Production of High-Energy Plasmas by Magnetically Driven Shock Waves*

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High-voltage discharges (5–100 kilovolts) with peak currents in 0.3 to 5 microseconds have been used to produce shock waves in deuterium plasmas. The discharge is struck in a transverse magnetic field and the resultant Lorentz force drives the plasma out of the region of the discharge into a quartz side arm. Velocity measurements of the magnetically driven plasma are discussed in relation to the parameters of the discharge. These measurements indicate that deuterium plasma with ion energies >100 ev/ion at densities $>5\times10^{16}$ ions/cm^3 can be produced by this method.

I. INTRODUCTION

THE generation of high-velocity waves in hydrogen or deuterium affords a possible method for producing high transient temperatures. Because of the low ionization potential of these atoms, complete ionization occurs at temperatures above ~30 000°K. At higher temperatures only the translational modes of excitation are affected by the addition of energy to the fully ionized plasma.

Fowler *et al.*[1–3] have studied shock waves produced by the ohmic heating of various gases in a T-shaped tube. In these experiments a discharge was struck between a pair of electrodes in a quartz tube and shock waves were observed to propagate down a side arm perpendicular to the current path.[4] With this apparatus plasma temperatures of the order 30 000°K were obtained in hydrogen for discharge voltages in the 1–10 kilovolt range. Because the electrical resistivity of a completely ionized gas depends inversely on the three-halves power of the temperature, the transfer of electrical energy to a plasma by resistive losses becomes inefficient with increasing temperature.

These experiments were extended by the use of external magnetic fields to accelerate the plasma as well as by utilizing higher voltages and lower inductance in the discharge circuit. The direction of the external magnetic field was orientated perpendicular to both the discharge and the side arm of the shock tube. As a result, there is a Lorentz force in the direction of propagation of the shock front. With this technique for magnetically driving a plasma, shock waves in deuterium with ion energies greater than 100 ev/ion have been produced.

II. HYDRODYNAMIC CONSIDERATIONS

A necessary condition for a hydrodynamic description of the gas flow in a shock tube at high temperatures is that the ion and electron densities be sufficiently high so that (a) their translational mean free paths are small compared to the dimensions of the tube and (b) the translational relaxation times must be small compared to cooling times. These conditions must be satisfied in order to establish local equilibrium in a fully ionized plasma. The relaxation time, τ_d, for deuterium ions to reach translational equilibrium is given approximately by the conventional formula,[5]

$$\tau_d=(16T^{\frac{3}{2}}/N_i\ln\Lambda)\ \text{sec};\qquad(1)$$

τ_d is approximately 2×10^{-8} second when $N_i=10^{17}$ ions/cm^3 and $T=10^6$ °K. The quantity $\ln\Lambda$ has been tabulated by Spitzer[5] and is of the order 10 for temperatures $T\sim10^5$–10^7 °K and for electron densities in the neighborhood of 10^{17} ions/cm^3. The translational mean free path λ_d, is found from the product of τ_d and the mean thermal velocity, v_d:

$$\begin{aligned}\lambda_d&=\tau_d v_d\\&=(1.6\times10^5T^2/N_i\ln\Lambda)\ \text{cm}\\&=0.2\ \text{cm for } N_i=10^{17}\ \text{ions/cm}^3 \text{ and } T=10^6\ \text{°K}.\end{aligned}\qquad(2)$$

As a consequence, for shock tubes a few centimeters in diameter, the ion density behind the shock front must exceed 10^{17} ions/cm^3 in the million-degree range if the mean free path is to be an order of magnitude smaller than the tube diameter.

The plasma temperatures and densities can be *estimated* from measurements of the shock velocity by utilizing the Rankine-Hugoniot equations for mass, momentum and energy conservation across the front. In the strong-shock approximation the pressure and internal energy ahead of the shock wave can be neglected in comparison to the pressure and internal energy in the high-temperature flow behind the shock front. By use of this approximation, the conservation equations for a completely ionized gas in thermal equilibrium can be written

$$\begin{aligned}\rho(V_s-u)&=\rho_0V_s.\\\rho u(V_s-u)&=P,\\\rho(\tfrac{1}{2}u^2+E)(V_s-u)&=Pu,\end{aligned}\qquad(3)$$

* This work was jointly supported by the U. S. Atomic Energy Commission and the Office of Naval Research.
1 Fowler, Goldstein, and Clotfelder, Phys. Rev. **82**, 879 (1951).
2 Fowler, Atkinson, and Marks, Phys. Rev. **87**, 966 (1952).
3 Fowler, Atkinson, Clotfelder, and Lee, University of Oklahoma Research Institute, Project Report, 1952 (unpublished).
4 In other experiments Fowler studied the production of shock waves with a coaxial discharge where the plasma is driven through a ring electrode into a quartz tube.
5 L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, New York, 1956).

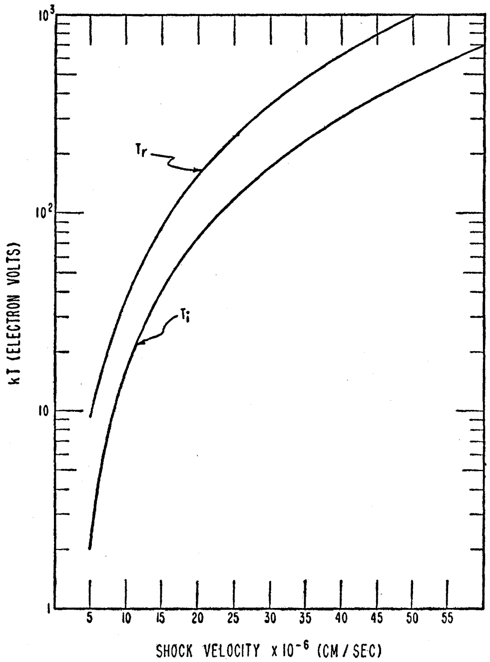

FIG. 1. Rankine-Hugoniot temperatures for deuterium in the strong-shock approximation, where T_i and T_r are the temperature behind the incident and reflected shock waves, respectively.

where V_s is the shock velocity, u the flow velocity, ρ the density, ρ_0 the ambient density, P the pressure, and E the internal energy per gram behind the front. E is related to P, ρ, and the dissociation and ionization energy per gram, I, by the relation

$$E=\tfrac{3}{2}(P/\rho)+I. \tag{4}$$

It follows from Eqs. (3) and (4) that E can be expressed in terms of V_s, u, and I:

$$E=\tfrac{1}{2}u^2=\tfrac{1}{2}\left\{\frac{9V_s^2}{32}\left[1+\left(1+\frac{32I}{9V_s^2}\right)^{\frac{1}{2}}\right]+\frac{I}{2}\right\}. \tag{5}$$

Also the density, ρ, behind the front is related to E, I, and the ambient density, ρ_0, by

$$\rho/\rho_0=(4E-I)/(E-I). \tag{6}$$

The translational temperature, T_i, behind the shock front is found from the relation

$$3kT_i+I'=E', \tag{7}$$

where E' and I' are the internal and dissociation energies per atom ($I'=15.8$ ev for a dissociated and ionized hydrogen or deuterium molecule). If the plasma behind the primary shock wave is brought to rest (by the collision of two strong shock waves for example) then the final temperature, T_r, can be estimated if one assumes that all the hydrodynamic flow energy is converted into internal energy. Since the internal energy, E, and the flow energy, $\frac{1}{2}u^2$, are equal, this leads to the expression

$$2E'=3kT_r+I'. \tag{8}$$

In Fig. 1 the temperatures T_i and T_r are plotted as a function of the shock velocity in deuterium using Eqs. (5), (7), and (8).

A more detailed theoretical analysis of the hydrodynamics has been made by Harris[6] and shows that behind the front the temperature increases and the density decreases relative to the conditions at the front. In this analysis it is assumed that the energy is deposited instantaneously in a slab of gas near the electrodes. It also assumes a γ-law gas and ignores wall effects and radiation losses. With these assumptions the position of the shock front, R, has the following dependence on the time, energy deposited in the slab of gas, W, and the ambient density

$$R\propto(t^2W/\rho_0)^{\frac{1}{3}}, \tag{9}$$

so that the shock velocity depends on W, ρ_0, t, and R according to the relation

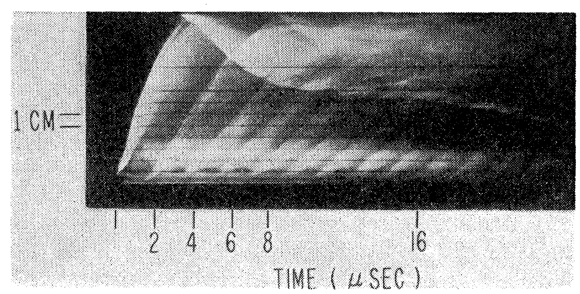

FIG. 2. Smear-camera photograph of magnetically driven shock waves. The successive shocks are due to current oscillations. The discharge is at the bottom of the photograph. Shocks reflected off the end of the tube are visible at the top of the photograph. (28 kv, 0.8 μf, 500 kc/sec, 3.64 mm hydrogen; maximum velocity 6.5 cm/μsec.)

$$V_s=\dot{R}\propto(W/\rho_0t)^{\frac{1}{3}}\propto(W/\rho_0R)^{\frac{1}{2}}. \tag{10}$$

III. EFFECT OF EXTERNAL MAGNETIC FIELDS

The simplest arrangement for accelerating a plasma in a T tube with magnetic fields is to place the lead to one of the electrodes at the base of the tube, parallel to the current path and perpendicular to the side arm. With this configuration the direction of the magnetic field produced by the strap behind the tube is such that there is a Lorentz force on the conducting plasma in the direction of the expansion chamber. A smear-camera photograph of shock waves generated in this fashion is presented in Fig. 2. The shock waves appearing after the primary shock are due to the ringing of the underdamped discharge. The high-temperature region behind the shock waves reflected from the end of the tube can be seen at the top of the photograph. The weakly

[6] E. G. Harris, Naval Research Laboratory Report NRL-4858, 1956 (unpublished).

luminous regions near the discharge, at the base of the shock tube, are caused by the magnetic field of the strap which expels the plasma from the vicinity of the electrodes. When the strap is removed, this effect is not observed on the photographs and the region near the discharge is filled with luminosity.

The nature of the luminous fronts in hydrogen have been investigated to confirm the existence of hydrodynamic shock waves. Smear-camera photographs (Fig. 3) with a slit imaged perpendicular to the direction of gas flow for ambient pressures from 0.3–3.0 mm of deuterium and for discharge voltages up to 50 kv show that the fronts are reasonably plane even at one tube diameter from the discharge, from which it follows that the luminous front is probably a hydrodynamic shock wave. The successive shock waves which appear are again due to the ringing of the discharge circuit. These

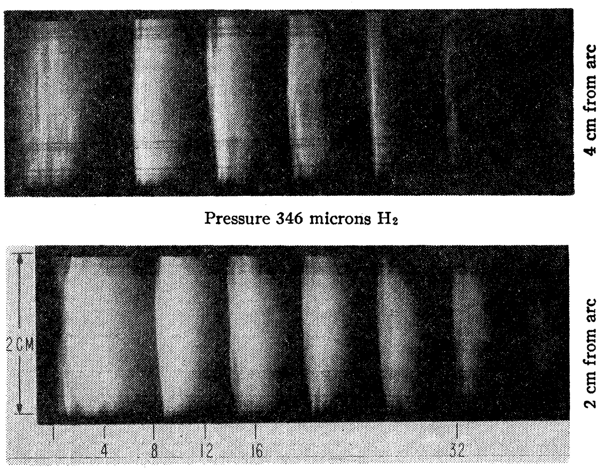

FIG. 3. Smear-camera photograph with the camera slit normal to the direction of shock propagation, showing the degree of planarity of the luminous fronts at one and two tube diameters from the discharge.

observations are presently being extended to higher voltages. Spectroscopic observations of the luminous fronts with a time-resolution of 10^{-7} second have also been made. These spectra show that the luminosity is primarily due to a continuum as well as spectral lines of silicon and oxygen from the walls of the tube. For shocks in deuterium or hydrogen the Balmer lines may or may not be prominent depending on the number density of neutral atoms which, in turn, depends sensitively on the temperature. Impurity lines from the metallic electrode material are either absent or very weak a few centimeters up the tube and, in any event, occur late in time. The latter observations eliminate the possibility that the high-velocity fronts consist of a jet of material boiled off the electrodes.

The influence on the shock velocity of the strap's magnetic field was demonstrated by the following method: the strap was rotated in such a manner that the

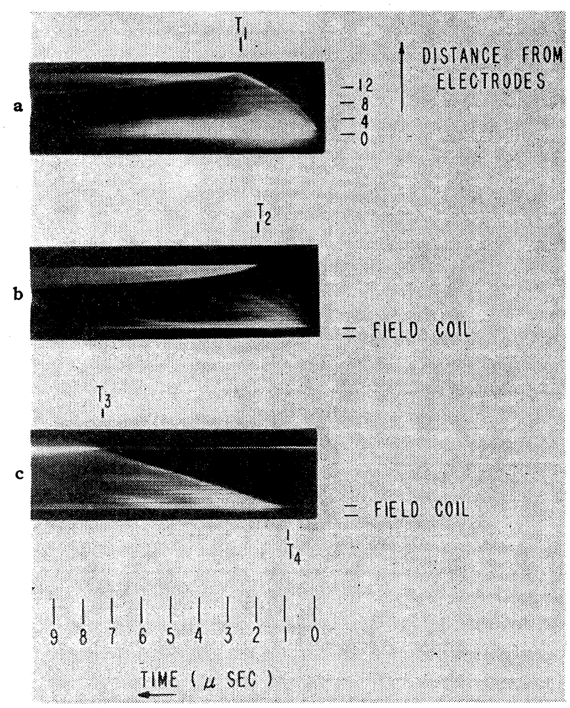

FIG. 4. Shock waves produced by (a) no external field coil, (b) external field coil aiding strap field, and (c) external field opposing strap field. The shock velocity for case b>a>c (i.e., $T_2<T_1<T_3$). The external field was generated with an 18-turn solenoid of one-centimeter radius. See also Table I.

Lorentz force was perpendicular to the direction of the shock propagation, i.e., perpendicular to the side arm of the T-tube. With this arrangement the velocity of the luminous front in deuterium was reduced by a factor of four at an ambient pressure of 2.1 mm Hg and by a factor of two at 10 mm Hg. The voltage, capacitance, and ring frequency were the same in both cases ($V=20$ kv, $C=5$ μf, $f=115$ kc/sec). The same experiment with $P=0.7$ mm, $V=50$ kv, $C=0.52$ μf, $f=700$ kc/sec showed that the strap field held the plasma in the discharge chamber for 0.3 μsec which was also the rise time to maximum current.

An obvious method for increasing the magnetic forces on the plasma is by the use of external field coils arranged so that there is a magnetic field perpendicular to both the discharge current and the expansion tube. In experiments with external fields the current in the coils is held essentially constant during the rise-time of the primary discharge.

Figures 4(a), (b), (c), show the effects of (a) no external field coils, (b) external field aiding the strap field, and (c) external field opposing the strap field. If the net magnetic forces are not directed along the expansion chamber, then the hot plasma is held in the region of the discharge for approximately one microsecond as shown in Fig. 4(c).

In Table I typical velocity data are given for a 50-kv discharge with and without an external field of 15 000

TABLE I. Shock velocity as a function of distance from electrodes for a 50-kv, 0.52-μf, 700-kc/sec discharge in (A) 15 000-gauss external field, and (B) zero field. Ambient pressure: 0.7 mm deuterium.

R (cm)	V_A (cm/μsec)	V_B (cm/μsec)	$(V_A/V_B)^2$
3.5	9.0	10.3	0.9
4.5	9.0	8.8	1.0
5.5	8.8	7.0	1.6
6.5	8.5	6.1	1.9
7.5	8.1	5.8	2.0
9	7.5	5.6	1.8
11	6.7	5.5	1.5

gauss. In this particular experiment the field was generated with a solenoid of one centimeter radius. The electrodes in the shock tube were at the center of the solenoid.

The external field increases the over-all velocity of the shock wave [compare also Figs. 4(a) and 4(b)] so that the plasma energy some 8 centimeters from the electrodes is doubled, as measured from the ratio of the square of the shock velocities. In other experiments with slower discharges (40 kv, 3 μf, 115 kc/sec) in a 20 000-gauss field, the plasma energy could be increased by a factor seven. However, not very reproducible results were obtained by attempting to strike a discharge with a slow rise time in a transverse field. There was a tendency for the external field to blow out the coronal current before an appreciable electron density could be produced in the electrode gap.

The preferential driving of the plasma in the direction determined by $[\mathbf{J}_{\text{internal}}\times\mathbf{H}_{\text{external}}]$ was also graphically demonstrated by placing the electrodes some four centimeters from the bottom of the tube and inside a solenoid. The transverse external field and the primary discharge were out of phase so that the Lorentz force changed direction some 16 μsec after the initiation of the primary discharge in the tube. As a result, the first two shock waves were directed up the tube and the next two waves were directed down the tube as shown in Fig. 5. These photographs (Figs. 4 and 5) demonstrate that the acceleration of the plasma is due principally to the existence of external magnetic fields rather than to the resistive heating of a cold gas with subsequent expansion of the hot plasma.

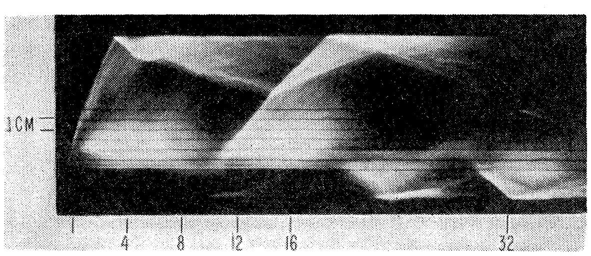

FIG. 5. Smear-camera photograph showing the preferential driving of hydrogen plasma in the direction of the Lorentz force produced by the field of a solenoid normal to the current path. The electrodes were 4 cm from the base of the tube.

IV. DEPENDENCE OF THE SHOCK VELOCITY ON THE PARAMETERS OF THE DISCHARGE

Measurements of the dependence of the shock velocity on ambient pressure, discharge voltage, inductance, capacitance, and the distance from the discharge have been made. Because of the large number of parameters which can be varied, the experimental results to be reported in this section have been selected to illustrate some of the main features of the high-temperature flow processes associated with the acceleration of ionized gases by the magnetic field of a strap at the base of a T-tube.

The shock velocity depends sensitively on the distance from the electrodes. This behavior is illustrated in Fig. 6 for a 15-kv discharge in deuterium. The error flags represent probable errors derived from eight experiments under the same conditions and indicate the degree of reproducibility of the velocity measurements.

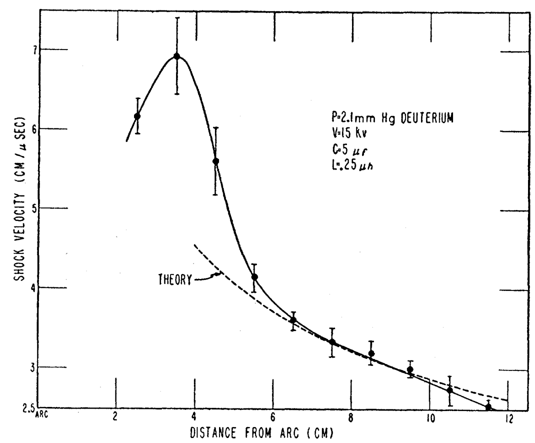

FIG. 6. Typical dependence of the shock velocity on the distance from the electrodes and comparison with a blast-wave theory as discussed in the text.

The dashed curve is the theoretical[6] velocity dependence arbitrarily normalized at 9 cm from the discharge. The velocity dependence agrees fairly well with the theoretical $V_s \propto R^{-\frac{1}{2}}$ for $R > 5.5$ cm. Between 3.5 and 5.5 cm the velocity falls off much more rapidly with distance than the theory predicts. This is in line with the general observation that the higher the maximum velocity the faster the initial attenuation. This arises from the higher rate of energy loss at high temperatures by radiation and by heat conduction to the walls. The theoretically predicted variation of the ion density with position from the front has also been qualitatively verified by time-resolved spectroscopic observations of the continuum intensity and the broadening of spectral lines for temperatures in the range ~30 000°K–60 000°K. The continuum intensity and the line broadening are greatest at the front and decrease with distance from the front. This implies that the ion densities are highest just behind the shock discontinuity.

The shock velocities also increase with decreasing ambient pressure. The experimental curve in Fig. 7 is compared with a theoretical curve[6] in which the velocity depends inversely on the square root of the ambient density. The velocity is not a sensitive function of the pressure at high velocities and low densities. This behavior is also ascribed to the loss of energy by heat conduction to the tube walls during the initial acceleration.

It has been found that an important parameter for producing high-velocity shock waves by the techniques described in Sec. III is the initial rate of rise of the current, $dI/dt = V/L$ amp/sec, where V and L are the discharge voltage and the total inductance of the circuit respectively. This is not an unexpected result since the magnetic driving is effective for only a few tenths of a microsecond for velocities of the order of 10^7 cm/sec. At later times the ejected plasma is too far from the discharge to couple efficiently to the driving field. In Fig. 8 the maximum velocity of the front (generally 2–5 cm from the electrodes) is plotted against $dI/dt = V/L$. The inductance was measured from the ring frequency for the first full cycle of the discharge. From these observations it is seen that the shock velocity increases more rapidly with voltage at constant inductance than with L^{-1} at constant voltage. Since the initial rate of rise of the magnetic energy density in an inductive load is proportional to V^2/L, the greater sensitivity of the shock velocity on the voltage than on the inductance can probably be explained qualitatively on this basis.

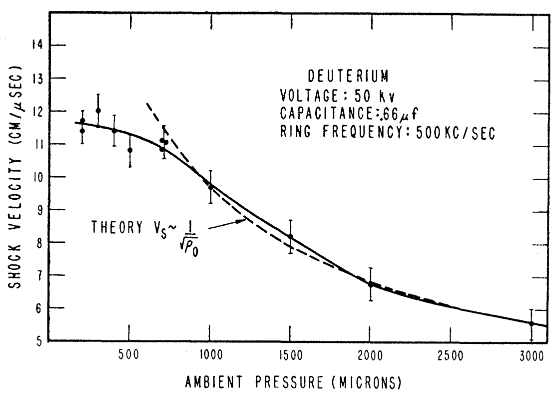

FIG. 7. The shock velocity *versus* ambient pressure and comparison with a blast-wave theory.

From measurements of the shock velocity with a smear camera the kinetic energy in the ordered motion of the deuterium ions can be estimated with the aid of Eq. (5). For a 0.52-μf, quarter-damped discharge, with a rise-time of 0.3 μsec to maximum current, the ion energy depends linearly on the discharge voltage as shown in Fig. 9. These data were obtained with deuterium at an ambient pressure of 0.7 mm.

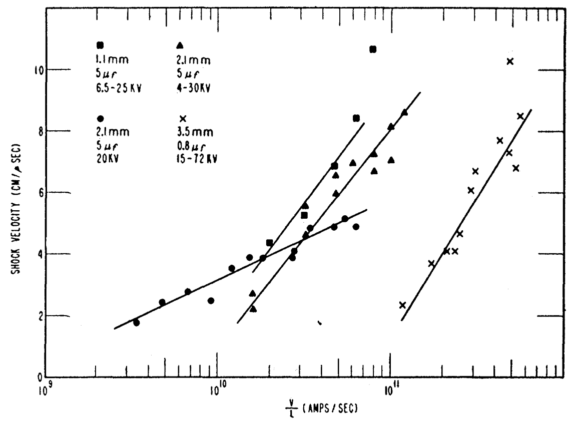

FIG. 8. Dependence of the shock velocity on the discharge voltage and inductance at various ambient pressures. The velocity increases more rapidly with increasing voltage than with decreasing inductance.

V. SUMMARY

These experiments have shown that high-energy, shock-heated plasmas can be produced by magnetic acceleration in a shock tube. At high densities, where the collision rate is high, the flow processes can probably be described hydrodynamically. However, it still remains to be demonstrated that the plasma in the shock tube actually reaches the high temperatures indicated by the velocity measurements. Spectroscopic temperature measurements as well as experiments at higher V/L and higher densities, in an attempt to more nearly achieve quasi-equilibrium conditions, will be reported in a subsequent publication. It is not expected, how-

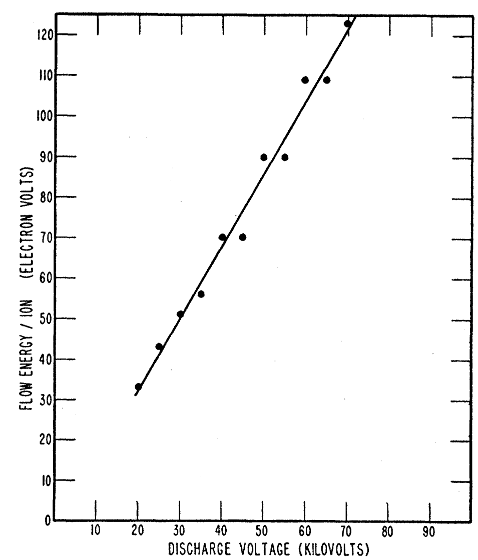

FIG. 9. The kinetic energy in the ordered motion of deuterium ions as a function of the discharge voltage. (0.52 μf, 0.3 μsec to maximum current, 0.7-mm deuterium.)

ever, that the shock velocity and temperature can be substantially increased by merely raising the discharge voltage since the effect of wall cooling becomes increasingly important as the plasma temperature is raised. However, it is not improbable that longitudinal magnetic fields along the expansion chamber can be used to inhibit the heat conduction to the tube walls.

ACKNOWLEDGMENTS

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Relaxation of a System of Particles with Coulomb Interactions*

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The relaxation to a Maxwellian distribution of a system of particles interacting through inverse-square-law forces is investigated in the approximation of two-particle interactions resulting in small-angle deflections of particle trajectories. The time required for the relaxation of the distribution in the neighborhood of the average energy is found to agree with the self-collision time defined by Spitzer. The time required for the distribution to become Maxwellian throughout the range from zero energy to several times the average energy is found to be nearly ten times the self-collision time. Filling of the high-energy portion of the Maxwell distribution is also discussed.

I. INTRODUCTION

THE relaxation of the electron or ion component of an ionized gas to a Maxwellian distribution has been of some astrophysical interest. Spitzer[1] has analyzed various aspects of the relaxation phenomenon such as (1) removal of angular anisotropy, (2) energy exchange, and (3) loss of energy of a particle by "dynamical friction." Bohm and Aller[2] have presented a detailed analysis on the relative importance of electron-electron collisions in establishing the velocity distribution of electrons in gaseous nebulae and stellar atmospheres. Although the general conclusions reached by these authors is almost certainly correct, the discussions were based on the rates of change of the distribution function and not on an explicit solution of the time-dependent problem.

In this paper we present an equation for the effect of particle interactions on the one-particle distribution function and the results of a numerical integration of the equation on an electronic digital computer for a distribution initially peaked about a particular energy. The filling of the high-energy portion of the Maxwell distribution is treated approximately.

II. TIME-DEPENDENT EQUATION

In obtaining an equation which describes the effect of interactions between particles of charge e and mass m upon the velocity distribution, we assume that (1) all interactions are a superposition of two-body interactions resulting in (2) small-angle deflections of particle trajectories. Although the validity of these two approximations is not rigorously established, the work of Spitzer, Cohen, and Routly[3] and of Gasiorowicz, Neuman, and Riddell[4] indicate their essential correctness for many phenomena. We shall use the Rutherford scattering law to determine the probability of deflections of a given magnitude. Restricting ourselves to isotropic angular distributions, we can obtain an equation for the time rate of change of the distribution function, either from an expansion of the integrand of the Boltzmann collision integral in powers of the angle of deflection,[5] or from the Fokker-Planck equation,[6]

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[1] L. Spitzer, *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956).

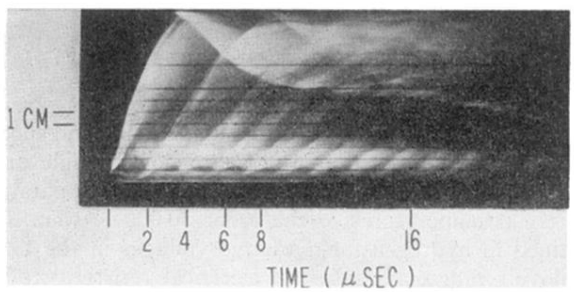
[2] D. Bohm and L. Aller, Astrophys. J. **105**, 131 (1947).

[3] Cohen, Spitzer, and Routly, Phys. Rev. **80**, 230 (1950).

[4] Gasiorowicz, Neuman, and Riddell, Phys. Rev. **101**, 922 (1956).

[5] The development by this procedure was considered too lengthy to be given here since the equation given in reference 6 yields the same result much more easily.

[6] Rosenbluth, MacDonald, and Judd, Phys. Rev. (to be published).



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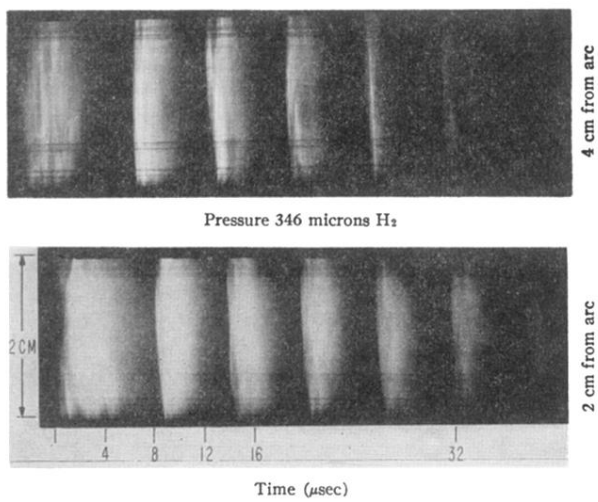

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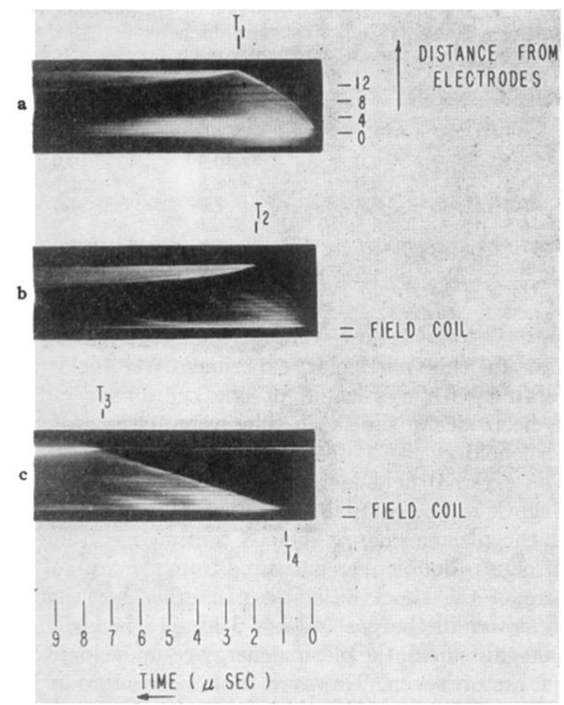

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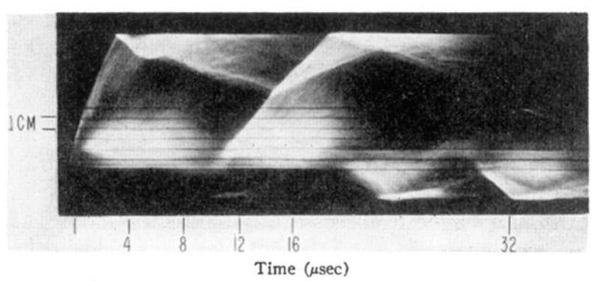

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