which Slater screening constants in the usual formula[7] for bound electronic wave functions were used. The use of Reitz's wave functions (instead of Slater screening constants) reduces the branching ratios by about 1% at intermediate $Z(\sim 50)$ and by about 2% at high Z.

It should be stressed that the Z in Table I refers to the charge of the *parent* nucleus.

The decrease of the branching ratio due to screening depends on W_0 and Z. For $Z=16$, it varies between $5\frac{1}{4}\%$ at $W_0=1.6\,mc^2$ to $2\frac{3}{4}\%$ at $W_0=4.8\,mc^2$. For $Z=29$, for the same range of W_0, the correction varies between $7\frac{3}{4}\%$ and $3\frac{1}{2}\%$; for $Z=49$ between $14\frac{1}{2}\%$ and $5\frac{3}{4}\%$; for $Z=84$ between $26\frac{3}{4}\%$ and $8\frac{3}{4}\%$; and for $Z=92$ between $29\frac{1}{2}\%$ and $10\frac{1}{4}\%$. These numbers are in essential agreement, within the 1 to 2% probable error in the numerical calculations, with those presented at the New York Meeting.[8] (Most of this error stems from the screening correction.)

* Operated by the General Electric Company for the U. S. Atomic Energy Commission.
[1] P. F. Zweifel, Phys. Rev. **96**, 1572 (1954).
[2] A. H. Wapstra (private communication).
[3] M. L. Perlman (private communication).
[4] H. Brysk and M. E. Rose, Oak Ridge National Laboratory Report ORNL-1830 (unpublished).
[5] M. E. Rose and D. K. Holmes, Oak Ridge National Laboratory Report ORNL-1022 (unpublished).
[6] J. R. Reitz, *Relativistic Electron Wave Functions for a Thomas-Fermi-Dirac Statistical Atom* (University of Chicago Press, Chicago, 1949).
[7] H. A. Bethe, *Handbuch der Physik* (Verlag Julius Springer, Berlin, 1933), second edition, Vol. 24, Part II, p. 311 ff.
[8] Wolfsberg, Welker, and Perlman, Bull. Am. Phys. Soc. Ser. II, **2**, 24 (1957).

Circular Polarization of Internal Bremsstrahlung

R. E. CUTKOSKY
Department of Physics, Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received May 17, 1957)

LEE and Yang's suggestion[1] that parity might not be conserved in weak interactions and the experimental proof[2–4] of the correctness of their suggestion have made desirable a re-examination of all processes in which weak interactions take part. The internal bremsstrahlung associated with K capture will be examined in this paper for properties which are consequences of parity nonconservation, with particular attention being given to the two-component neutrino theory.[5,6]

This paper will follow in detail the notation and methods introduced previously for studying the spectra and angular correlations of the internal bremmstrahlung gamma ray.[7] It follows directly from a theorem proved in IB that the asymmetry of the gamma rays emitted by an oriented electron-capturing nucleus[8] will be the same as the asymmetry which would be obtained if the nucleus were to emit positrons of zero mass; therefore, this paper will deal with the polarization of the bremsstrahlung, which has not been studied before. It is possible to envisage the bremsstrahlung as arising from the emission of a virtual positron, which annihilates one of the K electrons,[9] the momentum (and angular momentum) of the positron being carried off by the gamma ray. Since, as was predicted theoretically,[6,10,11] and as has been verified experimentally,[12,13] beta particles are circularly polarized, it is not implausible that the gamma ray might also be polarized.[14]

The matrix element for capture of a K electron, with emission of a gamma ray, is given by Eq. (12) of IB:

$$X_{K\gamma}=ie(2\omega)^{-\frac{1}{2}}\phi(0)\bar{u}_\nu(\mathbf{q},\rho)\bar{\Lambda}\left(\frac{i\gamma_\mu g_\mu-m}{g^2+m^2}\right)\mathbf{e}^*\cdot\gamma u_e(0,\sigma). \tag{1}$$

Since we are now interested in circular polarizations, it is necessary to introduce complex polarization vectors,

$$\mathbf{e}=2^{-\frac{1}{2}}(\boldsymbol{\varepsilon}_1\pm i\boldsymbol{\varepsilon}_2).$$

The plus sign refers to right circular polarization,[15] the minus sign to left circular polarization ($\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ are two perpendicular real unit vectors, which, with a unit vector $\boldsymbol{\kappa}$ having the direction of the photon propagation, form a right-handed triad). The square of $X_{K\gamma}$, summed over the electron and neutrino spins, is

$$\sum|X_{K\gamma}|^2=-\frac{2\pi\alpha}{\omega}[\phi(0)]^2\tfrac{1}{4}\,\mathrm{Tr}\left\{\frac{i\gamma_\mu g_\mu-m}{g^2+m^2}\boldsymbol{\gamma}\cdot\mathbf{e}^*(1+i\gamma_0)\times\boldsymbol{\gamma}\cdot\mathbf{e}\left(\frac{i\gamma_\mu g_\mu-m}{g^2+m^2}\right)N(-\mathbf{k},-\mathbf{q},\xi)\right\}. \tag{2}$$

Note that

$$\begin{aligned}\boldsymbol{\gamma}\cdot\mathbf{e}^*\boldsymbol{\gamma}\cdot\mathbf{e}&=\tfrac{1}{2}(\gamma_1\mp i\gamma_2)(\gamma_1\pm i\gamma_2)\\&=1\mp\boldsymbol{\kappa}\cdot\boldsymbol{\sigma}.\end{aligned} \tag{3}$$

Again, the upper sign refers to right circular polarization, the lower sign to left circular polarization. Equation (2) can now be reduced in a straightforward way to

$$\sum|X_{K\gamma}|^2=\pi\alpha\omega^{-1}m^{-2}[\phi(0)]^2\times\tfrac{1}{4}\,\mathrm{Tr}\{(i\gamma_0-i\boldsymbol{\gamma}\cdot\boldsymbol{\kappa})(1\mp\boldsymbol{\kappa}\cdot\boldsymbol{\sigma})N\}. \tag{4}$$

The factor $(i\gamma_0-i\boldsymbol{\gamma}\cdot\boldsymbol{\kappa})(1\mp\boldsymbol{\kappa}\cdot\boldsymbol{\sigma})\times\frac{1}{4}$ is equal to the projection operator (in hole theory) for a massless positron, traveilng in the direction of $\boldsymbol{\kappa}$, which is right (minus sign) or left (plus sign) circularly polarized. Equation (4) therefore shows that the polarization of the gamma ray is the same as the polarization of a zero-mass positron; this relation does not depend on the nature of the beta interaction. In the two-component theory, with scalar and tensor interactions, positrons have a degree of right circular polarization equal to v/c, and therefore the gamma ray will be 100% right circularly polarized, in the approximation considered here. [See IB (13), et seq.]

The method of the first part of IB can be used to give a simple proof that the polarization of massless beta particles, and hence of the internal bremsstrahlung, is independent of the degree of forbiddenness of the transition (in the two-component theory with scalar and tensor interactions), provided terms of order αZ are neglected. However, the large spin-orbit coupling terms, which are of order $\alpha Z/R$ (R is the radius of the nucleus), are kept. The proof depends on the fact that the electron wave function can be represented approximately by the form $\Psi(r)=A(r)u(\mathbf{p},\sigma)$, where $u(\mathbf{p},\sigma)$ is a plane wave spinor, and $A(r)$ is a matrix which commutes with γ_5.[16] In the two-component neutrino theory, the beta emission matrix element has the form

$$X_\beta=\bar{u}_e(\mathbf{p},\sigma)\Lambda'(1-\gamma_5)v_\nu(\mathbf{q},\rho), \tag{5}$$

where Λ' is the matrix which would describe the interaction at the nucleus in the old, parity-conserving, theory. If the beta-decay Hamiltonian is a combination of scalar and tensor terms, γ_5 will commute with Λ', even with the spin-orbit coupling effect included. Thus one can write

$$X_\beta=\bar{u}_e(\mathbf{p},\sigma)(1-\gamma_5)\Lambda' v_\nu(\mathbf{q},\rho). \tag{6}$$

Since the projection operator $(1-\gamma_5)$ now stands next to the electron spinor $\bar{u}_e$, it is evident that an electron will be emitted with a 100% polarization, if it has zero mass.

It will be noted that 100% polarization of the internal bremsstrahlung requires that the electron which is left in the K shell also be polarized 100% along the direction opposite to that of emission of the gamma ray. It should be pointed out that the 100% polarization of the gamma radiation is based on an approximation which assumes $\hbar\omega\gg\frac{1}{2}\alpha^2Z^2mc^2$. In particular, the bremsstrahlung associated with capture of p electrons is largely unpolarized, as can be inferred from the close relation between these gamma rays and the x-rays which follow ordinary K capture.[17]

The same techniques can be used to study the polarization of the internal bremsstrahlung accompanying beta emission. The general result is very complicated, but it is found that energetic gamma rays are right (left) circularly polarized if they accompany slow positrons (electrons), while low-energy gamma rays have a linear polarization correlated with the direction of the beta particle.

Finally, it may be observed that the theory of internal bremsstrahlung of K-electron capture can be applied without change to the absorption of μ mesons from a mesonic K orbit. A study of the internal-bremsstrahlung gamma rays associated with μ meson capture would provide one of the few ways of investigating that process. If the muon-nucleon interaction is the same as the electron-nucleon interaction, the internal bremsstrahlung will also be right circularly polarized. However, it may be easier to examine the correlation in direction between the meson spin and the gamma ray. The gamma rays which accompany the absorption of oriented muons must have an angular distribution proportional to $1+\cos\theta$, if they are 100% right circularly polarized.

I should like to thank Dr. L. Wolfenstein for many illuminating discussions of the Yang-Lee theory and its consequences. I should also like to thank Dr. S. DeBenedetti and Dr. L. Grodzins for their interest and their communications on the possibility of experimental detection of circular polarization of gamma rays.

1 T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
2 Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
3 Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).
4 J. I. Friedman and V. L. Telegdi, Phys. Rev. **106**, 1681 (1957).
5 T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
6 L. Landau, Nuclear Phys. **3**, 127 (1957); A. Salam, Nuovo cimento **5**, 299 (1957).
7 R. E. Cutkosky, Phys. Rev. **95**, 1222 (1954). (Hereafter referred to as IB.)
8 Such as, e.g., Fe^{55}.
9 C. Møller, Physik. Z. Sowjetunion **11**, 9 (1937).
10 L. Wolfenstein (private communication).
11 Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957).
12 Frauenfelder, Bobone, von Goeler, Levine, Lewis, Peacock, Rossi, and De Pasquali, Phys. Rev. **106**, 386 (1957).
13 L. A. Page and M. Heinberg, Phys. Rev. **106**, 1220 (1957).
14 It has been shown recently that the *external* bremsstrahlung from beta particles is partially circularly polarized [Goldhaber, Grodzins, and Sunyar, Phys. Rev. **106**, 826 (1957)].
15 This definition of circular polarization is opposite to the usual optical definition.
16 The following proof was suggested, for allowed transitions, by L. Wolfenstein.
17 R. J. Glauber and P. C. Martin, Phys. Rev. **95**, 572 (1954).

S-Wave Pions in the Reaction $p+p\rightarrow\pi^++d$*

FRANK S. CRAWFORD, JR., AND M. LYNN STEVENSON
Radiation Laboratory, University of California, Berkeley, California
(Received May 13, 1957)

IN their phenomenological analysis of the reaction

$$p+p\rightarrow\pi^++d, \tag{1}$$

Gell-Mann and Watson[1] use for the center-of-mass differential cross section the expression

$$4\pi\frac{d\sigma}{d\Omega}=\alpha\eta+\beta\eta^3\left(\frac{X+\cos^2\theta}{X+\frac{1}{3}}\right), \tag{2}$$

where η is the pion c.m. momentum in units $m_\pi c$, and α, β, and X are constants. The first term, $\alpha\eta$, is supposed to arise from S-wave pions, and the second from P-wave pions.

Integration of Eq. (2) yields the total cross section

$$\sigma=\alpha\eta+\beta\eta^3. \tag{3}$$

Lichtenberg has recently made a theoretical calculation[2] of the differential cross section for reaction (1) at