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Galvanomagnetic Theory for *n*-Type Germanium and Silicon: Hall Theory and General Behavior of Magnetoresistance*

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A general expression for the resistivity tensor, appropriate to *n*-type germanium and silicon, is deduced from which the magnetoresistance $\Delta\rho/\rho$ and Hall coefficient R_H relations are evaluated. The angular dependence of $\Delta\rho/\rho$ in germanium shows precisely the qualitative features noted in the experiments of Pearson and Suhl. Additional details emerge, however, for silicon that were not detected by Pearson and Herring—presumably because of the restricted range of $\omega\tau$ they employed. The field dependence of $\Delta\rho/\rho$ for both germanium and silicon is examined for a number of high-symmetry orientations of the current **J** and magnetic **B** vectors with the finding of a departure from the square law at high fields. A detailed study of the R_H field dependence is made for the combinations J_{100}, B_{010} and J_{110}, $B_{1\bar{1}0}$. A minimum is observed in germanium for the latter case and in silicon for the former case. The minima occur between the limiting values for $b\to\infty$, $R_H=1/nqc$, and $b\to 0$, $R_H=(1/nqc)[3K(K+2)/(2K+1)^2]$; these limiting values are invariant for all alignments of **J** and **B**.

THIS paper deals with the magnetoresistance of *n*-type germanium and *n*-type silicon in the approximation of a constant scattering time. In a previous paper,[1] we obtained an expression for the conductivity tensor for the appropriate combination of ellipsoidal energy surfaces. The resistivity tensor was then evaluated in the simpler case of the saturation limit only. The magnetoresistance calculations are now extended to intermediate fields, and the Hall coefficient is determined. Some preliminary interesting aspects of the Hall coefficient have been pointed out elsewhere.[2]

The approximation of a constant scattering time τ is used here because of the extreme difficulty in making intermediate-field calculations of the magnetoresistance using an energy-dependent τ. Furthermore, we avoid the problems involved in a detailed treatment of the scattering processes, with which we are not concerned here. Because of this simplification, we shall not present quantitative comparison between our calculations and experimental results. It will be found, however, that there is qualitative agreement between the results presented here and the experimental data now available.

THE RESISTIVITY TENSOR

The conductivity tensor σ for either *n*-germanium or *n*-silicon can be expressed in the form

$$\sigma/\sigma^*=\begin{pmatrix} a_1 & c_3+d_3 & c_2-d_2 \\ c_3-d_3 & a_2 & c_1+d_1 \\ c_2+d_2 & c_1-d_1 & a_3 \end{pmatrix}, \tag{1}$$

where

$$\sigma^*=\frac{nq^2\tau}{m^*};\quad \frac{3}{m^*}=\frac{2}{m_2}+\frac{1}{m_1}. \tag{2}$$

The expression for the quantities in Eq. (1), derived in Part I (which should be consulted for meaning of all other symbols and general notation) are reproduced for reference in Appendix I.

The resistivity tensor ρ, from which the magnetoresistance derives, is the inverse of Eq. (1). This may be written as follows:

$$\frac{\rho}{\rho^*}=\frac{1}{\det(\sigma/\sigma^*)}\|A_{jk}\|, \tag{3}$$

where

$$\begin{aligned}
A_{11}&=a_2a_3-c_1^2+d_1^2,\\
A_{12}&=(c_1-d_1)(c_2-d_2)-a_3(c_3+d_3),\\
A_{13}&=(c_1+d_1)(c_3+d_3)-a_2(c_2-d_2),\\
A_{21}&=(c_1+d_1)(c_2+d_2)-a_3(c_3-d_3),\\
A_{22}&=a_1a_3-c_2^2+d_2^2,\\
A_{23}&=(c_2-d_2)(c_3-d_3)-a_1(c_1+d_1),\\
A_{31}&=(c_1-d_1)(c_3-d_3)-a_2(c_2+d_2),\\
A_{32}&=(c_2+d_2)(c_3+d_3)-a_3(c_1-d_1),\\
A_{33}&=a_1a_2-c_3^2+d_3^2,
\end{aligned} \tag{3a}$$

and

$$\det(\sigma/\sigma^*)=a_1a_2a_3+2c_1c_2c_3+2(c_1d_2d_3+c_2d_1d_3+c_3d_1d_2) -[a_1(c_1^2-d_1^2)+a_2(c_2^2-d_2^2)+a_3(c_3^2-d_3^2)]. \tag{4}$$

THE MAGNETORESISTANCE

The double scalar product of the resistivity tensor with the unit current vector gives the resistance ρ; $\rho/\rho^*=1+\Delta\rho/\rho^*$, where $\Delta\rho/\rho^*$ is the magnetoresistance.

For selected orientations of the current **J**, we find

J	ρ/ρ^*
100	$\rho_{11}/\rho^*=(a_2a_3-c_1^2+d_1^2)/\det(\sigma/\sigma^*)$
110	$\tfrac{1}{2}(\rho_{11}+\rho_{22}+\rho_{12}+\rho_{21})/\rho^* =[a_3(a_1+a_2)-(c_1-c_2)^2+(d_1+d_2)^2-2a_3c_3]/\det(\sigma/\sigma^*)$, (5)

where ρ_{11}, ρ_{12}, etc., are the components

$$\rho_{jk}=\rho^*A_{jk}/\det(\sigma/\sigma^*).$$

* The research reported in this document was supported jointly by the Army, Navy, and Air Force, under contract with Massachusetts Institute of Technology.

[1] L. Gold and L. M. Roth, Phys. Rev. 103, 61 (1956); hereafter called "Part I."

[2] L. Gold, Phys. Rev. 99, 596 (1955).

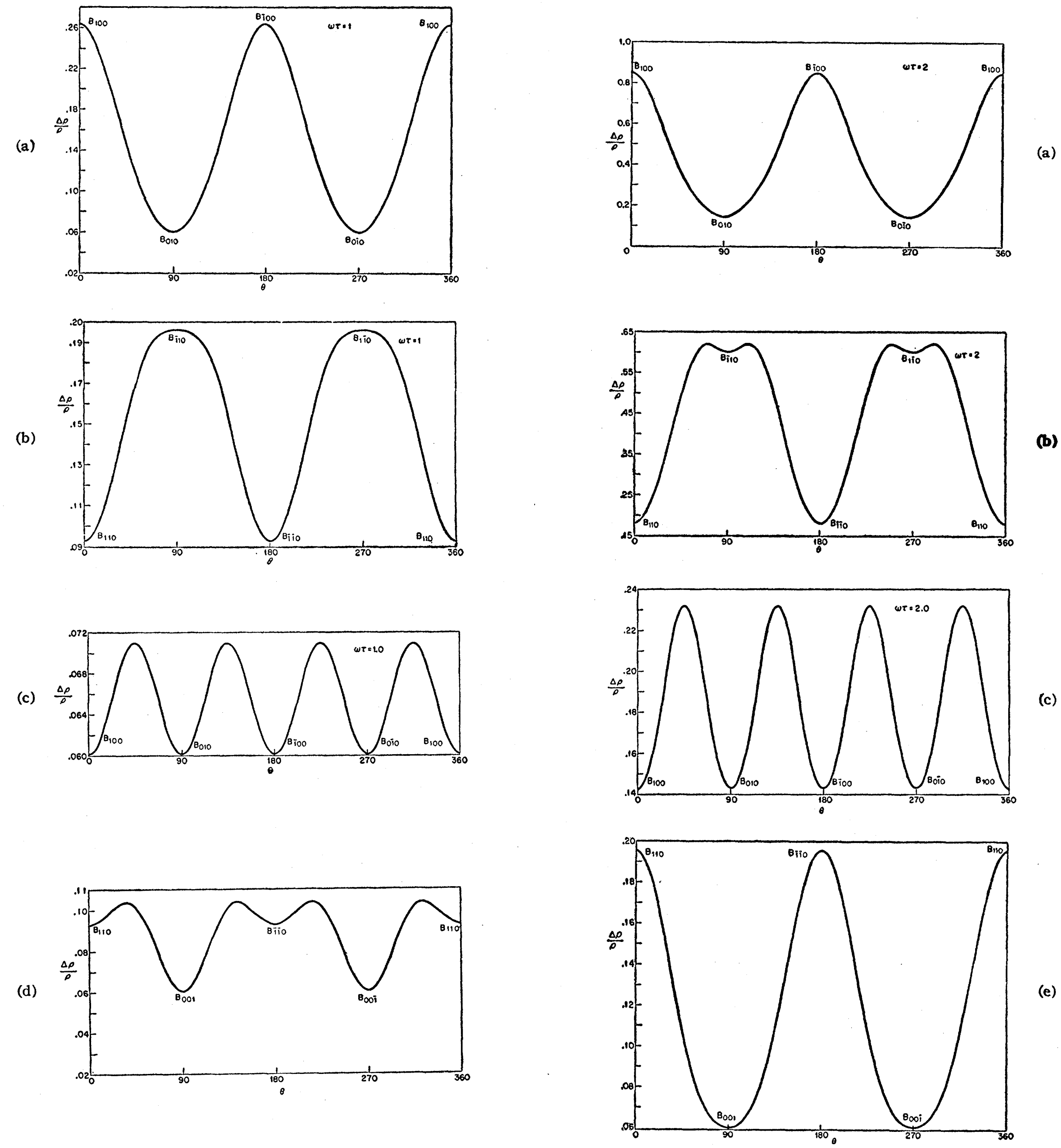

FIG. 1. Directional behavior of the magnetoresistance $\Delta\rho/\rho$ in *n*-germanium for some cardinal current orientations and typical values of $\omega\tau=qB\tau/m_2c$. (a) **B** rotated in (001) plane, J_{100} with $\omega\tau=1.0$ and 2.0; (b) **B** rotated in (001) plane and J_{110} with $\omega\tau=1.0$ and 2.0; (c) **B** rotated in (001) plane and J_{001} with $\omega\tau=1.0$ and 2.0 (transverse magnetoresistance); (d) **B** rotated in $(1\bar{1}0)$ plane and J_{110}, $\omega\tau=1.0$; (e) **B** rotated in $(1\bar{1}0)$ plane and $J_{1\bar{1}0}$, $\omega\tau=1.0$ (transverse magnetoresistance).

A. Angular Dependence of $\Delta\varrho/\varrho$ in *n*-Germanium

(The expressions for ρ/ρ^* for particular directions of **J** and planes of rotation of **B** are given in Appendix II.) The angular dependence for intermediate values of $b=\omega\tau$ is shown in Fig. 1 for comparison with the saturation case and with the experimental results of Pearson and Suhl.[3] For numerical evaluation we have used, as in Part I, the value of the mass ratio $K=m_1/m_2=16.9$ from cyclotron resonance.[4]

[3] G. L. Pearson and H. Suhl, Phys. Rev. **83**, 768 (1951).

[4] It has been pointed out that if one takes into account the anisotropy of τ, K should be replaced by $K=(m_1/m_2)(\tau_2/\tau_1)$. [See Benedek, Paul, and Brooks, Phys. Rev. **100**, 1129 (1955), and L. Gold, Phys. Rev. **104**, 1580 (1956).]

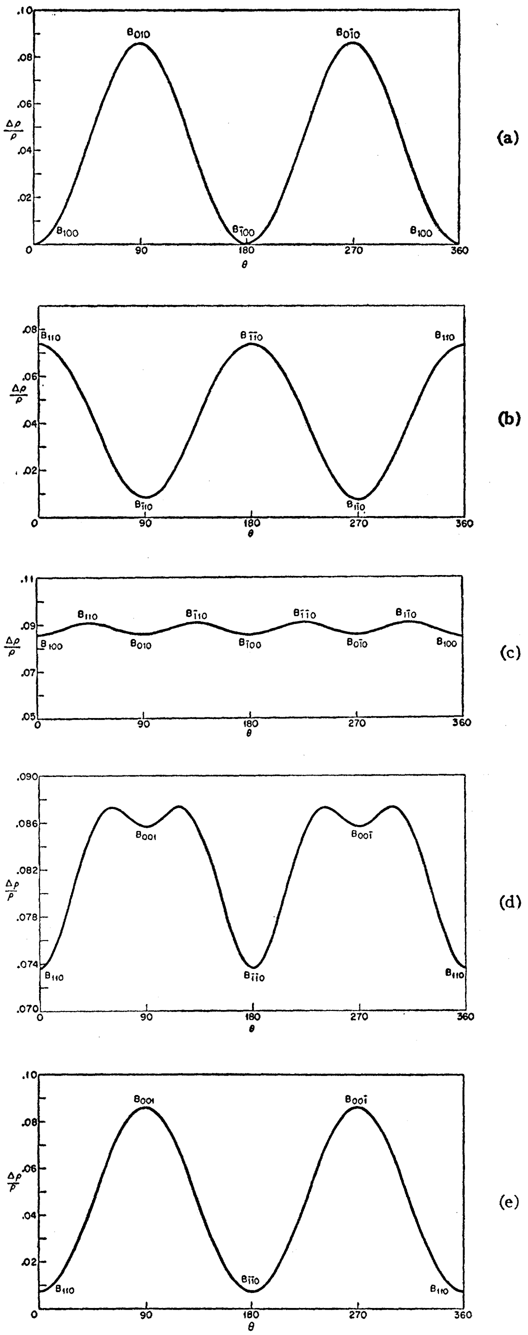

FIG. 2. Directional behavior of the magnetoresistance $\Delta\rho/\rho$ in n-silicon for some cardinal current orientations and typical $\omega\tau$ values. (a) **B** rotated in (001) plane and J_{100} with $\omega\tau=0.8$; (b) **B** rotated in (001) plane and J_{110}, $\omega\tau=0.8$; (c) **B** rotated in (001) plane and J_{001}, $\omega\tau=0.8$ (saturation result constant, $\Delta\rho/\rho=0.745$; transverse magnetoresistance); (d) **B** rotated in $(1\bar{1}0)$ plane, J_{110}, $\omega\tau=0.8$; (e) **B** rotated in $(1\bar{1}0)$ plane, $J_{1\bar{1}0}$, $\omega\tau=0.8$ (transverse magnetoresistance).

When **B** is rotated in the (001) plane, with **J** in the [100] direction [Fig. 1(a)] there is little change in the appearance of the curves as $\omega\tau$ is varied. The magnitude of the $\Delta\rho/\rho$ is, of course, less than the saturation case, and the ratio of longitudinal to transverse magnetoresistance increases as $\omega\tau$ is increased. Similarly, for the transverse magnetoresistance with J_{001} [Fig. 1(c)], the only essential change is an increase in the anisotropy as $\omega\tau$ is increased.

However, the arrangement J_{110}, $B_3=0$ [Fig. 1(b)] exhibits new features in the angular dependence as $\omega\tau$ is varied. For $\omega\tau=1$, the π symmetry is distinguished from the saturation limit in that the maxima are decidedly flatter. If $\omega\tau$ is increased somewhat, as Fig. 1(b) shows for $\omega\tau=2$, intermediate minima appear as observed by Pearson and Suhl (open circles in their Fig. 8).

The case of J_{110} with B rotated in the $(1\bar{1}0)$ plane [Fig. 1(d)] also shows new features as $\omega\tau$ is varied. For $\omega\tau=1.0$, the appearance is quite different from the saturation result, which showed peaks at 45° and equivalent minima at B_{001} and $B_{\bar{1}\bar{1}0}$. Here the minima have become unequal, while the peaks have shifted in position. This is in agreement with Pearson and Suhl results as shown by the **X**'s in their Fig. 8.

Finally, the angular variation of transverse magnetoresistance, with $J_{1\bar{1}0}$ [Fig. 1(e)] is relatively insensitive to changes in $\omega\tau$. This case corresponds to the closed circles of Fig. 8 in Pearson and Suhl.

In the qualitative features of the angular dependence of the magnetoresistance in n-germanium, our results are, therefore, entirely consistent with experiment.

B. Angular Dependence of $\Delta\rho/\rho$ in n-Silicon[5]

The angular dependence of the magnetoresistance in silicon for $\omega\tau=0.8$ is shown in Fig. 2. Numerical evaluation is based upon $K=5.2$.[4] For J_{100} and $B_3=0$ [Fig. 2(a)], the curve differs essentially only in magnitude from the saturation case. For J_{110} and $B_3=0$ [Fig. 2(b)], the intermediate peaks found at saturation disappear as $\omega\tau$ is lowered, having only maxima at B_{110} and minima at B_{001}. The transverse magnetoresistance with $B_3=0$ and J_{001} shows no anisotropy at saturation. In the intermediate region, the anisotropy is small, with $\pi/2$ symmetry as shown in Fig. 2(c).

The arrangement with J_{110} with B rotated in the $(1\bar{1}0)$ plane [Fig. 2(d)] shows intermediate minima not found at saturation. Finally, the transverse case with J_{110} shows nothing essentially different from the saturation case.

In the case of silicon there is insufficient experimental data to give an adequate test of the theoretical work. The work of Pearson and Herring[6] indicates the features characteristic of the saturation limit, but fails to delineate the details found here in the intermediate region. It would be interesting to see further experimental results.

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[5] See Appendix II-B for elaboration of ρ/ρ^*.

[6] G. L. Pearson and C. Herring, Physica **20**, 975 (1954).

FIELD DEPENDENCE OF MAGNETORESISTANCE

In Fig. 3 the magnetoresistance for the 4- or 8-ellipsoid model for particular orientations of **J** and **B** is plotted as a function of magnetic field, showing the departure from the square law at high fields. The expressions for the field dependence are, for the simplest cases with J in the (100) direction,

$$\frac{\rho}{\rho^*}=\left(1+\frac{K+2}{3K}(\omega\tau)^2\right)\Big/\left(1+\frac{3}{2K+1}(\omega\tau)^2\right) \quad \text{(longitudinal, } \mathbf{B}_{100}\text{)}, \quad (6)$$

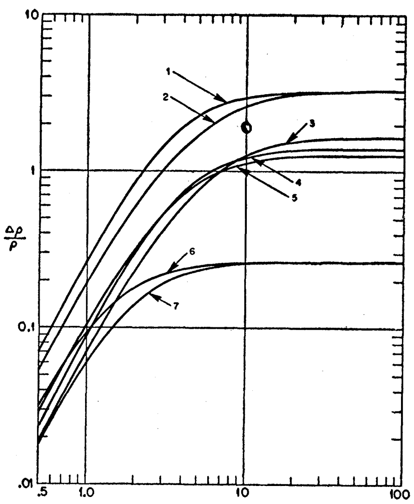

FIG. 3. Field dependence of magnetoresistance $\Delta\rho/\rho$ for n-germanium. The cardinal combinations of current and magnetic field are (1) J_{100}, B_{100}; (2) $J_{\bar{1}10}, B_{110}$; (3) J_{001}, B_{110}; (4) $J_{\bar{1}10}, B_{111}$; (5) J_{110}, B_{111}; (6) J_{110}, B_{110}; (7) $J_{100}, B_{010}=J_{110}, B_{001}$.

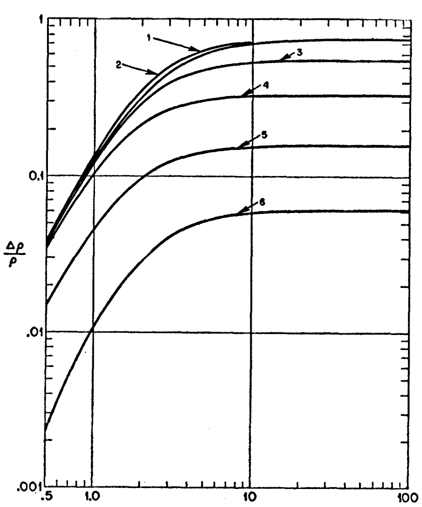

FIG. 4. Field dependence of magnetoresistance $\Delta\rho/\rho$ for n-silicon The cardinal combinations of current and magnetic field are (1) J_{001}, B_{110}; (2) $J_{100}, B_{010}=J_{110}, B_{001}$; (3) J_{110}, B_{111}; (4) J_{110}, B_{110}; (5) $J_{\bar{1}10}, B_{111}$; (6) $J_{\bar{1}10}, B_{110}$.

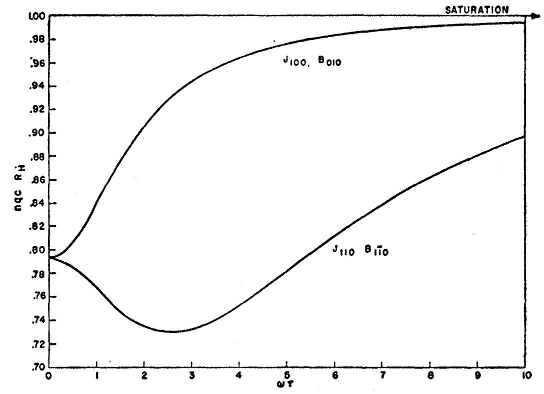

FIG. 5. Field dependence of Hall coefficient R_H (in reduced units) for n-germanium for J_{001}, B_{010} and $J_{110}, B_{1\bar{1}0}$.

$$\frac{\rho}{\rho^*}=\left(1+\frac{K+2}{3K}(\omega\tau)^2\right)\Big/\left[1+\left(\frac{K+2}{2K+1}\right)^2(\omega\tau)^2\right] \quad \text{(transverse, } \mathbf{B}_{010}\text{)}, \quad (7)$$

where again K is the mass ratio m_1/m_2.[4]

Similarly for the 3- or 6-ellipsoid model, the field dependence is shown in Fig. 4. In the longitudinal case with J_{100}, the magnetoresistance vanishes identically in our approximation; the transverse case cannot be simply expressed as for germanium and it is given in Appendix II-B.

THE HALL COEFFICIENT

Our considerations are restricted to a single-carrier system, i.e., we suppose that only electrons contribute significantly to the Hall fields. We define the Hall coefficient in the following way:

$$\begin{aligned} R_H &= \frac{1}{2}\frac{1}{|\mathbf{B}\times\mathbf{J}|^2}[\mathbf{E}(\mathbf{B})\cdot\mathbf{B}\times\mathbf{J}-\mathbf{E}(-\mathbf{B})\cdot\mathbf{B}\times\mathbf{J}] \\ &= \frac{1}{2}\frac{(\mathbf{B}\times\boldsymbol{\alpha})\cdot(\rho-\tilde{\rho})\cdot\boldsymbol{\alpha}}{|\mathbf{B}\times\boldsymbol{\alpha}|^2}, \end{aligned} \quad (8)$$

where $\boldsymbol{\alpha}$ is a unit vector in the **J** direction.

This definition holds even if **B** and **J** are not perpendicular. Also since it includes only the antisymmetric part of ρ, it corresponds to the experimental situation in which the transverse voltage is measured before and after reversing the magnetic field, and the magnitude of the two results averaged. For symmetrical current directions we have

J	R_H
100	$\frac{1}{2}[(\rho_{21}-\rho_{12})B_3-(\rho_{31}-\rho_{13})B_2]/(B_2+B_3)^2$
110	$\frac{1}{2}[2B_3(\rho_{21}-\rho_{12})+(B_1-B_2)(\rho_{31}-\rho_{13}+\rho_{32}-\rho_{23})]/[2B^2-(B_1+B_2)^2]$. (9)

The detailed expressions are given in Appendix III for

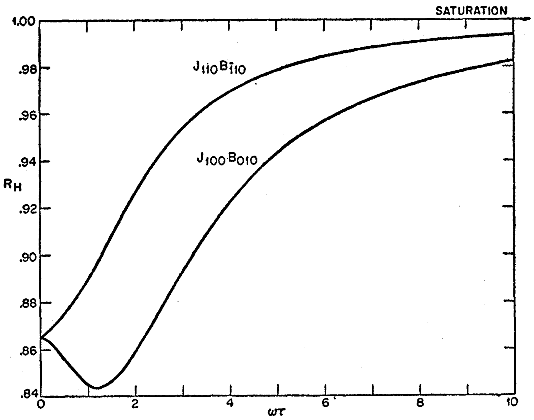

FIG. 6. Field dependence of Hall coefficient R_H for n-silicon for J_{110}, $B_{\bar{1}10}$ and J_{100}, B_{010}.

the same planes of rotation of the magnetic field as were considered for the magnetoresistance.

The field dependence of the Hall coefficient for germanium is displayed in Fig. 5 for J_{100}, B_{010} and J_{110}, $B_{1\bar{1}0}$. The former coefficient can be written explicitly

$$R_H=\frac{1}{nqc}\frac{3K(K+2)}{(2K+1)^2}\times\left(1+\frac{2+K}{3K}b^2\right)\Big/\left[1+\left(\frac{K+2}{2K+1}\right)^2 b^2\right], \quad (10)$$

and clearly has the limits:

$$b\rightarrow\infty,\quad R_H=1/nqc;$$

$$b\rightarrow 0,\quad R_H=\frac{1}{nqc}\left(\frac{3K(K+2)}{(2K+1)^2}\right).$$

It does not exhibit a minimum as does J_{110}, $B_{1\bar{1}0}$.

The parallel situation for silicon appears in Fig. 6, but now the minimum occurs for J_{100}, B_{010}, for which the Hall coefficient is given explicitly:

$$R_H=\frac{3K}{nqc}\times\frac{(1+b^2)(1+b^2/K)[K+2+3b^2]}{[(2K+1)+(K+2)b^2]^2+b^2[K+2+3b^2]^2}. \quad (11)$$

Here $b\rightarrow\infty$ leads to $R_H=1/nqc$ as required, and for $b\rightarrow 0$ we find precisely the same result as for germanium. Indeed, it is seen that for $b\rightarrow 0$, i.e., the low-field limit, the Hall coefficient becomes independent of orientation. The factor $3K(K+2)/(2K+1)^2$ properly has a value of unity for $K=1$ and in the limits of $K\rightarrow 0$ and $K\rightarrow\infty$ approaches zero and $\frac{3}{4}$, respectively.

Finally, the symmetry properties for R_H are observed with such typical equivalences as J_{110}, $B_{001}=J_{100}$, $B_{010}=J_{00\bar{1}}$, B_{100}, etc.

ACKNOWLEDGMENTS

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APPENDIX I. CONDUCTIVITY TENSOR

A. Germanium

For the quantities in Eq. (1), we have from Part I:

$$a_1=\left[\frac{1}{4}\sum_i\frac{1}{\Delta_i}\right]\left(1+\frac{3}{2K+1}b_1^2\right),$$

$$c_1=\left[\frac{1}{4}\sum_i\frac{1}{\Delta_i}\right]\frac{3}{2K+1}b_2b_3-\frac{K-1}{2K+1}\left[\frac{1}{4}\sum_i\frac{S_1^i}{\Delta_i}\right],$$

$$d_1=\left[\frac{1}{4}\sum_i\frac{1}{\Delta_i}\right]\frac{K+2}{2K+1}b_1+\frac{K-1}{2K+1}\left[\frac{1}{4}\sum_i\frac{S_2^ib_3+S_3^ib_2}{\Delta_i}\right],$$

with the remaining quantities given by cyclic permutations. Here

$$\Delta_i=1+\frac{1}{K}b^2+\frac{K-1}{3K}(S_1^ib_1+S_2^ib_2+S_3^ib_3)^2,\quad \mathbf{b}=\frac{q\mathbf{B}\tau}{m_2c},$$

$K=m_1/m_2$, and the S^i are, for the four sets of ellipsoids,

	I	II	III	IV
S_1^i	1	1	−1	−1
S_2^i	1	−1	−1	1
S_3^i	1	−1	1	−1

B. Silicon

The conductivity tensor is again given by Eq. (1), where now

$$a_1=\frac{1}{2K+1}\left(\frac{1}{\Delta_1}+\frac{K}{\Delta_2}+\frac{K}{\Delta_3}+\sum_i\frac{1}{\Delta_i}b_1^2\right),$$

$$c_1=\frac{b_2b_3}{2K+1}\sum_i\frac{1}{\Delta_i},$$

$$d_1=\frac{b_1}{2K+1}\left(\frac{K}{\Delta_1}+\frac{1}{\Delta_2}+\frac{1}{\Delta_3}\right),\ \text{etc.,}$$

$$\Delta_i=1+\frac{1}{K}b^2+\frac{K-1}{K}b_i^2.$$

APPENDIX II. SPECIALIZATION OF THE MAGNETORESISTANCE [EQS. (5)] FOR PARTICULAR PLANES OF ROTATION OF THE MAGNETIC VECTOR

A. n-Germanium

1. $\mathbf{B}$ in a (100) plane ($B_3=0$)

(a) For $\mathbf{J}_{100}$ (*longitudinal*),

$$\rho/\rho^*=\Delta^{-1}[1+yb^2\sin^2\theta+b^2\cos^2\theta(z-xb^2\sin^2\theta)],$$

where

$$x=\frac{2(K-1)^2}{3K(2K+1)}\left(1+\frac{K+2}{3K}b^2\right)^{-1},$$

$$y=\frac{3}{2K+1},\quad z=\frac{K+2}{2K+1},$$

$$\Delta=\{(1+yb^2)(1+z^2b^2)-\tfrac{1}{4}b^4\sin^2 2\theta x[(4z+x)(1+yb^2)-2(z-1)^2]+\tfrac{1}{8}b^8\sin^4 2\theta(2y+x)x^2\}\times\left\{1+\frac{K+2}{3K}b^2-\frac{x}{2yK}\sin^2 2\theta\right\},$$

and θ is the angle between **B** and the [100] axis.

(b) For $\mathbf{J}_{001}$ (*transverse*),

$$\rho/\rho^*=\Delta^{-1}[1+yb^2-\tfrac{1}{4}b^4\sin^2 2\theta\cdot(x+2y)x],$$

(c) For $\mathbf{J}_{110}$,

$$\rho/\rho^*=\tfrac{1}{2}\Delta^{-1}[2+yb^2-(y+x)b^2\sin 2\theta+b^2(z-\tfrac{1}{2}xb^2\sin 2\theta)^2(1+\sin 2\theta)].$$

2. **B** in a (110) plane ($B_1=B_2$, $B_3\neq 0$)

The calculation is facilitated by rotation of the coordinate axes to $x'\rightarrow 110$, $y'\rightarrow\bar{1}10$, $z'\rightarrow 001$.

(a) For $\mathbf{J}_{110}$ (*longitudinal*),

$$\rho/\rho^*=P\,(\det')^{-1}(F_1F_3+F_5^2),$$

(b) For $\mathbf{J}_{110}$ (*transverse*),

$$\rho/\rho^*=P(\det')^{-1}(F_2F_3-F_4^2),$$

where

$$P=(A-xb^2\sin^2\theta)[(A+xb^2\sin^2\theta)^2-2x^2b^4\sin^2 2\theta],$$

$$A=1+\left(\frac{K+2}{3K}\right)b^2,\quad x=\frac{K-1}{3K},$$

$$\det'=F_1F_2F_3+2F_4F_5F_6-F_1F_4^2+F_2F_5^2+F_3F_6^2,$$

$$F_1=S_0-S_2,$$

$$F_2=S_0\left(1+\frac{3}{2K+1}b^2\sin^2\theta\right)+S_2,$$

$$F_3=S_0\left(1+\frac{3}{2K+1}b^2\cos^2\theta\right),$$

$$F_4=S_0\frac{3}{2K+1}\,\tfrac{1}{2}b^2\sin 2\theta+S_1,$$

$$F_5=S_0\frac{K+2}{2K+1}b\sin\theta-S_1b\cos\theta-S_2b\sin\theta,$$

$$F_6=S_0\frac{K+2}{2K+1}b\cos\theta-S_1b\sin\theta,$$

$$S_0=A(A+xb^2\sin^2\theta)-x^2b^4\sin^2 2\theta,$$

$$S_1=\frac{K-1}{2K+1}xb^2\sin 2\theta(A-xb^2\sin^2\theta),$$

$$S_2=\frac{K-1}{2K+1}xb^2\sin^2\theta(A-4xb^2+5xb^2\sin^2\theta),$$

θ is angle between **B** and z' axis.

B. *n*-Silicon

1. **B** in a (100) plane ($B_3=0$)

(a) For $\mathbf{J}_{100}$,

$$\frac{\rho}{\rho^*}=\frac{\Delta_1\Delta_2\Delta_3}{\det'}(F_2F_3+F_5^2),$$

(b) For $\mathbf{J}_{001}$,

$$\frac{\rho}{\rho^*}=\frac{\Delta_1\Delta_2\Delta_3}{\det'}(F_1F_2-F_4^2),$$

(c) For $\mathbf{J}_{110}$,

$$\frac{\rho}{\rho^*}=\frac{1}{2}\frac{\Delta_1\Delta_2\Delta_3}{\det'}[(F_1+F_2)F_3-2F_3F_4+(F_5+F_6)^2],$$

where

$$\Delta_1=1+\frac{1}{K}[b^2+(K-1)b_1^2],$$

$$\Delta_2=1+\frac{1}{K}[b^2+(K-1)b_2^2],\quad \Delta_3=1+\frac{b^2}{K},$$

$$\det'=F_1F_2F_3+2F_4F_5F_6+F_1F_5^2+F_2F_6^2-F_3F_4^2,$$

$$F_1=A+\tfrac{1}{4}Kxb^4\sin^2 2\theta+b^2\cos^2\theta[E+\tfrac{1}{4}xb^4\sin^2 2\theta],$$

$$F_2=A+\tfrac{1}{4}Kxb^4\sin^2 2\theta+b^2\sin^2\theta[E+\tfrac{1}{4}xb^4\sin^2 2\theta],$$

$$F_3=A+\tfrac{1}{4}xb^4\sin^2 2\theta,$$

$$F_4=\tfrac{1}{2}b^2\sin 2\theta(C+\tfrac{1}{4}xb^4\sin^2 2\theta),$$

$$F_5=b\cos\theta(D+\tfrac{1}{4}xb^4\sin^2 2\theta+yb^2\sin^2\theta),$$

$$F_6=b\sin\theta(D+\tfrac{1}{4}xb^4\sin^2 2\theta+yb^2\cos^2\theta),$$

$$x=\frac{1}{2K+1}\left(\frac{K-1}{K}\right)^2,\quad y=\frac{\Delta_3}{2K+1}\frac{(K-1)^2}{K},$$

$$A=\frac{1}{2K+1}\left[\Delta_3^2(1+2K)+\frac{K^2-1}{K}\Delta_3b^2\right],$$

$$C=\frac{1}{2K+1}\left[3\Delta_3^2+2\Delta_3\left(\frac{K-1}{K}\right)b^2\right],$$

$$D=\frac{1}{2K+1}\left[\Delta_3^2(2+K)+2\Delta_3\left(\frac{K-1}{K}\right)b^2\right],$$

$$E=\frac{1}{2K+1}\left[3\Delta_3^2+2\Delta_3\frac{K-1}{K}b^2+\Delta_3\frac{(K-1)^2}{K}\right].$$

2. **B** in a (110) plane ($B_1=B_2$, $B_3\neq 0$)

The calculation is facilitated by the same rotation of axes as for *n*-germanium.

(a) For $\mathbf{J}_{110}$,

$$\frac{\rho}{\rho^*}=\frac{\Delta\Delta_3}{\det'}(F_1F_3+F_5^2),$$

(b) For $\mathbf{J}_{\bar{1}10}$,

$$\frac{\rho}{\rho^*}=\frac{\Delta\Delta_3}{\det'}(F_2F_3-F_4^2),$$

where

$$\Delta=1+\frac{1}{K}b^2+\frac{K-1}{2K}b^2\sin^2\theta,$$

$$\Delta_3=1+b^2-\frac{K-1}{K}b^2\sin^2\theta,$$

$$\det'=F_1F_2F_3+2F_4F_5F_6-F_1F_4^2+F_2F_5^2+F_3F_6^2,$$

$$F_1=1+zb^2-zxb^2\sin^2\theta,$$

$$F_2=1+zb^2+b^2\sin^2\theta(y+b^2/K-zx)-xyb^4\sin^4\theta,$$

$$F_3=(1+b^2)(1+b^2/K)-b^2\sin^2\theta[y+b^2/K+x(v+yb^2)]+xyb^4\sin^4\theta,$$

$$F_4=\tfrac{1}{2}b^2\sin 2\theta(y+b^2/K-xyb^2\sin^2\theta),$$

$$F_5=b\sin\theta(z+ub^2-xb^2\sin^2\theta),$$

$$F_6=b\cos\theta(z+yb^2-xwb^2\sin^2\theta),$$

$$x=\frac{K-1}{2K},\quad y=\frac{3}{2K+1},\quad z=\frac{K+2}{2K+1},$$

$$u=\frac{1+K+K^2}{K(2K+1)},\quad v=\frac{4K-1}{2K+1},\quad w=\frac{4-K}{2K+1}.$$

3. Field dependence of ρ/ρ^* for J_{100} and B_{010}

The field dependence of ρ/ρ^* for J_{100} and B_{010} is given by

$$\frac{\rho}{\rho^*}=\frac{A(A+Eb^2)}{A^2+D^2b^2},$$

where A, D, and E are the expressions in Sec. 1.

APPENDIX III. SPECIALIZATION OF EQS. (9) FOR PARTICULAR PLANES OF ROTATION OF THE MAGNETIC VECTOR B

We use here the same notation as in Appendix II.

A. *n*-Germanium

1. **B** *in a (100) plane ($B_3=0$)*

(a) For $\mathbf{J}_{100}$,

$$R_H=\frac{b_2}{B_2}\rho^*\Delta^{-1}[b_1^2(y+x)(z-xb_2^2)+(1+yb_2^2)(z-xb_1^2)],$$

(b) For $\mathbf{J}_{001}$,

$$R_H=\frac{\rho^*}{Bb}\Delta^{-1}\{zb^2(1+yb^2)-b_1^2b_2^2[x(y+2x)b^2-2x(z-1)]\},$$

(c) For $\mathbf{J}_{110}$,

$$R_H=\frac{b\rho^*}{B}\Delta^{-1}[z(1+yb^2)-b_1b_2x(z-1)-b_1^2b_2^2x(2y+x)].$$

2. **B** *in a (110) plane ($B_1=B_2$, $B_3\neq 0$)*

(a) For $\mathbf{J}_{110}$,

$$R_H=\frac{b}{B}\rho^*\frac{P(\det')^{-1}}{b\cos\theta}(F_4F_5+F_3F_6),$$

(b) For $\mathbf{J}_{110}$,

$$R_H=\frac{\rho^*}{B}P(\det')^{-1}[(F_4F_6+F_2F_5)\sin\theta+(F_4F_5+F_2F_6)\cos\theta],$$

B. *n*-Silicon

1. **B** *in a (100) plane ($B_3=0$)*

(a) For $\mathbf{J}_{100}$,

$$R_H=\frac{\rho^*}{B}\left(\frac{\Delta_1\Delta_2\Delta_3}{\det'}\right)\left(\frac{F_4F_5+F_2F_6}{\sin\theta}\right),$$

(b) For $\mathbf{J}_{001}$,

$$R_H=\frac{\rho^*}{B}\left(\frac{\Delta_1\Delta_2\Delta_3}{\det'}\right)[(F_4F_5+F_2F_6)\sin\theta+(F_4F_6+F_1F_5)\cos\theta],$$

(c) For $\mathbf{J}_{110}$,

$$R_H=\frac{\rho^*}{B}\left(\frac{\Delta_1\Delta_2\Delta_3}{\det'}\right)\left(\frac{1}{\cos\theta-\sin\theta}\right)\times(F_4F_6+F_1F_5-F_4F_5-F_2F_6).$$

2. **B** *in a (110) plane ($B_1=B_2$, $B_3\neq 0$)*

(a) For $\mathbf{J}_{110}$,

$$R_H=\frac{\rho^*}{B}\left(\frac{\Delta\Delta_3}{\det'}\right)\left(\frac{F_4F_5+F_3F_6}{\cos\theta}\right),$$

(b) For $\mathbf{J}_{110}$,

$$R_H=\frac{\rho^*}{B}[(F_4F_6+F_2F_5)\sin\theta+(F_4F_5+F_3F_6)\cos\theta].$$