statz, de Mars, Davis, and Adams, Phys. Rev. **101**, 1272 (1956).

[17] It is tacitly assumed, in this paragraph, that the circumstances are such that the observed low-frequency field-effect mobility is not lower than what it would be in the absence of fast states by more than, say, an order of magnitude. This allows us to say $\tau_2\approx\Omega^{-1}$ [see (58)].

show that all the fast states present in significant numbers are relatively distant from the center of the gap; or else, that the states close to the center of the gap have unexpectedly large cross sections.

How, then, can we account for the low-frequency field-effect experiment, which cannot be interpreted simply in terms of a pair of discrete levels, one high and one low, but seems to demand a continuous distribution of fast states near the center of the gap? The most reasonable assumption is that there are inhomogeneities in Y from point to point on the surface. This, as pointed out previously,[6] would have the effect of smoothing out the simple Boltzmann expressions giving the dependence on mean Y of the charge trapped in fast states. It is possible that a detailed analysis of the high-frequency measurements, taken in conjunction with field-effect and surface photovoltage measurements of the sort reported previously,[5] would lead to an estimate of the magnitude of the inhomogeneities demanded.

9. FINAL REMARKS

This paper has presented a theoretical treatment of the two relaxation processes observed by Montgomery in the high-frequency field-effect experiment. The salient results are the following:

1. The lower-frequency dispersion phenomenon, associated with thermal recombination processes, is characterized by a time τ_1, which, under certain restrictions, is equal to the fundamental decay-mode lifetime.

2. The higher-frequency dispersion phenomenon, associated with the relaxation of the fast states themselves, is characterized by a time τ_2, which is principally affected by the position of the fast-states trapping level in the gap, and the hole and electron capture cross sections. With reasonable values for these latter, the experimental values for τ_2 suggest that the fast states are distant $\sim$0.18 volt from the center of the gap.

3. The field-effect mobility at $\omega\tau_2 \gg 1$ should be equal in magnitude to the mobility of the majority carrier in the bulk, except insofar as the Schrieffer correction is important. As suggested by Montgomery, measurements in this range should afford a direct experimental determination of the magnitude of the Schrieffer correction.

4. The field-effect mobility at frequencies intermediate between τ_1^{-1} and τ_2^{-1} has a value which depends on the trapping parameters, surface potential, etc., in a rather complicated way. For the case of a well-developed inversion layer, the mobility in this range should have a value intermediate between the mobility of the bulk majority carrier and the sum of the mobilities of the bulk majority and minority carriers. At the other extreme of surface potential, the mobility in this frequency range will be equal to that observed at low frequencies.

5. The difficulties usually encountered in trying to fit low-frequency field-effect data to a model including only a pair of discrete fast-state levels are more likely to be associated with inhomogeneities in surface potential on the surface studied than with the existence of a continuous distribution of fast-state levels near the center of the gap.

6. The extension of field-effect measurements to still higher frequencies—into the wave-guide range, for example— would furnish an interesting tool for studying transit-time effects.

My thanks are due to H. C. Montgomery and W. L. Brown for many interesting discussions.