Λ^0-Nucleon Potential and the Binding of Hyperfragments*

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(Received March 20, 1957)

The volume integral Ω of the effective interaction potential between the Λ hyperon and the nucleon has been calculated from the observed binding energies of hyperfragments of mass number 3, 4, 5, 7, and 9. An assumption of short-range, central, spin-independent forces leads to a single value of $\Omega=(240\pm20)\times10^{-39}$ Mev cm^3, independent of experimental uncertainties in the size and shape of nuclei and in the observed binding energies, except for the case of $_\Lambda He^5$. The additional freedom gained by assuming central spin-dependent forces does not permit the fitting of $_\Lambda He^5$ and the other hypernuclei. It is concluded that the explanation of the anomalously low value of Ω for $_\Lambda He^5$ is to be found in the nuclear properties of its alpha-particle core.

I. INTRODUCTION

THE binding energies of the hyperfragments provide the most quantitative information now available concerning the interaction between the nucleon (N) and the Λ hyperon.[1] These binding energies increase smoothly with mass numbers from about 0.4 Mev for $_\Lambda H^3$ with about 1 Mev extra for each additional nucleon. Since these energies are small compared to the mass defects of the hyperfragments, it seems reasonable to suppose that the hyperfragments possess comparatively undistorted nuclear cores, as has been assumed by previous authors,[2,3] and to consider the dynamics of the Λ^0 hyperon moving in a given distribution of nuclear matter. A further simplification is obtained by assuming that the effective Λ^0-N interaction in hyperfragments is a short-range, spin-independent, central potential.

Under these assumptions, we have calculated the volume integral Ω of the effective Λ^0-N interaction which must be assumed for each hyperfragment. From the constancy of Ω we have concluded that these assumptions are in good agreement with all the observed hyperfragment data excepting only the case of $_\Lambda He^5$. While the assumption of spin independence seems rather drastic, we shall show that no central spin-dependent force is in agreement with all the data. This conclusion is insensitive to the assumption of very short range and to the experimental uncertainties of the binding energies.

The effect of distortion has been calculated in the case of $_\Lambda H^3$ and will be discussed in the concluding section, together with the anomalous case of $_\Lambda He^5$.

* A preliminary account of this work was presented at the Midwest Conference on Theoretical Physics, March 15 and 16, 1957 at Iowa City, Iowa.

† Research supported in part by the National Science Foundation.

[1] Binding energies of hyperfragments have been measured in nuclear emulsion by various investigators, especially Fry, Schneps, and Swami, Phys. Rev. **101**, 1526 (1956), and Slater, Silverstein, Levi-Setti, and Telegdi, Bull. Am. Phys. Soc. Ser. II, **1**, 319 (1956). A graph summarizing the measurements is given by L. M. Brown, Phys. Rev. **106**, 354 (1957).

[2] R. H. Dalitz, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics, Rochester, 1956* (Interscience Publishers, Inc., New York, 1956), p. V-40.

[3] R. Gatto, Nuovo cimento **1**, 372 (1955).

II. CALCULATIONS

The wave function in the relative coordinate $\mathbf{r}$ of Λ^0 and nuclear core satisfies

$$\{(-\hbar^2/2\mu)\nabla^2-\Omega(A-1)\rho(r)+|E|\}\psi(\mathbf{r})=0, \quad (1)$$

where Ω is considered to be the eigenvalue.

Exact numerical solutions have been obtained for the ground states of the hyperfragments ($_\Lambda H^4$, $_\Lambda He^4$), $_\Lambda He^5$, $_\Lambda Li^7$, $_\Lambda Be^9$ with the binding energies $|E|$ and nucleon densities $\rho(r)$ given in Table I. For $_\Lambda He^5$ and $_\Lambda Li^7$, $\rho(r)$ is obtained from the Stanford electron scattering data[4] for He^4 and Li^6. For $_\Lambda Be^9$, the electron scattering data for normal Be^9 was used. Its rms radius is in satisfactory agreement with the Coulomb energy radius for Be^8. For the charge doublet ($_\Lambda H^4$, $_\Lambda He^4$) a Gaussian shape was used, adjusted to give the observed Coulomb energy difference of H^3 and He^3. The form of the potential term in (1) corresponds to assuming for the Λ^0-N potential

$$V_{\Lambda N}=-\Omega\delta(\mathbf{r}_\Lambda-\mathbf{r}_N). \quad (2)$$

Thus Ω is the volume integral of the Λ^0-N interaction potential energy.

We have also considered in some detail the problem of the hypertriton, for which we write

$$(H-E)\psi(\mathbf{q},\mathbf{r})=0, \quad (3)$$

$$H=-(\hbar^2/m)\nabla_q^2-(\hbar^2/2\mu)\nabla_r^2+V(q) -\Omega[\delta(\mathbf{r}-\tfrac{1}{2}\mathbf{q})+\delta(\mathbf{r}+\tfrac{1}{2}\mathbf{q})]. \quad (3a)$$

Here $-E$ is the energy needed to dissociate the hypertriton into three particles, $\mathbf{q}$ is the vector from neutron to proton, $\mathbf{r}$ is the Λ^0 coordinate with respect to the deuteron center of mass, m is the proton mass, and μ is the reduced mass of the Λ^0-d system. $V(q)$ is the Yukawa potential with parameters adjusted to give the correct deuteron binding energy and effective range,[5] namely

$$V(q)=51.19(\alpha q)^{-1}\exp[-\alpha q]\text{ Mev}, \quad (4)$$

with $\alpha=0.7185\times10^{13}$ cm^{-1}. If $\psi(\mathbf{q},\mathbf{r})$ is written as a

[4] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).

[5] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

TABLE I. Data used in the calculations for $A>3$. $|E|$ is the observed binding energy of the hyperfragment; a is the electron scattering or Coulomb radius as described in the text, while a' is reduced for finite proton size. The nuclear shape $\rho(r)$ is taken from reference 4. Note that the normalization given there for the modified exponential shape ($_\Lambda Li^7$, $_\Lambda Be^9$) is incorrect.

Hyperfragment	$4\pi a^3\rho(r)$	a (10^{-13} cm)	a' (10^{-13} cm)	$\lvert E\rvert$ (Mev)
($_\Lambda H^4$, $_\Lambda He^4$)	$3(6/\pi)^{\frac{1}{2}}\exp[-\frac{3}{2}(r/a)^2]$	1.84	1.67	1.45
$_\Lambda He^5$	$3(6/\pi)^{\frac{1}{2}}\exp[-\frac{3}{2}(r/a)^2]$	1.61	1.40	2.50
$_\Lambda Li^7$	$27(8)^{-\frac{1}{2}}[1+(18)^{-\frac{1}{2}}r/a]\exp[-(18)^{\frac{1}{2}}r/a]$	2.78	2.67	5.45
$_\Lambda Be^9$	$27(8)^{-\frac{1}{2}}[1+(18)^{-\frac{1}{2}}r/a]\exp[-(18)^{\frac{1}{2}}r/a]$	3.04	2.94	6.35

product of deuteron and hyperon wave functions,

$$\psi(\mathbf{q},\mathbf{r})=\psi_d(q)\psi_\Lambda(r), \tag{5}$$

then upon multiplication on the left by $\psi_d(q)$ and integration over the space of $\mathbf{q}$, Eq. (1) is obtained. We have, however, chosen to obtain Ω for this case by a Thomas variation procedure in which ψ_d and ψ_Λ are each represented by Hulthén trial functions[6]

$$\psi_\Lambda=r^{-1}(e^{-\eta r}-e^{-cr}),$$
$$\psi_d=q^{-1}(e^{-aq}-e^{-bq}), \tag{6}$$

and the parameters are obtained by joint variation to minimize Ω. In this case we have therefore included the principal effects of distortion. For the undistorted deuteron, $a=0.2316$, $b=1.37$. The variation principle yields $c=1.69$, $\eta=0.045$, $\Omega=290\times10^{-39}$ Mev cm^3. Simultaneous variation of the four parameters yields $a=0.2763$, $b=1.52$, $c=2.04$, $\eta=0.062$, $\Omega=255\times10^{-39}$ Mev cm^3. The variation parameters are in units of 10^{13} cm^{-1}. The linear distortion of the deuteron is of the order of ten percent.‡

The results of our calculation for Ω are given in Table II. Excluding $_\Lambda He^5$, all the other values of Ω lie within about 10% of their average value which is 240 $\times10^{-39}$ Mev cm^3. The smaller radius given for each species of hyperfragment is obtained by unfolding from the charge density a Gaussian proton shape of radius 0.8×10^{-13} cm as deduced by Hofstadter *et al.*[4] At the same time it displays the insensitivity of the agreement of the model (always excepting $_\Lambda He^5$) to reasonable changes of the assumed nuclear radii. The principal effect of a finite range of the Λ^0-N force will be to *fold in* the Λ^0-N potential, so that assumption of a range having the order of magnitude of the proton size cannot affect our conclusion. In the course of our calculations it was ascertained that present experimental uncertainties in the binding energies do not sensibly affect our calculated values of Ω.

An additional piece of information can be obtained from the apparent nonexistence of the hyperdeuteron. Representing the interaction as a square well whose volume integral is $\Omega=240\times10^{-39}$ Mev cm^3, we conclude that to have no bound state the square-well radius must exceed 0.60×10^{-13} cm. If a spin dependence of the interaction were assumed, such that a favored spin orientation would give a larger value of Ω, this minimum range would be increased proportionally.

At the heavy end of the mass scale, we expect the binding energies to rise smoothly with mass number and to saturate at about 19 Mev.[7]

III. DISCUSSION

Our principal conclusion is that a central, spin-independent Λ^0-N potential having a volume integral $\Omega=(240\pm20)\times10^{-39}$ Mev cm^3 leads to agreement with the observed binding energies of the hyperfragments,

TABLE II. Calculated values of the volume integral Ω of the Λ^0-N interaction, and nuclear volume per nucleon. The smaller radius given for each hyperfragment core is the value after correction for the finite size of the proton.

Hyperfragment	Root-mean-square radius (10^{-13} cm)	Ω (10^{-39} Mev cm^3)	Volume per nucleon (10^{-39} cm^3)
$_\Lambda H^3$	(undistorted)	290	...
	(distorted)	255	...
$_\Lambda H^4$, $_\Lambda He^4$	1.84	275	8.70
	1.67	240	6.50
$_\Lambda He^5$	1.61	179	4.37
	1.40	148	2.89
$_\Lambda Li^7$	2.78	247	15.0
	2.67	232	13.3
$_\Lambda Be^9$	3.04	239	14.7
	2.94	225	13.3

[6] It is interesting to note in this connection that the trial function must be given the flexibility to peak at short distances independently of its asymptotic form. For example, the use of an exponential wave function results in a serious overestimate of Ω. The Hulthén wave functions deduced here bring the nucleon and hyperon peaks very close together in spite of their very different binding energies.

‡ *Note added in proof.*—B. W. Downs has remarked (in a private communication) that the zero range Λ^0-N force, taken literally, leads to a zero value for Ω if the deuteron core is permitted to collapse to an arbitrarily small size. This zero is, however, spurious from the physical point of view; our Ω, corresponding to the local minimum obtained by continuous variation of the parameters from the undistorted case is the one of physical interest (and rather insensitive to the range of forces). We are greatly indebted to Dr. Downs for pointing out a small error in the value of the Yukawa well depth which we had originally used. The corrections for $_\Lambda H^3$ have been incorporated in the text and lead to no changes in the conclusions. Our results for $_\Lambda H^3$ agree with those of G. H. Derrick, Nuovo cimento 4, 565 (1956) and with Downs, who is undertaking more extensive three-body calculations.

[7] This is based on the assumption of a constant Ω and an asymptotically constant nuclear density. If the Λ^0 possesses a parity-doublet structure [T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956)] the effective value of Ω will probably be somewhat larger, owing to the admixture of higher orbital states. See L. M. Brown, reference 1.

if one assumes that the alpha-particle core of $_{\Lambda}He^5$ is responsible for the anomalously low effective value of Ω for that case. It is especially important that this conclusion holds independent of variations of the experimental parameters going beyond present experimental uncertainties.

Since the difference between the effective values of Ω for $A=4$ and $A=5$ has been previously attributed to a strong spin dependence of the force,[2] it is interesting to observe that assignment of spin-dependent central potentials to explain $A=5$ and any other hyperfragment is inconsistent with at least two other hyperfragments. For example, $_{\Lambda}Be^9$ has a spin-saturated core; nevertheless, it has an effective Ω of 239×10^{-39} Mev cm^3, not 179×10^{-39} Mev cm^3. To bring it into agreement with $_{\Lambda}He^5$ would require a reduction of its radius by more than 30%. Similarly, from $_{\Lambda}He^4$ and $_{\Lambda}He^5$ we would deduce that $\Omega=460\times10^{-39}$ Mev cm^3 for a spin-favored alignment of the Λ^0 with a nucleon. This is in evident disagreement with $\Omega=255\times10^{-39}$ Mev cm^3 for $_{\Lambda}H^3$, including effects of distortion.

To explain the $_{\Lambda}He^5$ anomaly as a distortion effect, we would have to suppose that all the hyperfragment cores except the alpha particle must be reduced in radius by about 35%. In view of the small calculated distortion of $_{\Lambda}H^3$, this appears to be impossible.

The anomalously low effective Ω for $_{\Lambda}He^5$ must then find an explanation other than those mentioned above. Such an explanation could relate to the large nucleon density of He^4 (see Table II), or to its closed nucleon shells. A detailed interpretation of this anomaly may well contribute greatly to our understanding of the Λ^0-N force.

ACKNOWLEDGMENTS

We wish to thank Dr. D. G. Ravenhall for valuable suggestions and Mr. S. H. Ahn for assistance with numerical integrations.

PHYSICAL REVIEW VOLUME 107, NUMBER 1 JULY 1, 1957

Pair-Creation Cross Section of Spin One-Half Particles Possessing an Anomalous Magnetic Moment*†

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(Received March 21, 1957)

The electromagnetic pair-production cross section of spin-$\frac{1}{2}$ particles possessing an anomalous magnetic moment λ (in units of $e\hbar/2mc$) is calculated. The result is compared with the experimental measurement of the pair production of mu mesons, and it is found that $-0.4<\lambda<0.2$.

INTRODUCTION

Very recently the pair production of μ mesons by gamma rays was measured at Stanford University.[1] The experimental cross section is used in the present paper to estimate the magnetic moment of the μ meson.

Since the μ meson does not seem to possess any strong nuclear interaction, and since its spin[2] is most likely $\frac{1}{2}$, its magnetic moment is expected to be close to the value $e\hbar/2mc$, predicted by the Dirac equation. The deviation from the Dirac value is expressed in terms of a dimensionless parameter λ, such that the value for the total magnetic moment is given by $(1+\lambda)e\hbar/2mc$. Fowler[3] reviewed the various sources of experimental information concerning the μ meson, with the conclusion that $\lambda\ll1$. This result is based mainly on the high-energy bremsstrahlung processes which give rise to cosmic-ray bursts. However, the method of calculation used in Fowler's analysis has questionable validity[4] at these high energies. The pair-creation experiment mentioned above, being done near threshold, therefore provides welcome additional information on the μ meson.

The assumptions used in this calculation are the following: The μ meson is treated as a spin-$\frac{1}{2}$ particle and its anomalous magnetic moment, taken as being small but different from zero, is described phenomenologically by means of a Pauli term

$$\lambda(e\hbar/2mc)(\tfrac{1}{2}F_{\mu\nu}\gamma_\mu\gamma_\nu). \quad (1)$$

The effect of this additional interaction upon the pair-creation cross section is calculated in first Born

* Supported in part by the U. S. Air Force through the Air Force Office of Scientific Research, Air Research and Development Command.
† This work contains part of the research the account of which has been submitted by George H. Rawitscher in partial fulfillment of the requirements for the degree of Doctor of Philosophy to the Department of Physics, Stanford University.
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[1] Masek, Lazarus, and Panofsky, Phys. Rev. **103**, 374 (1956).
[2] H. A. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, 1955), Vol. II, p. 365.
[3] G. N. Fowler, Nuclear Phys. **1**, 119 and 125 (1955); see also Hirokawa, Komori, and Ogawa, Nuovo cimento **4**, 736 (1956).
[4] W. Pauli, Revs. Modern Phys. **13**, 223 (1941).