

Charged Σ Hyperons from 1001 K^- Meson Stars*

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1001 K^- -meson stars were found by area-scanning in four successive stacks of emulsion pellicles exposed to momentum-analyzed negative channels at the Berkeley Bevatron. Systematic following of the prongs showed that there emerged from these stars 319 charged π mesons, 46 hyperfragments, and 158 identified charged hyperons. There were 26 $\Sigma^+ \rightarrow p + \pi^0$ decays at rest; 20 $\Sigma^+ \rightarrow \pi^+ + n$ decays at rest; 14 $\Sigma^+ \rightarrow p + \pi^0$ decays in flight; 26 $\Sigma^\pm \rightarrow \pi^\pm + n$ decays in flight; 50 Σ^- stars at rest, and 22 Σ^- zero-prong stars with Auger electrons.

The branching ratio of the Σ^+ hyperon is found to be $R = (\Sigma^+ \rightarrow p) / (\Sigma^+ \rightarrow \pi^+) = 1.18 \pm 0.32$. The best single value for the Σ^+ lifetime, as determined from all $\Sigma^+ \rightarrow p$ decays, is $(0.96_{-0.21}^{+0.37}) \times 10^{-10}$ sec. For the Σ^- hyperons, a lifetime of

$(2.5 \pm 0.8) \times 10^{-10}$ sec is deduced. However, the lifetime obtained from the mixture of $\Sigma^\pm \rightarrow \pi^\pm + n$ decays in flight alone is $(0.32_{-0.07}^{+0.11}) \times 10^{-10}$ sec.

The angular distribution of $\theta_{\Sigma\pi}$, the angle between the decay π meson in the Σ rest system and the initial direction of motion of the Σ , was determined from 85 Σ decay events. Of these, 50 events have $|\cos\theta_{\Sigma\pi}| > 0.5$, and 53 events have $\theta_{\Sigma\pi} > 90^\circ$. Hence this sample of data suggests, but does not prove, that the spin of the Σ is greater than $\frac{1}{2}$ and that there is parity-doubling for each Σ .

Other topics are presented, including the energy distribution of the Σ hyperons, an analysis of stars produced by Σ^- hyperons, and the apparent nonvalidity of the isotopic-spin selection rule $\Delta T = \pm \frac{1}{2}$.

I. INTRODUCTION

THE nuclear captures of K^- mesons in emulsion are a fruitful source of charged Σ hyperons, whose characteristics can then be studied. The rapid development of beams of K^- mesons from high-energy accelerators has been the primary factor in making such studies practical. In this paper an analysis is presented of 1001 stars produced by K^- mesons at rest. A detailed report¹ of the first 30 of these K^- -meson stars and a brief summary² of the first 207 have already been given. Many other experimental groups have also been studying K^- -meson stars.³

Many properties of the K^- mesons and charged hyperons can be studied in this type of investigation. In this paper we shall consider primarily the following: The general characteristics of K^- stars (Sec. III); the masses of the Σ^+ and Σ^- hyperons (Sec. IV A); the branching ratio of the Σ^+ decay (Sec. IV B); the characteristics of stars produced by the capture of Σ^- hyperons (Sec. IV C); the lifetimes of the Σ hyperons (Sec. IV D); and the angular correlations in the Σ decay processes (Sec. IV E).

The experimental procedure and further discussions are presented in Secs. II and V, respectively.

II. EXPERIMENTAL PROCEDURE

Four stacks of Ilford G-5, 600-micron-thick nuclear emulsion pellicles were exposed, at different times, to momentum-analyzed K^- -meson beams from the Berkeley Bevatron. Some of the details of these exposures are given in Table I. The three exposures at 90° made use of a channel similar in design to that originally built by members of Professor Richman's group.⁴ The fourth exposure, at 0° , was made in a channel designed by members of Professor Barkas' group.⁵

The plates were area-scanned under a magnification of $100\times$ for negative K^- -meson stars. Only that region of each plate, usually a strip about 2 cm in width, was scanned where the K^- mesons were expected to stop. When a star was found, sufficient observations were made on the incoming track to establish that it was due to a K^- meson. Only stars produced by K^- -mesons at rest are included in this summary.

TABLE I. Description of K^- exposures.

Stack No.	No. of pellicles ^a	Size of pellicles	θ_{p,K^-} b	\bar{E}_{K^-} mesons (Mev) ^c	No. of K^- mesons per cm ²	K^-/π^- d	No. of K^- stars found ^e
I	66	2 in. \times 3 in.	90°	53	1.4	0.8×10^{-4}	27
II	106	2 in. \times 3 in.	90°	68	5.5	0.6×10^{-4}	177
III	119	3 in. \times 4 in.	90°	80	15	2×10^{-4}	400
IV	68	3 in. \times 4 in.	0°	94	50	3.4×10^{-4}	400

^a In stacks III and IV only about half of the pellicles were scanned.

^b θ_{p,K^-} is the angle at which the K^- mesons were produced relative to the direction of the incident 6-Bev proton beam.

^c \bar{E}_{K^-} is the average kinetic energy of the K^- -meson beam. The spread in energy in stacks I, II, and III was about 20 Mev, and in stack IV was about 8 Mev.

^d This number is the ratio of K^- -meson flux to the flux of lightly ionizing tracks in the beam direction (no attempt was made to distinguish π mesons from μ mesons and electrons among these tracks).

^e Included in the total number of 1001 K^- stars are four which were found in plates exposed to energetic mesons and protons.

⁴ Kerth, Stork, Birge, Haddock, and Whitehead, Phys. Rev. **99**, 641 (A) (1955).

⁵ Heckman, Inman, Mason, Nichols, Smith, Barkas, Dudziak, and Giles, Bull. Am. Phys. Soc. Ser. II, **1**, 386 (1956).

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¹ Fry, Schneps, Snow, and Swami, Phys. Rev. **100**, 1448 (1955). Hereafter called (I).

² Fry, Schneps, Snow, and Swami, Phys. Rev. **100**, 950 (1955).

³ See references in I for earlier work. Also see S. Goldhaber, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics, 1956* (Interscience Publishers, New York, 1956); S. C. Freden and H. K. Ticho, Phys. Rev. **99**, 1057 (1955); George, Herz, Noon, and Solntseff, Nuovo cimento **10**, 95 (1956); Haskin, Bowen, and Schein, Phys. Rev. **103**, 1512 (1956); White, Gilbert, and Violet, Bull. Phys. Soc. Ser. II, **2**, 20 (1957).

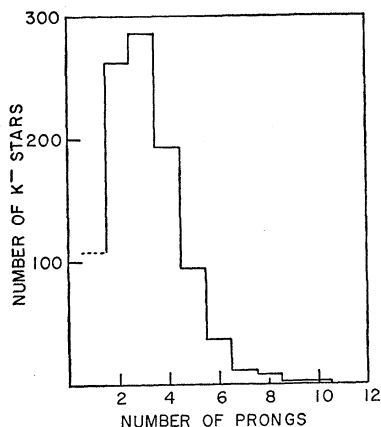


Fig. 1. Histogram of prong distribution of K^- stars (excluding zero-prong stars and stars with one lightly ionizing track only).

We believe that this method of scanning does not introduce much bias toward finding large stars in comparison to those small ones that have at least one black or gray prong. On the other hand, there is a large bias against finding zero-prong stars (K_p 's) or stars which have only one lightly ionizing prong (K_π 's). Consequently, no attempt was made to find all such stars, and those which were found were not included in the 1001 K^- stars analyzed here.

After a K^- star was found, every track was followed to the end of its range, to the point of decay in flight, or to where it left the stack,⁶ except for lightly ionizing tracks. These latter were all assumed to have been made by π mesons, and were not followed since almost all of them would leave our small stacks. The end point of each track was carefully scrutinized in order to detect the possible presence of a star, a decay particle, or an Auger electron.

TABLE II. Summary of 1001 K^- stars.

Prong No.	No. of stars	No. of stars with a π meson	No. with a Σ^+ hyperon	No. with identified Σ^- hyperon ^a	No. with $\Sigma^\pm \rightarrow \pi^\pm$ in flight ^b	No. with hyper-fragments	No. of (Σ, π) events ^c
1	108		10	8	2	1	
2	262	111	27	25	7	3	44
3	286	95	14	21	10	19	27
4	193	65	7	10	3	11	5
5	94	32	1	7	2	9	2
6	36	10	1	1	2	2	1
7	11	4				1	
8	8	1					
9	2	0					
10	1	1					
Totals:	1001	319	60	72	26	46	79

^a Identified Σ^- hyperons are those that produce a star or an Auger electron when they come to rest. Undoubtedly many Σ^- hyperons make zero-prong stars with no Auger electrons. (See Sec. IV for further discussion.)

^b The charge of the Σ 's listed in column 6 cannot be determined because the mesons from their decay always leave the stack.

^c A (Σ, π) event is a star from which both a charged Σ and a charged π emerge.

⁶ Only about 40 tracks out of 3000 left the stacks.

III. GENERAL CHARACTERISTICS OF K^- STARS

The prevailing theories⁷ of the interactions of strange particles predict that the basic interaction between a K^- meson and a nucleon (N) is

$$K^- + N \rightarrow Y + \pi, \quad (1)$$

where Y can be a Λ or Σ hyperon. In nuclear matter the π meson of reaction (1) can be virtually emitted by one nucleon and reabsorbed by an adjacent nucleon, giving rise to the reaction

$$K^- + N + N \rightarrow N + Y. \quad (2)$$

It should be also noted that when a Σ hyperon and a real meson are produced in a nucleus, either or both of them can be absorbed before emerging. This process converts a Σ hyperon into a Λ hyperon via the reaction

$$\Sigma + N \rightarrow \Lambda + N. \quad (3)$$

A further phenomenon that can take place is the capture of a Λ^0 into a nuclear fragment from the K^- star, forming a hyperfragment.

The frequency of charged π mesons, charged hyperons, hyperfragments, and K^- -star prong numbers are listed in Table II. A histogram of the prong distribution is shown in Fig. 1. Zero-prong stars (K_p 's) and one-prong stars with only a lightly ionizing particle (K_π 's) are not included in this histogram because of our experimental bias against finding them. (In the course of this scan we found 7 K_p 's and 9 K_π 's.)

The frequency with which charged π mesons emerge from K^- -meson stars (not including K_p 's and K_π 's) is found to be 32%. As pointed out in I, if one assumes that isotopic spin is a good quantum number (and $Z = \frac{1}{2}A$ for the nucleus), the number of π^0 mesons that emerge from these stars should equal one-half of the total number of charged π mesons. Hence the frequency with which π mesons in all three charge states emerge is 48%.

The number of identified charged Σ hyperons that emerge from these 1001 K^- stars is 158. The identification criteria used were the following. An event in which a singly charged particle comes to rest and gives rise to a proton of range about 1670μ is interpreted as a $\Sigma^+ \rightarrow p + \pi^0$ decay from rest. One in which a singly charged particle comes to rest and gives rise to a lightly ionizing track is interpreted as a $\Sigma^+ \rightarrow \pi^+ + n$ decay from rest. Since the π meson leaves the stack we cannot determine its sign, but we interpret all of these events as $\Sigma^+ \rightarrow \pi^+$ decays because a Σ^- hyperon would almost certainly cascade down through atomic orbits and be captured by the nucleus, via the reaction of Eq. (3), in a time much shorter than its lifetime. This argument does not help to distinguish the sign of the charge in

⁷ M. Gell-Mann, Phys. Rev. **92**, 833 (1953); K. Nishijima, Progr. Theoret. Phys. Japan **13**, 285 (1955); M. Gell-Mann and A. Pais, *Proceedings of the Glasgow Conference* (Pergamon Press, London, 1955); R. G. Sachs, Phys. Rev. **99**, 1573 (1955); M. Goldhaber, Phys. Rev. **101**, 433 (1956).

decay events in flight into π mesons. Such events could be either $\Sigma^+ \rightarrow \pi^+$ or $\Sigma^- \rightarrow \pi^-$ decays. Events in which a Σ^+ decays into a proton in flight must be carefully distinguished from proton scattering events. If the velocity of the decay proton is found to be greater than the velocity of the Σ^+ at the point of decay, there is no ambiguity with a proton scattering. On the other hand, when the proton is slower than the Σ^+ in the laboratory system one cannot automatically rule out a proton scattering. In all such cases the Q value for the event was calculated under the assumption that it was a $\Sigma^+ \rightarrow p$ decay. If this Q value agreed with the established value ($Q=116$ Mev), we called the event a $\Sigma^+ \rightarrow p$ decay in flight. Out of about 20 such cases, 6 had the correct Q value. There were 8 other cases in which the proton velocity was greater than the Σ^+ velocity. A Σ^- hyperon that comes to rest and makes a star of one or more prongs is readily identifiable except for those of very short range ($\sim 40\mu$) which cannot often be distinguished from nonmesonic decays of hyper-

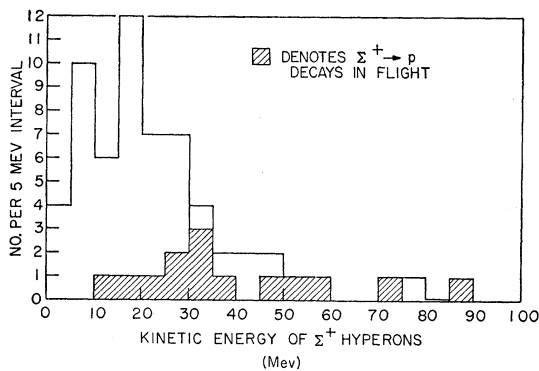


FIG. 2. Energy distribution of identified Σ^+ hyperons.

fragments. We estimate that this ambiguity leads to an uncertainty of $\lesssim 15\%$ in the number of Σ^- stars. In addition, Σ^- hyperons which do not make visible stars at the end of their range can be identified if they produce a visible Auger electron. Out of the 72 identified Σ^- hyperons that came to rest, 22 had one or more Auger electrons but no nuclear prongs.

One of the interesting characteristics of K^- stars is the ratio of Σ^- to Σ^+ hyperons produced. To determine this ratio we must estimate the number of zero-prong Σ^- stars which are not accompanied by Auger electrons. In Sec. IV C this problem is discussed in some detail, and we conclude that the total number of Σ^- hyperons that come to rest is about 115. The best guess for the breakdown of the 26 $\Sigma^\pm \rightarrow \pi^\pm$ decay events in flight is 12 $\Sigma^+ \rightarrow \pi^+$ and 14 $\Sigma^- \rightarrow \pi^-$. This is obtained by applying the branching ratio of the Σ^+ decay (see Sec. IV B). Hence the ratio of Σ^- to Σ^+ hyperons that emerge from these K^- stars is $129/72 = 1.8 \pm 0.4$.

The best estimate for the total number of charged

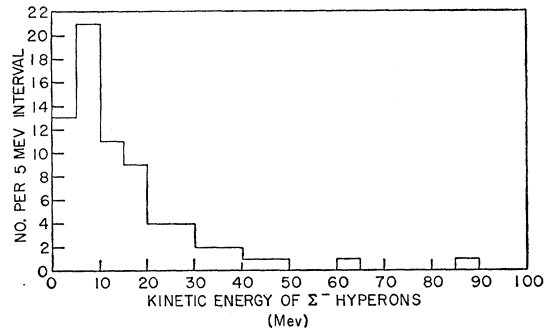


FIG. 3. Energy distribution of identified Σ^- hyperons.

hyperons emerging from these 1001 K^- stars is 201. Again under the assumption that isotopic spin is a good quantum number for strong interactions,⁸ the number of Σ^0 hyperons should equal one-half of the total number of charged Σ 's, i.e., 101 (see I). Hence the total number of Σ hyperons that emerge is about 302.

It is interesting to note that out of 158 stars with identified Σ 's, 79 stars (50%) also had a charged π meson. These (Σ, π) events are a direct reflection of the basic absorption mechanism of a K^- meson by a nucleon, as given in Eq. (1), which probably has occurred near the surface of the nucleus. The prevalence of this reaction is also exhibited by the shape of the energy distribution of the charged Σ 's. Figures 2, 3, and 4 illustrate the energy distributions of the identified Σ^+ hyperons, the identified Σ^- hyperons and the Σ^\pm hyperons which decay in flight into π^\pm . The kinetic energy of a Σ^+ or Σ^- hyperon from the capture of a K^- meson by a nucleon at rest is about 14 Mev. It is expected that the Fermi distribution of momenta for the nucleons inside nuclear matter will smear out the kinetic energy of the Σ hyperon over a region from 0 to 60 or 70 Mev, in a manner qualitatively similar to the observed energy distributions. The few fairly energetic Σ hyperons observed are probably due to the two-nucleon absorption reaction of the K^- meson [Eq. (2)]. A comparison of Figs. 2 and 3 indicates a much larger proportion of low-energy Σ^- hyperons than Σ^+ hyperons. This can be simply explained in terms of the classical nuclear Coulomb-barrier effect.

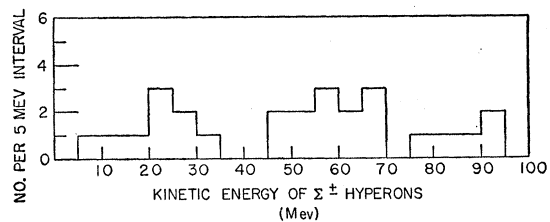


FIG. 4. Energy distribution of Σ^\pm hyperons that decay in flight into π^\pm mesons.

⁸ The Coulomb force can, of course, alter the predictions based on isotopic-spin invariance.

TABLE III. Characteristics of 26 $\Sigma^+ \rightarrow p + \pi^0$ decays from rest. R is the range of the Σ^+ hyperon; E_Σ , the kinetic energy of the Σ at the K^- star; T , the moderation time of the Σ ; and $\theta_{\Sigma\pi}$, the angle in the rest system of the Σ , between the direction of the π meson and the initial direction of the Σ .

Event No.	R_Σ (μ)	E_Σ (MeV)	T (10^{-10} sec)	$\theta_{\Sigma\pi}$ (degrees)	$\cos\theta_{\Sigma\pi}$
K 3	227	6.52	0.106	40°	+0.77
41	5700	42.4	1.00	151	-0.87
89	412	9.38	0.160	63	+0.45
113	1130	16.9	0.320	104	-0.24
168	1670	21.0	0.402	68	+0.37
211	290	7.59	0.125	106	-0.27
413	1159	17.13	0.331	128	-0.62
440	615	11.9	0.210	88	+0.03
512	1045	16.1	0.300	90	0.0
607	208	6.19	0.099	34	+0.83
631	2500	26.5	0.560	78	+0.21
646	580	11.5	0.202	137	-0.73
703	1050	16.2	0.304	61	+0.49
753	2263	25.0	0.522	95	-0.09
762	786	13.75	0.248	146	-0.83
774	1640	20.85	0.416	80	-0.17
788	1050	16.2	0.304	38	+0.79
812	3050	29.7	0.644	151	-0.87
815	2235	24.9	0.515	117	-0.45
826	118	4.36	0.068	143	-0.80
845	203	6.05	0.097	139	-0.75
846	7300	48.7	1.20	130	-0.64
973	193	5.91	0.095	89	+0.02
979	2030	23.6	0.483	107	-0.29
991	273	7.3	0.120	84	+0.10
1023	3070	29.8	0.646	44	+0.72

The bulk of the 46 hyperfragments observed have a very short range ($<10\mu$) and decay nonmesonically. A detailed report of these hyperfragments will appear in a separate publication.⁹

IV. PROPERTIES OF Σ HYPERONS

In Tables III through VIII we list the essential characteristics of all the identified charged Σ hyperons.

TABLE IV. Characteristics of 20 $\Sigma^+ \rightarrow \pi^+ + n$ decays from rest. See Table III for explanation of symbols.

Event No.	R_Σ (μ)	E_Σ (MeV)	T (10^{-10} sec)	$\theta_{\Sigma\pi}$ (degrees)	$\cos\theta_{\Sigma\pi}$
15	280	7.4	0.122	106	-0.27
101 ^a	12 023	80	1.30
105	114	4.3	0.066	55	+0.43
109	4100	35.1	0.794	144	-0.81
189	95	3.8	0.058	130	-0.64
215	1448	19.5	0.379	133	-0.68
524	2740	27.9	0.594	70	+0.34
554	1250	17.9	0.340	139	-0.75
748	5300	40.6	0.952	75	+0.26
790	410	9.4	0.159	48	+0.67
791	29	1.8	0.026	136	-0.72
797	1980	23.2	0.471	151	-0.87
808	1125	16.8	0.317	106	-0.28
820	1039	16.1	0.302	123	-0.54
842	1075	16.4	0.309	128	-0.62
874	615	11.9	0.210	128	-0.62
950	3820	33.7	0.755	28	+0.89
993	1290	18.2	0.352	126	-0.59
1008	2400	25.9	0.544	34	+0.83
1026	227	6.5	0.105	110	-0.34

^a In this event the Σ^+ interacted in flight after traversing 1.2 cm [see Fry, Schneps, Snow, and Swami, Phys. Rev. **100**, 939 (1955)].

⁹ Schneps, Fry, and Swami, Phys. Rev. **106**, 1062 (1957).

The tables list the event number, the range, the kinetic energy, the moderation time T , the time t to the point of decay for all Σ 's that decay in flight, the angle $\theta_{\Sigma\pi}$ in the Σ rest system between the direction of the π meson and the initial direction of the Σ hyperon, and finally the cosine of $\theta_{\Sigma\pi}$. Various aspects of the data in these tables are discussed in the following subsections.

A. $\Sigma^- - \Sigma^+$ Mass Difference and the Mass of the K^- Meson

In the course of this scan, two events (No. 788 and 818) were found which are interpreted as the capture of K^- mesons at rest in hydrogen. These two events appear to be examples of the reactions

$$K^- + p \rightarrow \Sigma^+ + \pi^- + Q_1, \quad (4)$$

and

$$K^- + p \rightarrow \Sigma^- + \pi^+ + Q_2, \quad (5)$$

TABLE V. Characteristics of 14 $\Sigma^+ \rightarrow p + \pi^0$ decays in flight. The quantity t is the time spent by the Σ in traversing the distance from the star to the point of decay. See Table III for explanation of the other symbols.

Event No.	R_{Σ^+} (μ)	E_Σ (MeV)	t (10^{-10} sec)	T (10^{-10} sec)	$\theta_{\Sigma\pi}$ (degrees)	$\cos\theta_{\Sigma\pi}$
7	4200	58	0.48	1.49	104°	-0.24
39	3800	52.5	0.44	1.30	0	+1.0
48	1400	28	0.23	0.60	45	+0.71
172	900	36.9	0.15	0.84	62	+0.47
315	2650	30.5	0.19	0.67	153	-0.89
372	3500	32.3	0.69	0.72	115	-0.42
378	138	28.8	0.021	0.62	51	+0.64
452	330	20.6	0.061	0.41	35	+0.82
465	127	11.6	0.032	0.20	96	-0.10
640	1480	45.2	0.18	1.09	162	-0.95
681	2080	90.2	0.18	2.57	82	+0.17
707	3830	34.5	0.70	0.78	84	+0.10
714	3730	70.9	0.37	1.90	121	-0.52
800	1230	18.0	0.31	0.35	135	-0.70

respectively. The details of these two events have already been published.¹⁰ Taking the mass of the Σ^+ hyperon to be $2327.4 \pm 1.0 m_e$,¹¹ one deduces from event No. 788 that the mass of the K^- meson is $(966.7 \pm 2) m_e$. A comparison of the ranges of the Σ^+ and Σ^- hyperons from these two events yields for the mass difference $m_{\Sigma^-} - m_{\Sigma^+} = (15.9 \pm 2.9) m_e$. This mass difference is in excellent agreement with the value $(14 \pm 6) m_e$ given by Chupp *et al.*¹² and the value of $(16 \pm 5.4) m_e$ given by Budde *et al.*¹³

¹⁰ Fry, Schneps, Snow, Swami, and Wold, Phys. Rev. **104**, 270 (1956).

¹¹ Fry, Schneps, Snow, and Swami, Phys. Rev. **103**, 226 (1956).

¹² Chupp, Goldhaber, Goldhaber, and Webb, Nuovo cimento Suppl. **2**, 382 (1956). See also S. Goldhaber, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics, 1956* (Interscience Publishers, New York, 1956).

¹³ Budde, Chretien, Leitner, Samois, Schwartz, and Steinberger, Phys. Rev. **103**, 1827 (1956).

B. The Branching Ratio for the Σ^+ Decay

Several authors¹⁴ have indicated the importance of obtaining an unbiased estimate of the branching ratio between the two modes of decay of the Σ^+ ; namely

$$\Sigma^+ \rightarrow p + \pi^0, \quad (6)$$

and

$$\Sigma^+ \rightarrow n + \pi^+. \quad (7)$$

An estimate of this ratio is given by the ratio of the number of $\Sigma^+ \rightarrow p$ decays from rest to the number of $\Sigma^+ \rightarrow \pi^+$ decays from rest. Since each track from the K^- stars was followed to the end of its range, where it was carefully examined by two observers, no bias is

TABLE VI. Characteristics of 26 $\Sigma^+ \rightarrow \pi^+ + n$ decays in flight. See Tables III and V for explanation of the symbols.

Event No.	R_{Σ} (μ)	E_{Σ} (Mev) ^a	t (10^{-10} sec)	T (10^{-10} sec)	$\theta_{\Sigma\pi}$ (degrees)	$\cos\theta_{\Sigma\pi}$
66	4240	68	0.44	1.80	112	-0.37
85	3350	57	0.37	1.44	68	+0.37
170	2665	63	0.28	1.65	45	+0.71
186	51	10	0.013	0.172	102	-0.21
204	955	35	0.143	0.794	48	+0.67
222	68	21	0.012	0.40	97	-0.12
251	4900	76	0.48	2.08	135	-0.71
256	9700	95	0.89	2.70	138	-0.74
300	49	20	0.009	0.40	127	-0.60
414	2320	69	0.23	1.83	128	-0.62
432	3830	90	0.33	2.55	148	-0.85
449	410	51	0.047	1.27	8	+0.97
471	1700	27	0.30	0.580	134	-0.70
496	293	13	0.071	0.243	112	-0.37
536	1270	28	0.21	0.600	109	-0.33
548	173	16	0.032	0.305	153	-0.89
560	1700	56	0.18	1.42	25	+0.90
585	231	65	0.024	1.70	121	-0.52
604	455	52	0.050	1.28	144	-0.81
634	87	49	0.010	1.21	117	-0.45
667	1600	47	0.097	1.14	47	+0.68
731	4370	94	0.38	2.69	34	+0.83
889	2250	65	0.23	1.72	109	-0.32
928	482	23	0.084	0.47	138	-0.74
944	11 900	84	1.17	2.35	153	-0.89
1024	3215	56	0.65	1.73	94	-0.07

^a The kinetic energy of the Σ was estimated from the ionization of the track. The percentage errors varied from track to track being least for long gray tracks and most for short black tracks. Typical errors range from ± 5 Mev to ± 10 Mev.

introduced into the branching ratio other than that due to the slightly smaller efficiency for detecting the lightly ionizing π^+ meson as compared to the heavily ionizing proton. This inefficiency is estimated to be about 10%. We find for the ratio $R = (\Sigma^+ \rightarrow p) / (\Sigma^+ \rightarrow \pi^+)$ the value 26/20. When we correct for the 10% inefficiency, the best estimate for R is $26/22 = 1.18 \pm 0.32$. Since the theoretical significance of this ratio is closely linked with the ratio of Σ^- to Σ^+ lifetimes, further discussion is postponed until Sec. V.

¹⁴ R. Gatto, Nuovo cimento 3, 318 (1956); G. Wentzel, Phys. Rev. 101, 505 (1956); G. Takeda, Phys. Rev. 101, 1547 (1956); C. Iso and M. Kawaguchi, Progr. Theoret. Phys. Japan 16, 177 (1956); W. G. Holladay, Bull. Am. Phys. Soc. Ser. II, 1, 51 (1956); M. Kawaguchi and K. Nishijima, Progr. Theoret. Phys. Japan 15, 180, 182 (1956); D'Espagnat and J. Prentki, Nuovo cimento 3, 1045 (1956).

TABLE VII. Characteristics of 50 Σ^- hyperons that make stars with one or more prongs.

Event No.	R_{Σ^-} (μ)	E_{Σ^-} (Mev)	T (10^{-10} sec)	Event No.	R_{Σ^-} (μ)	E_{Σ^-} (Mev)	T (10^{-10} sec)
13	790	13.6	0.25	509	626	12.06	0.212
45	5700	42.3	1.0	547	2459	26.3	0.551
46	280	7.42	0.12	555	2387	25.8	0.54
56	4300	36	0.82	557	11 000	61.6	1.596
63	910	14.9	0.29	579	1200	17.5	0.334
73	880	14.6	0.265	580	1400	19.1	0.372
84	30	1.83	0.026	623	1136	17.0	0.324
116	7650	50	1.22	624	238	6.7	0.109
121	4050	34.9	0.78	627	388	9.13	0.153
123	290	7.6	0.125	653	507	10.6	0.184
174	1800	22	0.44	663	301	7.76	0.134
184	19	1.36	0.02	679	108	4.13	0.064
220	19	1.36	0.02	716	207	6.20	0.100
234	2800	28.3	0.603	755	3900	34.1	0.766
279	170	5.5	0.087	781	180	5.65	0.090
293	1110	16.6	0.31	799	4.7	0.47	0.006
303	284	7.49	0.122	828	837	14.3	0.260
346	582	11.50	0.202	849	5000	39.3	0.91
352	405	9.27	0.158	852	2500	26.5	0.560
357	50	2.55	0.037	853	320	8.05	0.134
396	34	2.0	0.029	856	350	8.5	0.143
466	390	9.20	0.157	861	48	2.48	0.036
482	1180	17.3	0.330	879	90	3.68	0.056
489	79	3.39	0.047	947	260	7.08	0.116
503	82	3.47	0.052	952	380	8.90	0.151

C. Characteristics of Σ^- Endings

When a Σ^- comes to rest in nuclear emulsion, it is first captured in an outer atomic orbit from which it cascades down to lower orbits until it is absorbed by the nucleus. The cascade process can give rise to Auger electrons and the nuclear capture to a star. We have observed 50 Σ^- stars of one or more prongs, 15 of which were accompanied by Auger electrons, and 22 stoppings where the Σ^- produced one or more Auger electrons but no visible stars. The probability of a random coincidence of a background electron with a proton ending is negligible in this stack. The ranges, energies and moderation times of these Σ^- hyperons are listed in Tables VII and VIII. The detailed characteristics of these 50 Σ^- stars are listed in Table IX.

As can be seen in Tables VII and VIII, many of the Σ^- have very short ranges which preclude the possi-

TABLE VIII. Characteristics of 22 Σ^- hyperons that have Auger electrons at their endings with zero nuclear prongs.

Event No.	R_{Σ^-} (μ)	E_{Σ^-} (Mev)	T (10^{-10} sec)	Event No.	R_{Σ^-} (μ)	E_{Σ^-} (Mev)	T (10^{-10} sec)
128	21 100	89	2.5	561	1796	22.0	0.44
135	256	7.1	0.115	564	483	10.3	0.18
202	1410	19.1	0.37	678	516	10.7	0.19
238	1520	20.0	0.39	697	448	9.8	0.17
301	465	10.1	0.17	719	182	5.7	0.09
402	2250	25.0	0.52	809	1824	22.1	0.045
497	1296	18.3	0.35	818	670	12.5	0.22
521	408	9.3	0.16	872	148	5.05	0.08
526	56.6	2.7	0.04	948	1346	18.7	0.36
530	198	6.05	0.10	976	731	13.1	0.23
552	911	14.9	0.28	982	235	6.7	0.11

TABLE IX. Characteristics of 50 Σ^- stars.

Event No.	Track No.	Range of track (μ)	Probable identity	Energy ^a (Mev)	Reliability ^b	Auger electrons
13	1	35	α	7.0	G	
45	1	5.4	recoil	1.6	G	
	2	9	α or heavier	2.7		
46	1	13	α or heavier	3.6	F	
56	1	8	α or heavier	2.4	F	
63	1	10	α or heavier	3.0	F	
73	1	250	$p, d, \text{ or } t$	6.3	G	15 kev
84	1				G	
116	1	500	$p, d, \text{ or } t$	9.5	G	
121	1	1708	$p, d, \text{ or } t$	19.2	G	18 kev
123	1	8	recoil	2.4	F	
174	1	2000	p	21.0	G	
184	1	200	$p, d, \text{ or } t$	5.5	G	
220	1	80	α or heavier	13	F	
234	1	4	α or heavier	1.2	G	
	2	8	α or heavier	2.4		
	3	550	$p, d, \text{ or } t$	10.1		
279	1	14	recoil	3.8	G	
293	1	short	recoil		F	
303	1	156	α	19	G	
	2	2400	$d \text{ or } t$	31.4		
	3	1200	p	15.7		
	4	1	recoil	0.3		
346	1	2123	p	21.8	G	
352	1	80	α	13	G	18 kev
357	1	3	α or heavier	0.8	F	
	2	8	α or heavier	2.4		
396	1	450	$p \text{ or } d$	8.9	G	22 kev, 33 kev
466	1	15	α or heavier	4.0	G	el. blob
482	1	12	?	3.5	F	
489	1	290	$p \text{ or } d$	6.9	G	
503	1	45	α	8.5	G	25, 25 kev
	2	120	p	4.0		
	3	370	$p, d, \text{ or } t$	8.0		
509	1	450	$p, d, \text{ or } t$	9.0	G	50 kev
547	1	14	recoil	4.0	G	16, 33 kev
557	1		p	30 Mev	G	
579	1	50	α	9.0	G	
	2	60	α	10.5		
	3	120	α	16		

bility of establishing the identity of the particle forming the track in question from measurements on that track alone. The principal sources of confusion are proton interactions in flight, either elastic or inelastic, and hyperfragment events. To distinguish these various possibilities for short-range tracks, an attempt was made to answer such questions as: Did the particle come to rest; was there an Auger electron; did the particle have charge one; was the energy released in the reaction larger than the incident kinetic energy of the connecting particle, etc. On this basis an over-all reliability estimate was made as to the probability that a given event listed in Table IX represented a Σ^- star. We estimate that 42 events have good reliability and 8 have fair reliability. A similar proportion of good and fair reliability prevails for the 22 Σ^- zero-prong events with Auger electrons.

All of the Σ^- stars observed here have a visible energy that is less than the Q (81.4 ± 2 Mev) of the reaction

$$\Sigma^- + p \rightarrow \Lambda^0 + n + 81.4 \text{ Mev.} \quad (8)$$

One might expect that after the capture of a Σ^- by a nucleus, a Λ^0 hyperfragment might be formed. However,

in no case was a hyperfragment seen to emerge from a Σ^- star. Of course, some of the stars may be a result of the nonmesonic decay of a Λ^0 that is trapped in the same nucleus that captured the Σ^- . The prong distribution of the visible Σ^- stars is 31 one-prong, 13 two-prong, 5 three-prong, and 1 four-prong.

These observations are consistent with the hypothesis that reaction (8) plays a dominant role in the Σ^- capture process. The charge-exchange reaction,

$$\Sigma^- + p \rightarrow \Sigma^0 + n, \quad (9)$$

can also take place without introducing any contradiction to these observations. If the Σ^0 were to interact with a nucleon before leaving the nucleus, similar stars would be produced to those made by reaction (8). On the other hand, if the Σ^0 emerges from the nucleus, only a zero-prong star could be made. Zero-prong stars are also very likely to result from reaction (8) (e.g., see discussion in I).

Since a substantial fraction of the Σ^- captures do not produce any visible stars, a theoretical correction must be made in order to determine the total number of Σ^- hyperons that emerged from the 1001 K^- stars. We

TABLE IX.—(Continued).

Event No.	Track No.	Range of track (μ)	Probable identity	Energy ^a (Mev)	Reliability ^b	Auger electrons
580	1	70	α	11.5	G	
623	1	10	recoil	3.0	G	15 kev
624	1	6	recoil	1.8	G	
	2	330	$p, d,$ or t	7.5		
627	1	185	p	5.3	G	
	2	3	Li^8	1.5		
	3	13	recoil	3.7		
653	1	6.5	recoil	2.0	G	
	2	25	α	6.0		
663	1	10	α or heavier	3.0	G	18 kev
	2	13	α or heavier	3.7		
679	1	85		13	G	
	2	short	recoil or electron			
716	1	6	α or recoil	1.8	G	22 kev
755	1	3	recoil	0.8	G	
	2	780	$p, d,$ or t	12.3		
781	1	110	$p, d,$ or t	3.8	G	12, 25 kev
799	1		p	40	G	17 kev
828	1	47	α or heavier	8.8	G	
	2	8	α or heavier	2.4		
849	1	10 000	p	50	G	
852	1	2000	$p, d,$ or t	21	G	30 kev
	2	5000	$p, d,$ or t	35		
853	1	5	recoil	1.5	G	
	2	12	p or α	0.9 or 3.5		
856	1	5	α	1.5	G	
	2	5	α	1.5		
	3	150	$p, d,$ or t	4.6		
861	1	30	$p, d,$ or t	1.7	G	22 kev
879	1	5	recoil	1.5	G	
	2	5	recoil	1.5		
947	1	130	$p, d,$ or t	4.2	G	
	2	28	α or heavier	6.4		
952	1	4	recoil	1.2	G	
	2	120	$p, d,$ or t	4.0		

^a The energies listed in this column are the energies corresponding to the minimum charge consistent with our observations on each track. Hence the range-energy relationship for protons was used on tracks identified as $p, d,$ or $t,$ and the range-energy relationship for α particles was used for all unidentified recoil tracks.

^b F and G denote fair and good reliability of identification of the listed event as a Σ^- star. Those that are marked fair cannot be conclusively distinguished from such events as a proton scattering near the end of its range or a nonmesonic decay of a hyperfragment.

observed 50 Σ^- stars with visible prongs and 22 Σ^- zero-prong stars with Auger electrons. To make this correction an estimate is needed of the probability that a Σ^- emit an Auger electron in the atomic cascade process. Experimentally, the probability of Auger emission from those Σ^- that make stars is $15/50=0.30\pm 0.09$. Hence an estimate of the total number of zero-prong Σ^- stars is $(22/0.30)=73\pm 26$, where the error given is purely statistical. The total number of Σ^- that come to rest would then be 123 ± 27 . For this estimate of the total number of Σ^- zero-prong events it was assumed that there is no correlation between the probability of a Σ^- making an Auger electron and making a visible star. Such an assumption is probably not valid. The Auger-electron emission probability is much higher in the heavy elements (Ag,Br) than in the light elements (C,N,O). On the other hand, the probability that a Σ^- capture produces a visible star may be higher in the light elements than in the heavy elements. The reduced Coulomb barrier in the light elements allows lower energy charged particles to emerge. Furthermore, the recoil nucleus can be visible only in the light elements. On the other hand, the heavier elements provide a

longer mean free path for the neutral particles produced in reaction (8) to transform into visible charged particles. It is difficult to decide *a priori* as to which of these competing arguments is more important.

Another estimate of the number of Σ^- zero-prong stars, which attempts to take into account some of the differences between Σ^- capture in light and heavy elements, can be made as follows. Morinaga and Fry¹⁵ have found that the probabilities for the capture of μ^- mesons in heavy and light elements is 60% and 40% respectively. We shall assume that the same capture probabilities hold for Σ^- hyperons. Fry¹⁶ and Cosyns *et al.*¹⁷ have investigated the Auger emission probability for μ^- mesons. When one uses the above capture probabilities, their data indicate that the probability for emission of Auger electrons in heavy elements is $\sim 35\%$. It is assumed that the Auger emission proba-

¹⁵ H. Morinaga and W. F. Fry, Nuovo cimento **10**, 308 (1953).

¹⁶ W. F. Fry, Phys. Rev. **83**, 594 (1951).

¹⁷ Cosyns, Dilworth, Occhialini, Schoenberg, and Page, Proc. Phys. Soc. (London) **62**, 801 (1949). These authors deduce that the Auger probability for μ^- mesons in heavy elements is $> 23\%$ (when corrected for the 60-40 ratio of capture in heavy and light elements).

bility in light elements is negligible. Theoretically¹⁸ one expects an increase in Auger probability with increasing mass of the negative particle.

Assuming that the Auger-electron emission probability for Σ^- capture in heavy elements is 50%, and recalling that 15 of the 50 Σ^- stars had Auger electrons (and therefore were definitely captures in heavy elements), we deduce that 30 of the 50 stars took place in heavy elements. The 22 zero-prong Σ^- events with Auger electrons imply 44 zero-prong Σ^- captures in heavy elements. This yields $44+30=74$ for the total number of Σ^- captures in heavy elements. Now invoking the 40/60 ratio found by Morinaga and Fry, we deduce that the total number of Σ^- captures in light elements must be 50, and that the total number of Σ^- stoppings is 124. This number is in quite good agreement with our earlier estimate of 123 ± 27 . Had we assumed that the Σ^- Auger emission probability in heavy elements was 60%, we would have obtained for the total number of Σ^- stoppings 104. On the other hand, if we assume that the Σ^- Auger emission probability is the same as the μ^- , namely 35%, we obtain 177 for the total number of Σ^- stoppings. However, this would also imply that only 7 of the 50 Σ^- stars were in light elements, which seems inconsistent with the observed number of Σ^- stars with two and three low-energy prongs. The assumptions of 50% and 60% would imply that 20 and 25 of the 50 Σ^- stars were in light elements. These two numbers are consistent with the prong distribution listed in Table IX.

It is clear from the above discussion that one cannot determine the number of Σ^- stoppings very precisely. A reasonable estimate of this number is 115 ± 25 .¹⁹

D. Lifetimes of the Σ^\pm Hyperons

A determination of the lifetimes of the Σ^+ and Σ^- hyperons could allow a decisive test of one of the theories set forth by Lee and Yang,²⁰ that is, the theory that postulates parity doublets for all hyperons with odd strangeness. If the Σ^+ and Σ^- hyperons each exhibited a time distribution of decay points that was not consistent with a single exponential (i.e., a single lifetime), but which was consistent with a linear combination of two or three exponential terms, we would have an indirect confirmation of the hypothesis of parity-doubling. If only one lifetime is found for each charged hyperon, then either each charged hyperon has a unique parity with a unique lifetime, or if there are two parities for each charge, the two parity states

have equal or nearly equal lifetimes. The last alternative would be similar to the apparent situation that prevails for the θ^+ and τ^+ mesons and might be understood in terms of a mechanism for slow decays involving a weak interaction that does not conserve parity.²¹ It is important to note that the observation of different lifetimes for the Σ^+ and Σ^- hyperons, which is *a priori* probable on theoretical grounds,¹⁴ has no bearing on the above discussion about parity-doubling.

It is clear that an analysis of the time distribution of decays in flight and at rest, listed in Tables III to VIII, can yield some information about the lifetimes of the Σ^+ and Σ^- hyperons. Limited statistics are the major obstacle to a definite answer to the questions raised in the previous paragraph. A further difficulty arises from the fact that for the 26 decays in flight into π mesons listed in Table VI, one does not know the sign of the charge of the decaying hyperon. Hence an average lifetime deduced from these 26 events alone must be a composite lifetime of Σ^+ and Σ^- , as well as of two parity states for each charge if these exist.

Because of this last-mentioned difficulty, let us first consider the decay events $\Sigma^+ \rightarrow p$ in flight and at rest listed in Tables V and III. We have used the method of maximum likelihood as described by Bartlett²² to determine the best single lifetime that fits our data. The maximum likelihood estimate for $\tau_{\Sigma^+ \rightarrow p}$ is given by

$$\bar{\tau}_{\Sigma^+ \rightarrow p} = \frac{1}{14} \left[\sum_{i=1}^{14} t_i + \sum_{j=1}^{26} T_j \right], \quad (10)$$

where t_i denotes the time to the point of decay for each of the 14 $\Sigma^+ \rightarrow p$ decays in flight listed in Table V, and T_j denotes the moderation time for each of the 26 Σ^+ hyperons that decay into a proton from rest, listed in Table III. We find²³

$$\bar{\tau}_{\Sigma^+ \rightarrow p} = (0.96_{-0.21}^{+0.37}) \times 10^{-10} \text{ sec.} \quad (11)$$

In order to estimate the best single lifetime for the Σ^- hyperon, we must estimate what fraction of the 26 $\Sigma^\pm \rightarrow \pi^\pm$ decay events are $\Sigma^- \rightarrow \pi^-$ events. Using the branching ratio $(\Sigma^+ \rightarrow p)/(\Sigma^+ \rightarrow \pi^+) = 1.18$ and the fact that there are 14 $\Sigma^+ \rightarrow p$ decay events in flight, we deduce that about $(14/1.18) = 12$ of the 26 $\Sigma^\pm \rightarrow \pi^\pm$ events are $\Sigma^+ \rightarrow \pi^+$ decays. Hence there are about 14 $\Sigma^- \rightarrow \pi^-$ decay events in flight. The average moderation time, T_j , for the observed Σ^- that come to rest, listed in Tables VII and VIII, is 0.268×10^{-10} sec. Using our estimate of 115 for the total number of Σ^- stoppings, we get

$$\sum_{j=1}^{115} T_j = 115(0.268) = 30.82 \times 10^{-10} \text{ sec.}$$

¹⁸ E. H. S. Burhop, *The Auger Effect* (Cambridge University Press, New York, 1952), Chap. 7. Burhop estimates that the Auger probability should increase by about 50% in going from a μ^- to a K^- meson.

¹⁹ The number 115 is obtained from an approximate average of the results derived from the assumptions of 50% and 60% for the Σ^- Auger emission probability.

²⁰ T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956), and **104**, 822 (1956).

²¹ T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

²² M. Bartlett, Phil. Mag. **44**, 249 (1953).

²³ The errors indicated here and in the subsequent discussion of lifetimes denote one standard deviation.

The sum of t_i for the 26 $\Sigma^\pm \rightarrow \pi^\pm$ decays is

$$\sum_{i=1}^{26} t_i = 6.74 \times 10^{-10} \text{ sec.}$$

Hence the best estimate of τ_{Σ^-} is²⁴

$$\begin{aligned} \bar{\tau}_{\Sigma^-} &= \frac{1}{14} \left\{ \left(\frac{14}{26} \right) 6.74 + 30.82 \right\} \\ &= (2.5 \pm 0.8) \times 10^{-10} \text{ sec.} \end{aligned} \quad (12)$$

The uncertainty in this value of $\bar{\tau}_{\Sigma^-}$ comes from the error in the Σ^+ branching ratio, the number of $\Sigma^\pm \rightarrow \pi^\pm$ decays observed, and the estimated number of Σ^- stoppings.

If we apply the same method to the $\Sigma^+ \rightarrow \pi^+$ decays, assuming that 12 of the 26 $\Sigma^\pm \rightarrow \pi^\pm$ decays in flight are $\Sigma^+ \rightarrow \pi^+$, we get

$$\begin{aligned} \bar{\tau}_{\Sigma^+ \rightarrow \pi^+} &= \frac{1}{12} \left\{ \frac{12}{26} (6.74) + 8.16 \right\} \\ &= (0.94 \pm 0.35) \times 10^{-10} \text{ sec.} \end{aligned} \quad (13)$$

This somewhat indirect result for the lifetime of the Σ^+ deduced from $\Sigma^+ \rightarrow \pi^+$ decays agrees very well with the value of $\bar{\tau}_{\Sigma^+ \rightarrow p}$ given above.²⁵

It is possible to obtain an independent estimate of lifetime from the time distribution of t_i for the decays in flight alone. Bartlett²² has shown that the maximum likelihood estimate for τ from n decays in flight alone is given by the solution of the equation

$$f(\tau) = \sum_{i=1}^n \left[\frac{t_i}{\tau} - 1 + \frac{T_i}{\tau} \frac{e^{-T_i/\tau}}{(1 - e^{-T_i/\tau})} \right] = 0. \quad (14)$$

[T_i is the available time to observe a decay in flight in the emulsion stack. Our stack was sufficiently large so that T_i is just the potential moderation time.] The error in τ is determined from the function $S(\tau)$, defined by the equation

$$S(\tau) = f(\tau) / \left[\sum_{i=1}^n \left\{ 1 - \left(\frac{T_i}{\tau} \right)^2 \frac{e^{-T_i/\tau}}{(1 - e^{-T_i/\tau})^2} \right\} \right]^{1/2}. \quad (15)$$

$S(\tau)$ has zero mean and unit variance and is assumed to be Gaussian.

We have applied this method to the 26 $\Sigma^\pm \rightarrow \pi^\pm$ decays in flight and to the 14 $\Sigma^+ \rightarrow p$ decays in flight. Figure 5 shows a plot of $S(\tau)$ versus $1/\tau$ for the $\Sigma^\pm \rightarrow \pi^\pm$

²⁴ Since the first term in the bracket is small compared to the second term, our approximation of neglecting the difference between the distributions of decay times for $\Sigma^- \rightarrow \pi^-$ and $\Sigma^+ \rightarrow \pi^+$ introduces only a small error in τ_{Σ^-} .

²⁵ In a sense this estimate is a test of the consistency of the assumption that the Σ^+ has a single characteristic lifetime since we have made that assumption in order to deduce that there were 12 $\Sigma^+ \rightarrow \pi^+$ decays in flight. The agreement between Eqs. (11) and (13) is consistent with this assumption.

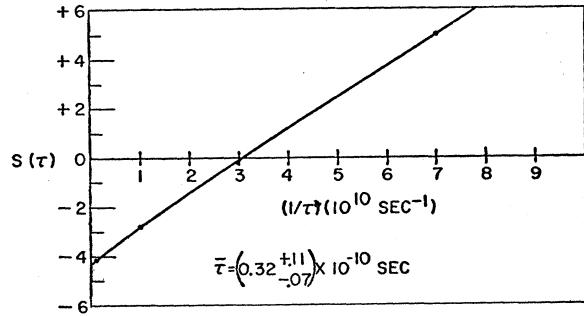


FIG. 5. $S(\tau)$ versus $1/\tau$ for the $\Sigma^\pm \rightarrow \pi^\pm$ decays in flight. The maximum-likelihood value for τ is the solution of the equation $S(\tau) = 0$. The standard-deviation estimates for τ are determined from the points $S(\tau) = \pm 1$.

decays. The results are

$$\bar{\tau}_{\Sigma^\pm \rightarrow \pi^\pm} = (0.32_{-0.07}^{+0.11}) \times 10^{-10} \text{ sec,} \quad (16)$$

and

$$\bar{\tau}_{\Sigma^+ \rightarrow p} = (0.47_{-0.15}^{+0.44}) \times 10^{-10} \text{ sec.} \quad (17)$$

This lifetime for the $\Sigma^+ \rightarrow p$ events, while smaller than the $\bar{\tau}_{\Sigma^+ \rightarrow p}$ of Eq. (11) and $\bar{\tau}_{\Sigma^+ \rightarrow \pi^+}$ of Eq. (12), is not in disagreement with these previous estimates because of its large error. On the other hand, the value of $\bar{\tau}_{\Sigma^\pm \rightarrow \pi^\pm}$ obtained by this method is significantly smaller than the previously obtained values of the lifetimes of both the Σ^+ and Σ^- hyperons. Under our previous assumption that 14 of the 26 $\Sigma^\pm \rightarrow \pi^\pm$ decays were $\Sigma^- \rightarrow \pi^-$ decays, we would have expected $\bar{\tau}_{\Sigma^\pm \rightarrow \pi^\pm}$ to be larger than the lifetime found for the Σ^+ . But instead $\bar{\tau}_{\Sigma^\pm \rightarrow \pi^\pm}$ is smaller than $\bar{\tau}_{\Sigma^+ \rightarrow p}$ of Eq. (11) by a factor of three. The probability that this difference is just a statistical fluctuation is less than one percent.

Several other experimental groups have obtained estimates of the Σ^+ and Σ^- lifetimes. Alvarez *et al.*,²⁶ from the study of K^- captures in a hydrogen bubble chamber, have obtained $\tau_{\Sigma^-} = (1.86 \pm 0.26) \times 10^{-10}$ sec and $\tau_{\Sigma^+} = (0.86 \pm 0.17) \times 10^{-10}$ sec. Budde *et al.*,¹³ from the study of Σ^- produced by energetic π^- mesons in a bubble chamber, have obtained $\tau_{\Sigma^-} = (1.4_{-0.5}^{+1.6}) \times 10^{-10}$ sec. Our best estimate of the Σ^+ and Σ^- lifetimes given in Eqs. (11) and (12) agree very well with these values. The results of Davies *et al.*,²⁷ from the study in emulsion of 11 $\Sigma^\pm \rightarrow \pi^\pm$ decays in flight, where the Σ 's come from energetic cosmic-ray stars, give $\tau_{\Sigma^\pm \rightarrow \pi^\pm} = (0.35_{-0.11}^{+0.15}) \times 10^{-10}$ sec. This last result agrees very well with our value for $\bar{\tau}_{\Sigma^\pm \rightarrow \pi^\pm}$ of Eq. (16), but is obviously in disagreement with the previously quoted lifetimes.

At the present time it is not clear how to resolve this discrepancy. One is tempted to consider this as evidence for the existence of two distinct lifetimes for each

²⁶ Alvarez, Bradner, Falk-Vairant, Gow, Rosenfeld, Solmitz, and Tripp (to be published).

²⁷ Davies, Evans, Fowler, Francois, Friedlander, Hiller, Iredale, Keefe, Menon, Perkins, and Powell, *Proceedings of the International Conference on Elementary Particles, Pisa, 1955*, Nuovo cimento Suppl. 2, 472 (1956).

charged hyperon. An analysis of our data in terms of the parity-doubling scheme of Lee and Yang²⁰ is very complicated, since in addition to the four lifetimes involved, there can be two distinct branching ratios for the two types of Σ^+ . (*A priori* there can be additional parameters for the relative amplitude of the two types of Σ^+ or Σ^- , but since the lifetimes of the τ^+ and θ^+ are nearly equal, the ratio of these amplitudes should be close to 1.) Despite this large number of parameters, it is not easy to determine a fit to all of the data of this experiment. If one assumes, for example, two lifetimes differing by a factor of about three for each charged hyperon and a branching ratio ($\Sigma^+ \rightarrow p$)/($\Sigma^+ \rightarrow \pi^+$) of 0 and ∞ for the short-lived and long-lived Σ^+ hyperons respectively, so as to make the lifetime measurements compatible, one encounters the difficulty that not enough $\Sigma^\pm \rightarrow \pi^\pm$ decay events were observed. (We found 26 whereas a number about 40 would be required.) It is apparent that better statistics are needed to resolve this question of the existence of two lifetimes or one lifetime for each charged hyperon.

E. Angular Distribution of Σ Decay Products

A study of the angular distribution of the Σ decay products may yield information as to the spin of the Σ hyperon and can provide a test of the hypothesis of parity doublets.

If we first assume that each Σ is not a parity doublet, then only even powers of $\cos\theta_{\Sigma\pi}$ can enter into the angular distribution, where $\theta_{\Sigma\pi}$ is the angle between the π -meson direction in the Σ rest system and the initial direction of flight of the Σ . If the spin of the Σ is $\frac{1}{2}$, the angular distribution must be isotropic. Treiman²⁸ has shown that if the K^- meson has spin zero and is captured by a nucleon from an s state, the angular distribution is uniquely determined by the spin of the Σ . For example, if the spin is $\frac{3}{2}$, the angular distribution is

$1+3\cos^2\theta$. When the K^- meson is captured by a nucleus the situation is somewhat more complicated. However, it may not be far removed from the ideal case discussed by Treiman, since most of the Σ 's are produced in single nucleon captures of the K^- meson, as indicated by the energy distribution of the Σ 's from K^- stars (Sec. III). Of course any scattering of the Σ as it leaves the nucleus will tend to smear out the observed angular distribution. On the other hand, the fact that the K meson be captured from a high orbital angular-momentum state relative to the center of mass of the nucleus, cannot change the angular distribution. One can easily see that the angular momentum pertinent to Treiman's discussion, is the relative angular momentum between the K^- meson and the nucleon by which it is captured. One might hope that the capture takes place from an s state of the K^- meson-nucleon system, since the energy in this system is of the order of 20 Mev, the characteristic Fermi energy in a nucleus. In this regard, the capture of a K meson in a nucleus may occur in an s state more often than it does in the capture by a free proton. In the case of the $K^- - p$ atomic system, the amount of p -state capture depends upon the lifetime of this process relative to the lifetime of the radiative transition from the $2p$ to the $1s$ state.²⁹ In any event, definite evidence for the presence of even powers of $\cos\theta$ other than zero, whether the Σ 's come from hydrogen captures or nuclear captures of K^- mesons, would prove that the spin of the Σ is greater than $\frac{1}{2}$.

Lee and Yang²⁰ have shown that if, and only if, there is parity doubling for each Σ , odd powers of $\cos\theta$ may appear in the angular distribution of the decay products. Since the amount of fore-aft asymmetry depends upon an interference term between two unknown amplitudes, no definite prediction is made as to the amount or sign of this asymmetry. In principle it can be different for Σ^+ and Σ^- decays and also different for $\Sigma^+ \rightarrow p$ decays as compared to $\Sigma^+ \rightarrow \pi^+$ decays.

We have measured the angle $\theta_{\Sigma\pi}$ for 85 Σ decays. These are listed in Tables III-VI. It is permissible to include Σ decays from rest in the angular distribution since, as Wolfenstein³⁰ has shown, the probability of changing the spin orientation of the Σ , via Coulomb scattering during the slowing-down process, is very small. Figure 6 is a histogram of the angular distribution for all 85 Σ decays. From the folded distribution we find that the number of events with $|\cos\theta_{\Sigma\pi}| > 0.5$ is 50, out of a total number of 85. A χ^2 test yields a probability of about 12% that this sample comes from a true distribution that is isotropic.

This sample has a folded angular distribution that lies in between an isotropic and a $(1+3\cos^2\theta)$ distribution, but it is certainly not conclusively different from isotropic. Alvarez *et al.*²⁶ found the number of Σ decay events with $|\cos\theta_{\Sigma\pi}| > 0.5$ to be 40 out of a total

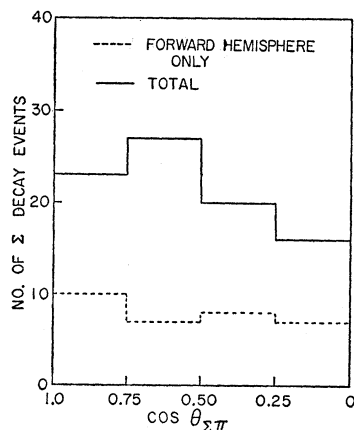


FIG. 6. Angular distribution of π mesons from Σ decays. ($\theta_{\Sigma\pi}$ is the angle between the direction of motion of the π meson in the rest system of the Σ and the initial direction of motion of the Σ .)

²⁸ S. B. Treiman, Phys. Rev. **101**, 1216 (1956).

²⁹ R. Gatto, Nuovo cimento **3**, 5 (1956).

³⁰ L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).

number of 65. The fact that both of these samples of data deviate from isotropy in the same direction suggests that the spin of the Σ may be greater than $\frac{1}{2}$.³¹

With respect to the possible existence of a forward-backward asymmetry, Table X gives a breakdown of our events for the various decay modes. The $\Sigma \rightarrow \pi$ decay modes taken together show a strong asymmetry in favor of the backward hemisphere (32 backward out of 45 events). The total number of Σ decays yield 53 backward out of 85 events. χ^2 tests on these two sets of numbers yield probabilities of about 0.7% and 2%, respectively, that the true distribution is symmetrical. This asymmetry would indicate the existence of parity doublets. However, the Brookhaven data³¹ show a large asymmetry in favor of the forward hemisphere (15 forward out of 22 events). Furthermore, the data of Alvarez *et al.*²⁶ show no asymmetry for $\Sigma \rightarrow \pi$ decays and a substantial forward asymmetry for $\Sigma^+ \rightarrow p$ decays. Combining all these data together tends to cancel out almost all the evidence for forward-backward asymmetries. On the other hand, the asymmetries do not cancel out if one examines the $\Sigma^+ \rightarrow p$ decays separately from the $\Sigma^+ \rightarrow \pi^+$ decays. The absence of a forward-backward asymmetry does not disprove the hypothesis of parity-doubling since the magnitude of this asymmetry cannot be quantitatively predicted.

Note added in proof.—A world-wide survey of data presented at the 1957 Rochester Conference on High Energy Physics on the angular distribution of Σ decays obtained from $\sim 10\,000$ K^- meson stars in emulsion showed *no* significant polar-equatorial or fore-aft asymmetries.

V. DISCUSSION

This set of data can help to illuminate many other questions concerning hyperons, besides those discussed in Sec. IV, including isotopic-spin selection rules for decay, matrix elements for K^- -nucleon capture processes, absorption cross sections for Σ 's in nuclear matter, and probability of hyperfragment formation.

A. Isotopic-Spin Selection Rule for Σ Decay

If one assumes the selection rule $\Delta T = \pm \frac{1}{2}$ for Σ decay, where T is the total isotopic-spin quantum number, then a relationship exists between the branching ratio $R = (\Sigma^+ \rightarrow p) / (\Sigma^+ \rightarrow \pi^+)$ and the ratio of lifetimes of the Σ^- and Σ^+ , $z = \tau_{\Sigma^-} / \tau_{\Sigma^+}$.¹⁴ Given R , z can take on either of two values for each assignment of spin and parity of the Σ . (For detailed discussion, see Iso and Kawaguchi¹⁴ and Alvarez *et al.*²⁶ Assuming time-reversal invariance, the spin and parity of the Σ determines the relative phase of the $T = \frac{3}{2}$ and $T = \frac{1}{2}$ matrix elements of the π , nucleon system.) We find $R = 26/22 = 1.18 \pm 0.32$ in good agreement with the

³¹ On the other hand, J. Hornbostel (private communication), informed us of preliminary Brookhaven results in which a sample of 22 Σ decays from K^- stars shows only a slight asymmetry in the folded angular distribution.

TABLE X. Forward-backward distribution of $\theta_{\Sigma\pi}$.

Type of event	No. in forward hemisphere	No. in backward hemisphere
$\Sigma^+ \rightarrow p$	19	21
$\Sigma^+ \rightarrow \pi^+$ at rest	6	13
$\Sigma^- \rightarrow \pi^\pm$ in flight	7	19
Total	32	53

ratio 14/14 obtained by Alvarez *et al.* Combining these two numbers, we get $R = 1.11 \pm 0.25$. If we assume one lifetime for each charged Σ , our best lifetime values, given in Eqs. (11) and (12), yield $\tau_{\Sigma^-} / \tau_{\Sigma^+} = 2.6 \pm 1.0$. Alvarez *et al.* give $\tau_{\Sigma^-} / \tau_{\Sigma^+} = 2.2 \pm 0.5$, again assuming one lifetime for each charged Σ . For $R = 1.11$, the predicted values for $\tau_{\Sigma^-} / \tau_{\Sigma^+}$ are 6.0 and 8.8 for spin and parity assignment $\frac{3}{2}+$ and $\frac{1}{2}+$ respectively ($\frac{1}{2}-$ yields a value just slightly lower than $\frac{1}{2}+$). These values for R , predicted by the $\Delta T = \pm \frac{1}{2}$ selection rule, are in clear disagreement with experiment. If the branching ratio R is as large as 1.40, the theory yields $\tau_{\Sigma^-} / \tau_{\Sigma^+} = 3.5$ and 5.3. (The second allowed solution for $\tau_{\Sigma^-} / \tau_{\Sigma^+}$ yields a value of $\sim \frac{1}{3}$ and hence is clearly ruled out by the data.)

In agreement with Alvarez *et al.*,²⁶ we conclude that the evidence is fairly strong for the *lack* of validity of a rigorous $\Delta T = \pm \frac{1}{2}$ selection rule. Only the assignment $\frac{3}{2}+$ to the Σ still has a non-negligible probability of fitting the data. This conclusion, however, depends in large measure on the assumption that each charged Σ hyperon has only one lifetime. The presence for each charged Σ of two lifetimes, which differed by as much as a factor of two, can significantly alter the predictions of the $\Delta T = \pm \frac{1}{2}$ theory. If each charged Σ has a unique lifetime but parity is not conserved in the decay, then again the predictions of the $\Delta T = \pm \frac{1}{2}$ assumption can be altered.

B. Matrix Elements for K^- -Nucleon Capture Process

Assume that isotopic spin is conserved in the K^- -nucleon capture process of Eq. (1). Then the number of $\Sigma^{+,0}$ hyperons can be expressed in terms of the two matrix elements M_0 and M_1 corresponding to the total isotopic-spin quantum numbers $T=0$ and 1.³² Both matrix elements enter in $K^- - p$ capture processes, while only M_1 plays a role in $K^- - n$ collisions. Alvarez *et al.*²⁶ have analyzed the relative number of Σ^+ , Σ^- , and Σ^0 hyperons observed from $K^- - p$ capture processes and have determined conditions on r and φ , where

$$r e^{i\varphi} = M_1 / M_0. \quad (18)$$

r and φ can be determined from the $K^- - p$ observations alone except for the uncertainty in the fraction,

³² For example, see S. Gasiorowicz, University of California Radiation Laboratory Report UCRL-3074, July, 1955 (unpublished); M. Koshiba, Nuovo cimento 4, 357 (1956).

α , of Λ^0 's (from Σ^0 's) decaying by the charged mode. From their observations that $\Sigma^-/\Sigma^+=2$, they obtain the inequality $r \geq 0.14$.

By combining the observations of Alvarez *et al.* on the relative number of Σ^+ and Σ^- hyperons from $K^- - p$ capture processes with similar observations from K^- -nucleus capture processes, one can obtain an estimate of r that is independent of the Σ^0 's.³³ We assume that inside the nucleus the predominant K^- capture process is by one nucleon, with protons and neutrons weighted according to Z/A and $(A-Z)/A$, respectively. One can easily show, by applying isotopic-spin arguments to Eq. (1), that

$$\left(\frac{N_{\Sigma^-} + N_{\Sigma^+}}{N_{\Sigma^+}} \right)_{K^- \text{-nucleus}} = \frac{\frac{1}{3}|M_0|^2 + \frac{1}{2}(A/Z)|M_1|^2}{P_{\Sigma^+}}, \quad (19)$$

and

$$\left(\frac{N_{\Sigma^-} + N_{\Sigma^+}}{N_{\Sigma^+}} \right)_{K^- - p} = \frac{\frac{1}{3}|M_0|^2 + \frac{1}{2}|M_1|^2}{P_{\Sigma^+}}, \quad (20)$$

where N_{Σ^\pm} are the number of Σ^+ and Σ^- hyperons observed, and P_{Σ^+} is the square of the matrix element for Σ^+ production in $K^- - p$ capture.

$$P_{\Sigma^+} = \frac{1}{6}|M_0|^2 - \left(\frac{1}{6}\right)^{\frac{1}{2}}|M_0 M_1| \cos \varphi + \frac{1}{4}|M_1|^2. \quad (21)$$

Taking the ratio of Eqs. (19) and (20), we obtain an expression for $r^2 = |M_1/M_0|^2$ in terms of the relative number of charged Σ hyperons in the two experiments, $K^- - p$ and K^- -nucleus.

$$r^2 = \frac{2(1-c)}{3[c(A/Z) - 1]}, \quad (22)$$

where

$$c = \left(\frac{N_{\Sigma^-} + N_{\Sigma^+}}{N_{\Sigma^+}} \right)_{K^- - p} / \left(\frac{N_{\Sigma^-} + N_{\Sigma^+}}{N_{\Sigma^+}} \right)_{K^- \text{-nucleus}}$$

Experimentally,

$$c = (83/28)(201/72) = 1.065 \pm 0.2. \quad (23)$$

In nuclear emulsion, the average value of A/Z , if one assumes a 60-40 distribution of K^- captures in heavy and light elements, respectively, is $\langle A/Z \rangle_{Av} = 2.17$. Hence Eq. (22) yields the result

$$r^2 = -0.03 \pm 0.14, \quad \text{or} \quad 0 < r^2 < 0.11. \quad (24)$$

Then

$$0 < r < 0.33.$$

Combining the lower limit on r obtained by Alvarez *et al.*, $r \geq 0.14$, we see that r is approximately in the range 0.14 to 0.33.³³ (From Fig. 9 of Alvarez *et al.*, this value of r corresponds to $0.35 \leq \alpha \leq 0.4$.)

The above analysis of K^- -nucleus captures has assumed that the K^- capture by two nucleons plays a

³³ The estimate of r obtained in this way can be seriously in error because of the large effect that the Coulomb force can have in changing the relative number of Σ^+ and Σ^- hyperons that actually emerge from the nucleus. See footnote 34.

very small role. The small number of high-energy Σ 's emitted from K^- stars indicates qualitatively that the two-nucleon capture is quite small, perhaps about 10%. Also the interactions of the Σ hyperons with other nucleons in the nucleus have been neglected in the above analysis. This is a second-order correction however, since if all the nuclei in the nuclear emulsion had $Z = \frac{1}{2}A$, then Eq. (19) would still hold rigorously, provided that isotopic spin were conserved in the Σ -nucleon interactions. Since $|A/2Z| > 1$, these neglected interactions have the effect of increasing the ratio (Σ^-/Σ^+) observed as compared to the ratio (Σ^-/Σ^+) initial. This tends to make r even smaller than the result of (24), hence reducing the upper limit on r slightly.

On the other hand the Coulomb forces between the Σ^+ or Σ^- hyperons and the nucleus do not commute with the total isotopic spin and hence can alter the observed ratio (Σ^-/Σ^+). Since the Σ^- must lose energy in passing from the surface of the nucleus to infinity, those Σ^- that arrive at the nuclear surface with less energy than the Coulomb barrier energy cannot emerge. This effect will decrease the observed ratio (Σ^-/Σ^+) as compared to the initial ratio and hence tend to increase the upper limit on r .³⁴

C. Absorption of Σ Hyperons

We have deduced (Sec. III) that the total number of Σ hyperons that emerge from these 1001 K^- stars is 302. If one knew how many Σ hyperons as compared to Λ^0 hyperons were formed in the initial K^- -nucleon capture process, then one could obtain a qualitative idea as to the strength of the absorption cross section for Σ 's via the reaction of Eq. (3).³⁵ When one uses the results of Alvarez *et al.*²⁶ that the relative probability of Λ^0 production to Σ production in a $T=1$ state is about $\frac{2}{3}$, and also the result of Eq. (24), it follows that the ratio of Λ^0 's to Σ 's produced inside the nucleus is $\lesssim 15\%$. (Again we ignore the two-nucleon production of hyperons.) Hence one expects that in at least 85% of K^- stars a Σ hyperon is produced, while we have deduced that a Σ (of any charge) emerges in only 30% of the K^- stars. For comparison, we note that a π meson (of any charge) emerges from K^- stars in about 50% of the cases, hence implying an escape probability of $\gtrsim 50\%$. Of course, the much lower energy distribution of the Σ 's increases the trapping probability of a Σ via multiple scattering so that a quantitative comparison of the π and Σ absorption probabilities is not

³⁴ The magnitude of this effect will depend on the unknown depth of the nuclear potential for Σ^+ and Σ^- inside the nucleus as well as on their energy distribution. The Σ^+ must penetrate a Coulomb barrier before emerging from the nucleus, but if the depth of the nuclear potential inside the nucleus were equal for Σ^+ and Σ^- , the probability for a Σ^+ to escape from the nucleus will be larger than for a Σ^- . White *et al.*³ has discussed this effect quantitatively.

³⁵ As discussed in the previous section, some additional absorption can be due to the Coulomb interaction between the Σ 's and the nucleus.

simple. Qualitatively, however, a comparison of these two results implies a large nucleon cross section (\sim geometric) for scattering and absorption of hyperons in the energy region 10 to 60 Mev.

D. Probability of Hyperfragment Formation

We observed 46 hyperfragments from the K^- stars and no hyperfragments from an estimated 115 Σ^- stars, including zero-prong events.³⁶ If one assumes that every Σ absorption yields a Λ^0 via reaction (3), then the hyperfragment formation probability is $(46/700)=6.6\%$ for K^- stars and $0/115$ for Σ^- stars. The Λ^0 's produced either directly or indirectly from the nuclear capture of K^- mesons appear to have a higher probability of emerging in the form of a hyperfragment than do the Λ^0 's, produced by the nuclear capture of Σ^- hyperons.

³⁶ There is some possibility of experimental bias in this comparison. The mean number of prongs from Σ^- stars is much less than from K^- stars, and hence the identification of hyperfragments that have very short ranges ("double centers" is more difficult in Σ^- events than in K^- events.

Since the total energy available to the nucleus in K^- absorption is much larger than in Σ^- absorption, this result implies that the process of hyperfragment formation is more like a boiling off of nuclear matter containing a Λ^0 than like a pickup process by the Λ^0 as it leaves the nucleus.

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Acceleration of Cosmic-Ray Particles among Extragalactic Nebulae

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It is proposed that cosmic-ray particles can be accelerated by the Fermi mechanism acting among galaxies in clusters in the same way that Fermi originally proposed for interstellar clouds in our own galaxy. When applied to the local group of galaxies this mechanism does not lead to an appreciable increase in energy over the limit attainable in our own galaxy. However, the conditions in a highly concentrated cluster such as the Coma cluster lead to maximum energies in the range 10^{18} – 10^{20} ev. Some implications of these results are discussed.

CURRENT theories of the acceleration of cosmic-ray particles suggest that if the original Fermi mechanism¹ or some variations and refinements of it²⁻⁴ are invoked, a reasonable upper limit to the energy which a particle can gain in our galaxy lies in the range 10^{15} – 10^{16} ev per nucleon. If the effects of diffusion and structure in the magnetic field are taken into account, Thompson⁵ has shown that energies of the order of 10^{17} ev may be attained. It is the purpose of this paper to point out that the Fermi mechanism may well operate among extragalactic nebulae,⁶ and the ultimate limit on the energy is probably determined only by the

conditions inside clusters of galaxies and to some extent by the age of the universe.

Observations from a number of directions can be used to estimate the probable conditions of acceleration. The clustering tendencies of galaxies have been realized in recent years to be of great importance (see the work of Shane and Wirtanen⁷ and Zwicky⁸). Also work by Zwicky⁹ has shown that much material exists in regions lying between galaxies. Detection of 21-cm radiation from the Coma cluster of galaxies¹⁰ and from the Cygnus radio source¹¹ which consists of two galaxies in interaction shows that there is a large amount of neutral hydrogen associated with these galaxies. These masses

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