Energies of $f_{7/2}{}^n$ Nuclear Configurations*

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An analysis of energies of excited states of proton configurations in the $f_{7/2}$ shell has recently been made.[1] These authors used, in accordance with previous work,[2–4] pure jj-coupling wave functions with the same radial functions for all the nuclei in the shell and assumed that the potential energy arises from two body forces. In order to express energies in the $f_{7/2}{}^n$ configuration in terms of those in $f_{7/2}{}^2$, they used this formula[5]:

$$\langle j^n\alpha J|\sum_{k<l}V_{kl}|j^n\alpha J\rangle$$

$$=\frac{n}{n-2}\sum_{\alpha_1\alpha_2J_1}(j^n\alpha J[\![j^{n-1}(\alpha_1J_1)jJ)(j^{n-1}(\alpha_2J_1)jJ]\!]j^n\alpha J)$$

$$\times\langle j^{n-1}\alpha_1J_1|\sum_{k<l}V_{kl}|j^{n-1}\alpha_2J_1\rangle,\quad(1)$$

where α, α_1, and α_2 stand for additional quantum numbers necessary to define the states. This equation holds also if energies relative to, say, the ground states are used on both sides.[2] The agreement obtained is reasonable and introduces some order among the many measured levels. It is worth while to point out that in addition to the proton configurations, which they consider, the corresponding neutron configurations (with 20 protons) have quite similar level schemes.[6,7] Using the proton levels as a guide, we can make the following tentative assignments. In Ca^{42} the $f_{7/2}{}^2$ levels could be the $J=0$ ground state, the 2+ level at 1.53 Mev, the level at 2.75 (or 2.42) Mev with $J=4$, and the level at 3.25 Mev (or thereabout) with $J=6$. The corresponding levels in Ca^{44} would be the ground state, at 1.16, at 2.66 (or 2.38) and at 3.30 (3.08) Mev. In Ca^{43} the $\frac{5}{2}$-level lies 0.37 Mev above the $\frac{7}{2}$-ground state and in Ca^{45} 0.18 Mev above it. The $\frac{3}{2}$-level at Ca^{43} lies only 0.59 Mev above the ground state (some other level is at 0.99), whereas the second excited state in Ca^{45} is at 1.43 Mev. The situation is very similar to that in the proton configurations. The variations are very probably due to configuration interaction. The consistency of this picture requires that the 1.84-Mev level in Ca^{42} and the 1.89-Mev level in Ca^{44} belong to a different configuration (this is possibly the case also with the 2.42-Mev level in Ca^{42} and the 2.38-Mev level in Ca^{44}). It also requires at least a large admixture of another configuration in the 0.59-Mev state of Ca^{43} and 1.16-Mev level in Ca^{44}. Various reactions might clarify the situation and the problem is being investigated in this Laboratory.

We can now check how these level assignments agree with the information on binding energies. The energy of the ground state can also be treated in the same way as the energies of excited states. For identical nucleons, which we consider now, the ground states of the $f_{7/2}{}^n$ configurations have $J=7/2$ for n odd and $J=0$ for n even. The seniority is a good quantum number in the $f_{7/2}$ shell of identical particles.[8] This is the reason why the levels of $f_{7/2}{}^4$ with seniorities 2 and 0 are predicted to have the same spacings as in the $f_{7/2}{}^2$ configuration.[1,8,9] We can, therefore, use Racah's methods[10] to write the energies of the ground states in terms of the $f_{7/2}{}^2$ energies. The $f_{7/2}{}^2$ excited states (with $J=2,4,6$) enter the result only in the energy difference between their center of mass and the ground state ($J=0$). If we define

$$-(16/9)c=(1/27)[5(E_2-E_0)+9(E_4-E_0)+13(E_6-E_0)],$$

we can deduce the following result which is a special case of Eq. (1) in reference 3:

$$\text{B.E.}-\text{B.E. (preceding closed-shells nucleus)}$$
$$=nA+\tfrac{1}{2}n(n-1)\alpha-[\tfrac{1}{2}n]2c\quad(2)$$

($[\frac{1}{2}n]$ is the largest integer not exceeding $\frac{1}{2}n$). A is the kinetic energy of an $f_{7/2}$ nucleon plus its interaction with the closed shells; α is a certain linear combination of the $f_{7/2}{}^2$ energies (it is equal to $a+\frac{1}{2}b$ of reference 3, whereas c has the same meaning). Unlike Eq. (1), reference 3, this formula is exact. Further, we deal here separately with protons and neutrons and so expect no difficulties from the difference in their radial functions.

Table I. Binding energies of $f_{7/2}{}^n$ configurations in Mev.

Nucleus	Binding energies[a] Experimental	Binding energies[a] Calculated	Nucleus	Binding energies[b] Experimental	Binding energies[b] Calculated
${}_{20}\text{Ca}_{21}{}^{41}$	8.36	8.38	${}_{21}\text{Sc}_{28}{}^{49}$	...	9.69
${}_{20}\text{Ca}_{22}{}^{42}$	19.83	19.86	${}_{22}\text{Ti}_{28}{}^{50}$	21.78	21.72
${}_{20}\text{Ca}_{23}{}^{43}$	27.75	27.78	${}_{23}\text{V}_{28}{}^{51}$	29.82	29.85
${}_{20}\text{Ca}_{24}{}^{44}$	38.89	38.80	${}_{24}\text{Cr}_{28}{}^{52}$	40.34	40.32
${}_{20}\text{Ca}_{25}{}^{45}$	46.31	46.26	${}_{25}\text{Mn}_{28}{}^{53}$	46.90	46.90
${}_{20}\text{Ca}_{26}{}^{46}$	56.72	56.81	${}_{26}\text{Fe}_{28}{}^{54}$	55.75	55.81
${}_{20}\text{Ca}_{27}{}^{47}$	...	63.81	${}_{27}\text{Co}_{28}{}^{55}$	60.85	60.82
${}_{20}\text{Ca}_{28}{}^{48}$	73.95	73.91	${}_{28}\text{Ni}_{28}{}^{56}$	...	68.17

[a] From these the binding energy of Ca^{40} was subtracted.
[b] From these the binding energy of Ca^{48} was subtracted.

Recent accurate values for binding energies[11] were used in the following analysis. Values for A, α, and c were obtained (by a least-squares fit) which reproduce best the experimental energies. Energies calculated with these values are compared with the experimental data in Table I. The agreement obtained is excellent. The rms deviation is 0.075 Mev (being 0.12% of the energy range considered) in the neutron case and 0.056 Mev (0.13% of the energy range) in the proton case. The best values of the parameters are (in Mev)

for the neutrons $A=8.380$, $\alpha=-0.230$, $c=-1.664$;

for the protons $A=9.691$, $\alpha=-0.780$, $c=-1.560$.

The values of A are different since the closed shells are different in the two cases and A for the protons contains some Coulomb energy. The values of α and c are different not only because the protons' two-body force contains the Coulomb force, but also because of the probable different radial functions for protons and neutrons. The values of $-(16/9)c$ thus obtained are in reasonable agreement with the energy of the center of mass of the levels with $J=2$, 4, 6 (the excited states lie near states of other configurations and so may be more perturbed than the ground state). The best agreement is in the ${}_{22}\mathrm{Ti}_{28}{}^{50}$ case. The excited levels lie at 1.59 ($J=2$), 2.76 ($J=4$) and 3.27 ($J=6$).[12] The center of mass is 2.79 Mev above the ground state as compared to the value of $-(16/9)c=(16/9)(1.56)=2.77$ Mev.

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Proposed Experiment to Determine the Direction of μ-Meson Polarization in Pion Decay*

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It is of considerable interest to determine directly the sign of the polarization of the μ meson emitted in π decay. This sign bears on the question of conservation of leptons and the possibility of a universal Fermi interaction.[1] We wish to propose here an experiment whereby the sign of the μ-meson polarization can be determined essentially directly and largely independent of theory[2] by using the known directional asymmetries in beta decay.[3] The experiment depends on the residual polarization of μ^- mesons when stopped and bound in K-shell orbits around nuclei.[4] Suppose the polarized μ^- meson is captured by the nucleus, with emission of a neutrino and formation of a "daughter" nucleus in a state of definite, nonzero spin. The daughter nucleus will then be partially polarized in the direction of the μ-meson spin. If the daughter now undergoes ordinary beta decay, the *direction* of its spin orientation can be determined by measuring the directional asymmetry of the emitted beta-particle, as in the experiments performed at the National Bureau of Standards,[3] and so the direction of the μ-meson spin can be established.

The feasibility of an experiment along these lines is suggested by the work of Godfrey.[5] He studied the nuclear absorption of negative cosmic-ray μ mesons in carbon. Most of the absorptions lead to nucleon emission, but Godfrey found that B^{12} is formed with probability 0.13 ± 0.05, the B^{12} being detected by observing its beta decay to C^{12} (half-life of ~0.025 sec and beta end point of 13.4 Mev).

Consider the idealized situation in which a polarized μ^- meson (polarization value $\langle\boldsymbol{\sigma}\rangle$) in a K orbit around a light nucleus is absorbed in an allowed transition, leaving the daughter nucleus in its ground state. It is easy to show that for a pure Gamow-Teller transition the polarization of the daughter nucleus is[6]

$$\frac{\langle\mathbf{J}\rangle}{J}=\left(\frac{J+1}{3J}\right)\lambda_{J'J}\langle\boldsymbol{\sigma}\rangle. \tag{1}$$

In the subsequent beta decay of the daughter nucleus, the directional asymmetry of the electrons determines this expectation value and hence determines the magnitude and direction of $\langle\boldsymbol{\sigma}\rangle$. This result is independent of any details of the theory of μ capture, except that it is a beta-type quadrilinear interaction.

For the capture process $\mu^-+\mathrm{C}^{12}\rightarrow\mathrm{B}^{12}+\nu$ with B^{12} in the ground state, the spin-parity assignments are $J'=0^+$, $J=1^+$, and $\lambda_{J'J}=1$. Thus $\langle\mathbf{J}\rangle=\frac{2}{3}\langle\boldsymbol{\sigma}\rangle$. Now it appears experimentally that $|\langle\boldsymbol{\sigma}\rangle|\simeq0.15$ for negative μ mesons stopped in carbon,[4] so the B^{12} nuclei would be about 10% oriented. In the subsequent beta decay of B^{12} the electrons will have an angular distribution relative to $\langle\mathbf{J}\rangle$ of the same form as that observed for Co^{60},[3] $(1+a\cos\theta)$, where the coefficient a is expected from both experiment[3] and theory[7] to be equal in magnitude to $|\langle\mathbf{J}\rangle|(v_e/c)$, i.e., $|a|\simeq0.1$, presumably an observable effect.[8]

The question now arises as to how far the actual state of affairs for μ^- mesons in carbon will depart from the idealized situation sketched above. First, the assumption of an allowed transition needs to be rationalized. With the neutrino carrying off ~100 Mev in energy, it is clear that an expansion in multipole order loses its character of an expansion in forbiddenness. For the case of no nuclear parity change, the neutrino is emitted, loosely speaking, as an s wave or a d wave, or higher. In spite of the lack of centrifugal-barrier inhibitions, Godfrey's calculations of the relevant nuclear matrix elements, based on the $j-j$ coupling model, show that d-wave emission is depressed by a factor 10^{-3} relative to s-wave emission.[5] It therefore