different in the two cases and A for the protons contains some Coulomb energy. The values of α and c are different not only because the protons' two-body force contains the Coulomb force, but also because of the probable different radial functions for protons and neutrons. The values of $-(16/9)c$ thus obtained are in reasonable agreement with the energy of the center of mass of the levels with $J=2$, 4, 6 (the excited states lie near states of other configurations and so may be more perturbed than the ground state). The best agreement is in the ${}_{22}\mathrm{Ti}_{28}{}^{50}$ case. The excited levels lie at 1.59 ($J=2$), 2.76 ($J=4$) and 3.27 ($J=6$).[12] The center of mass is 2.79 Mev above the ground state as compared to the value of $-(16/9)c=(16/9)(1.56)=2.77$ Mev.

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Proposed Experiment to Determine the Direction of µ-Meson Polarization in Pion Decay*

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It is of considerable interest to determine directly the sign of the polarization of the μ meson emitted in π decay. This sign bears on the question of conservation of leptons and the possibility of a universal Fermi interaction.[1] We wish to propose here an experiment whereby the sign of the μ-meson polarization can be determined essentially directly and largely independent of theory[2] by using the known directional asymmetries in beta decay.[3] The experiment depends on the residual polarization of μ^- mesons when stopped and bound in K-shell orbits around nuclei.[4] Suppose the polarized μ^- meson is captured by the nucleus, with emission of a neutrino and formation of a "daughter" nucleus in a state of definite, nonzero spin. The daughter nucleus will then be partially polarized in the direction of the μ-meson spin. If the daughter now undergoes ordinary beta decay, the *direction* of its spin orientation can be determined by measuring the directional asymmetry of the emitted beta-particle, as in the experiments performed at the National Bureau of Standards,[3] and so the direction of the μ-meson spin can be established.

The feasibility of an experiment along these lines is suggested by the work of Godfrey.[5] He studied the nuclear absorption of negative cosmic-ray μ mesons in carbon. Most of the absorptions lead to nucleon emission, but Godfrey found that B^{12} is formed with probability 0.13 ± 0.05, the B^{12} being detected by observing its beta decay to C^{12} (half-life of $\sim$0.025 sec and beta end point of 13.4 Mev).

Consider the idealized situation in which a polarized μ^- meson (polarization value $\langle\boldsymbol{\sigma}\rangle$) in a K orbit around a light nucleus is absorbed in an allowed transition, leaving the daughter nucleus in its ground state. It is easy to show that for a pure Gamow-Teller transition the polarization of the daughter nucleus is[6]

$$\frac{\langle\mathbf{J}\rangle}{J}=\left(\frac{J+1}{3J}\right)\lambda_{J'J}\langle\boldsymbol{\sigma}\rangle. \tag{1}$$

In the subsequent beta decay of the daughter nucleus, the directional asymmetry of the electrons determines this expectation value and hence determines the magnitude and direction of $\langle\boldsymbol{\sigma}\rangle$. This result is independent of any details of the theory of μ capture, except that it is a beta-type quadrilinear interaction.

For the capture process $\mu^-+\mathrm{C}^{12}\rightarrow\mathrm{B}^{12}+\nu$ with B^{12} in the ground state, the spin-parity assignments are $J'=0^+$, $J=1^+$, and $\lambda_{J'J}=1$. Thus $\langle\mathbf{J}\rangle=\frac{2}{3}\langle\boldsymbol{\sigma}\rangle$. Now it appears experimentally that $|\langle\boldsymbol{\sigma}\rangle|\simeq0.15$ for negative μ mesons stopped in carbon,[4] so the B^{12} nuclei would be about 10% oriented. In the subsequent beta decay of B^{12} the electrons will have an angular distribution relative to $\langle\mathbf{J}\rangle$ of the same form as that observed for Co^{60},[3] $(1+a\cos\theta)$, where the coefficient a is expected from both experiment[3] and theory[7] to be equal in magnitude to $|\langle\mathbf{J}\rangle|(v_e/c)$, i.e., $|a|\simeq0.1$, presumably an observable effect.[8]

The question now arises as to how far the actual state of affairs for μ^- mesons in carbon will depart from the idealized situation sketched above. First, the assumption of an allowed transition needs to be rationalized. With the neutrino carrying off $\sim$100 Mev in energy, it is clear that an expansion in multipole order loses its character of an expansion in forbiddenness. For the case of no nuclear parity change, the neutrino is emitted, loosely speaking, as an s wave or a d wave, or higher. In spite of the lack of centrifugal-barrier inhibitions, Godfrey's calculations of the relevant nuclear matrix elements, based on the $j-j$ coupling model, show that d-wave emission is depressed by a factor 10^{-3} relative to s-wave emission.[5] It therefore seems justifiable to assume that the transition to the B^{12} ground state is allowed in the usual sense.

The next question is that of transitions to excited, bound states of B^{12} in the μ-capture process. The fact that only 13% of all absorptions lead to bound states of B^{12} implies that high excitations are favored. Appreciable formation of excited states would wash out the orientation in the ground state because of the smearing over magnetic quantum numbers that occurs in the process of de-excitation by γ-ray emission. Fortunately, the situation here seems favorable. There are only four known excited states below the threshold for particle emission.[9] While no firm arguments can be made, what is known of the spins and parities of these states makes it seem probable that the large majority of μ-capture events leading to *bound* states of B^{12} actually go directly to the ground state.

Another effect which must be considered is possible depolarization of the B^{12} nucleus due to hyperfine interaction with the atomic electrons. Rough estimates indicate that the atom is probably ionized due to recoil at the instant of absorption of the μ meson. If the atom is always ionized and then re-forms again after it stops, we can calculate the depolarization under the assumption that the fine-structure substates are populated statistically. This gives, for the resultant B^{12} polarization,

$$\langle\mathbf{J}\rangle=\tfrac{2}{3}(0.54)\langle\boldsymbol{\sigma}\rangle=0.36\langle\boldsymbol{\sigma}\rangle. \tag{2}$$

Thus, if $|\langle\boldsymbol{\sigma}\rangle|$ equals 15%, the final polarization $|\langle\mathbf{J}\rangle|$ of the B^{12} is probably closer to 5% than to the value of 10% given above.

There is an additional depolarization due to the environment in which the B^{12} atom finds itself. But the relaxation time for this effect in graphite is presumably longer than the mean life of B^{12} since metals show relaxation times of the order of tens of milliseconds. In any event, such solid-state effects can be essentially eliminated by a suitable choice of organic material as target.

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Magnetic Dipole Moment of the Electron

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THE fourth-order radiative corrections to the magnetic dipole moment of the electron were calculated by Karplus and Kroll in 1949.[1] Their result is contained in the complete expression for the moment,

$$\mu_e/\mu_0=1+(\alpha/2\pi)-2.973(\alpha^2/\pi^2)=1.0011454, \tag{1}$$

where μ_0 is the Bohr magneton.

The calculation has been redone in the present instance using the mass-operator formalism of Schwinger.[2] We consider a single electron moving in a constant (in space and time) electromagnetic field. The expectation value of the mass operator in the lowest state represents the self or proper energy of the electron. The magnetic moment is identified from that part of the self-energy which is linear in the external field.

The electron Green's function G, the photon Green's function $\mathcal{G}$, and the interaction operator Γ, which appear in the symbolic expression for the mass operator,

$$M=m_e+ie^2\,\mathrm{Tr}\gamma G\Gamma\mathcal{G},$$

are computed in the presence of (as functions of) the external field. To do this it is sufficient to replace the electron's momentum operator, p, where it occurs, by by the combination $\Pi=p-eA$, provided that full account is taken of the commutation properties of Π. Units are such that $\hbar=c=1$. Renormalized quantities are used throughout the perturbation calculation.

The fourth-order contribution to the moment is found to be

$$\frac{\mu_e^{(4)}}{\mu_0}=\frac{\alpha^2}{\pi^2}\left(\frac{197}{144}+\frac{\pi^2}{12}+\tfrac{3}{4}\zeta(3)-\tfrac{1}{2}\pi^2\ln 2\right)=-0.328\frac{\alpha^2}{\pi^2}, \tag{2}$$

where $\zeta(3)$ is the Riemann zeta function of 3. Thus

$$\mu_e/\mu_0=1.0011596.$$

The discrepancy between (1) and (2) has been traced to the term $\mu^{\mathrm{I}}+\mu^{\mathrm{II}c}$ of Karplus and Kroll. In other words, terms $\mu^{\mathrm{II}e}$ and $\mu^{\mathrm{II}a}+\mu^{\mathrm{II}d}$ appear unchanged in the new result. A further point-by-point comparison of the two answers is not readily accomplished because the grouping of the terms differs markedly in the two cases. The present calculation has been checked several times and all of the auxiliary integrals have been done in at least two different ways.