

Polarization of Bremsstrahlung from Monocrystalline Targets

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We investigate the polarization of bremsstrahlung of high-energy electrons in a monocrystalline target. It is shown that, owing to interference effects, the polarization is considerably increased above its value when amorphous targets are used (by a factor of 1.5 for typical cases); moreover, the total emitted bremsstrahlung cone has an over-all net polarization with respect to the plane formed by the direction of incidence and the crystal axis. This latter fact provides a means of producing partially polarized radiation with an intensity sufficient for performing significant high-energy polarization experiments. The dependence on the emission angle of the interference radiation is also calculated. The influence of elastic scattering of electrons is estimated.

I. INTRODUCTION

PRESENT work in high-energy physics shows an increasing use of polarization as a tool for obtaining additional information on the interaction of elementary particles. Scattering of polarized protons was investigated,¹ and pion production with polarized protons has been considered.² Similarly, it would be desirable to perform experiments with polarized photons in the high-energy region, especially on photoproduction of pions. Very energetic photons can be obtained by producing bremsstrahlung from targets in electron accelerators. The polarization of bremsstrahlung has, therefore, been investigated theoretically.³⁻⁵ It was shown in these papers that bremsstrahlung is partly polarized, the degree of polarization depending on the target material, on the angle of emission of the radiation, and on the fraction of energy of the primary carried away by the photon. Polarization is largest for soft radiation and for emission angles of the order mc^2/E_{prim} , the maximum being of order 30-50%.

The experimentally obtainable percentage of polarization will be reduced by the fact that the angle and opening of a bremsstrahlung beam cannot be defined with arbitrary accuracy, and likewise by the multiple scattering of the electrons in the target which will cause a further uncertainty in emission angle. If we had any means of increasing the inherent degree of polarization, such effects could be partly overcome. It will be shown in the following that, by using cooled single-crystal targets for the production of radiation, this aim can indeed be achieved. It turns out in addition that upon integration over all the outgoing bremsstrahlung cone, a net polarization will remain by virtue of the fact that a reference plane containing the primary beam and the crystal axis is determined, whereas with amorphous targets the over-all polariza-

tion vanishes owing to the complete symmetry of the problem.

We refer to a recent investigation⁶ of bremsstrahlung from crystal targets, which predicts an enhancement of the soft part of the spectrum for certain angles of incidence θ of the electron beam with respect to a crystallographic axis; this is due to coherent emission of radiation from several lattice atoms. We shall see below that, at the same time, an increase of polarization will occur also. The notation of I will be used throughout.

II. ANGULAR DISTRIBUTION

We first want to give some consideration to the dependence on emission angle of the interference spectrum. In I, the spectrum was given after integration over this angle; but it can be seen that a demonstration of the angular dependence will exhibit more characteristic features of the interference effect. Imagine a Weizsäcker-Williams treatment of the process in the rest frame of the primary electron. The crystal is then represented by virtual photons possessing a discrete wave-number spectrum, the wave vectors being reciprocal-lattice vectors. The Compton scattered photons, appearing as bremsstrahlung in the laboratory frame, will also have a discrete spectrum if we fix the emission angle. In the laboratory system, the virtual-photon wave vectors correspond to the momentum transfer to the atoms, \mathbf{q} . In this frame, the \mathbf{q} vectors can be assumed to run continuously over those reciprocal-lattice planes which are approximately normal to the direction of incidence (see I). Then the lines in the emitted spectrum originating from one such lattice plane appear to be one broader line. Integration over the emission angle would broaden them even more, and this is avoided in the following treatment.

Introducing a reduced angle $U = \epsilon_1 \Theta_1$, and integrating the differential Bethe-Heitler cross section over electron angles after multiplying by the crystal factor of I, we find the spectrum of bremsstrahlung from a

¹ E.g., Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, *Phys. Rev.* **93**, 1430 (1954).

² F. Mandl and T. Regge, *Phys. Rev.* **99**, 1478 (1955).

³ M. M. May, *Phys. Rev.* **84**, 265 (1951).

⁴ Gluckstern, Hull, and Breit, *Phys. Rev.* **90**, 1026 (1953). R. L. Gluckstern and M. H. Hull, *Phys. Rev.* **90**, 1030 (1953).

⁵ R. Karplus and A. Reifman, University of California Radiation Laboratory Report UCRL-2686 (unpublished).

⁶ H. Überall, *Phys. Rev.* **103**, 1055 (1956); hereafter referred to as I.

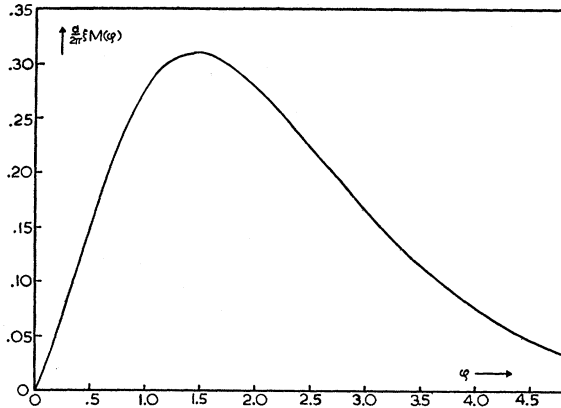


FIG. 1. $(a/2\pi)\xi M(\varphi)$ for Cu, $T=0^\circ$.

crystal:

$$\sigma(k, U, \psi_1) dk dU = N \bar{\phi} \frac{dk}{k} 4U dU \frac{d\psi_1}{2\pi} \left\{ \frac{1+(1-x)^2}{(1+U^2)^2} \right. \\ \times \left[\ln \frac{(1+U^2)^2}{\alpha} + (1+B)e^B \text{Ei}(-B) + M(\varphi) \right] \\ \left. - 2 \frac{1-x}{(1+U^2)^4} \left(2U^2 \left[\ln \frac{(1+U^2)^2}{\alpha} + (1+B)e^B \text{Ei}(-B) \right. \right. \right. \\ \left. \left. \left. + M'(\varphi, \psi_1) \right] + 1 - 4U^2 + U^4 \right) \right\}, \quad (1)$$

with the following notations:

$$\bar{\phi} = \frac{Z^2}{137} \left(\frac{e^2}{mc^2} \right)^2, \quad x = k/\epsilon_1, \quad \alpha = \xi^2 + \beta^{-2}, \\ \xi = \delta(1+U^2), \quad \delta = \frac{1}{2\epsilon_1} \left(\frac{x}{1-x} \right), \quad \beta = 111Z^{-1/2}, \\ B = \alpha A \quad (\text{with } A \text{ defined in I}), \quad \varphi = \xi/(\theta\alpha^{1/2}),$$

$$M(\varphi) = \frac{2\pi}{a\xi} \frac{\varphi}{(1+\varphi^2)^{1/2}} \left\{ \left(1+B - \frac{1}{2} \frac{1}{1+\varphi^2} \right) \right. \\ \left. \times \left[1 - \Phi((1+\varphi^2)^{1/2} B^{1/2}) \right] e^B - \left(\frac{B}{\pi} \right)^{1/2} \frac{\exp(-B\varphi^2)}{(1+\varphi^2)^{1/2}} \right\},$$

$$M'(\varphi, \psi_1) = 2 \sin^2 \psi_1 \frac{2\pi}{a\xi} \frac{\varphi}{(1+\varphi^2)^{1/2}} \\ \times \left\{ \left[1 + [1 + \varphi^2(1 - \cot^2 \psi_1)] \left(B - \frac{1}{2} \frac{1}{1+\varphi^2} \right) \right] \right. \\ \times \left[1 - \Phi((1+\varphi^2)^{1/2} B^{1/2}) \right] e^B - [1 + \varphi^2(1 - \cot^2 \psi_1)] \\ \left. \times \left(\frac{B}{\pi} \right)^{1/2} \frac{\exp(-B\varphi^2)}{(1+\varphi^2)^{1/2}} \right\},$$

$$\Phi(x) = 2\pi^{-1/2} \int_0^x \exp(-t^2) dt, \quad \text{Ei}(-x) = - \int_x^\infty t^{-1} e^{-t} dt;$$

a is the lattice constant and N the number of atoms in the crystal; ϵ_1 is the primary electron energy and k the quantum energy (both in units mc^2), U the reduced angle between the directions of the primary and the quantum, θ the angle between the primary direction and the crystal axis, and ψ_1 the dihedral angle between the plane containing the primary beam and the quantum and the plane containing the primary beam and the crystal axis. Equation (1) is exact for only those (soft) components of the spectrum which satisfy the condition of complete screening, $\xi^2 \ll \beta^{-2}$ (or rather the more stringent condition $\xi \ll 2\pi/a$), as we did not include the (small) effects arising from the harmonics of the lattice potential, $\hbar > 0$ (see I). The terms $M(\varphi)$, $M'(\varphi, \psi_1)$ represent the contribution from the fundamental, $\hbar = 0$, and the rest of the terms represent the "continuous" spectrum due to the temperature motion of the lattice.

To obtain the one-atom cross section of Schiff,⁷ we simply have to replace the two square brackets of Eq. (1) by

$$\ln \left(\frac{(1+U^2)^2}{\alpha} \right) - 1. \quad (2)$$

A plot of $(a/2\pi)\xi M(\varphi)$ is given in Fig. 1, and spectra are shown in Fig. 2 for $U=1$ and 2, both at $\epsilon_1=600$ Mev, and $\psi_1=\frac{1}{4}\pi$, for different crystal orientations θ , and with Schiff's one-atom spectrum for comparison. The θ values are so chosen that they correspond to $\varphi=1.5$ (maximum of Fig. 1) for $U=1$ and $x=0.2$ or 0.4, which gives $\theta=5$ or 13.4 milliradians. We see that the interference peak decreases as θ increases, in agreement with the behavior of the integrated spectrum given in I. Moreover, we recognize that the main part of the interference peak comes from small angles, $U \lesssim 1$.

We may remark that the observation of electrodynamic processes in crystals can provide a direct check on the reliability of dispersion formulas involving physical processes off the energy shell, for example in a recent derivation of first-order vacuum polarization⁸ which expresses the polarization coefficient as a dispersion integral over the cross section for pair production by off-the-energy shell photons in empty space. Now in the laboratory frame, the crystal field of force is endowed with a substantial wave number, $|\mathbf{k}| \sim 5$ kev, whereas it has no time variation, $k^4=0$. Thus, the equivalent photons are off the light cone, and by measuring bremsstrahlung from crystals, we are in fact observing a Compton effect of such photons. A check on the angular distribution will be necessary in order to identify the equivalent photons as much as possible.

III. POLARIZATION

The cross sections $\sigma_1(k, U, \psi_1)$ and $\sigma_{11}(k, U, \psi_1)$ express the emission probability for radiation with polariza-

⁷ L. I. Schiff, Phys. Rev. 83, 252 (1951).

⁸ R. N. Euwema and J. A. Wheeler, Phys. Rev. 103, 803 (1956).

tion perpendicular and parallel to the emission plane (\mathbf{p}_1, \mathbf{k}); we have $\sigma = \sigma_{\perp} + \sigma_{\parallel}$. Then the polarization is defined as

$$P = (\sigma_{\perp} - \sigma_{\parallel}) / (\sigma_{\perp} + \sigma_{\parallel}). \quad (3)$$

The differential polarized cross sections were taken from May (see reference 3) and integrated as before, with the result

$$\begin{aligned} \sigma_{\perp}(k, U, \psi_1) dk dU \frac{d\psi_1}{2\pi} &= N \bar{\phi} \frac{dk}{k} 4U dU \frac{d\psi_1}{2\pi} \\ &\times \left\{ \frac{1 + (1-x)^2}{(1+U^2)^2} \frac{1}{2} \left[\ln \frac{(1+U^2)^2}{\alpha} \right. \right. \\ &+ (1+B)e^B \text{Ei}(-B) + M(\varphi) \left. \right] - 2 \frac{1-x}{(1+U^2)^2} \\ &\left. \times \frac{1}{2} [1 + M'(\varphi, \psi_1) - M(\varphi)] \right\}, \quad (4) \end{aligned}$$

for complete screening again. In the case $\psi_1 = (2n+1) \times (\pi/4)$, ($\rightarrow M' = M$), we can write the polarization in a form similar to that of May, namely

$$P = \frac{4U^2}{(1+U^2)^2 F - 4U^2},$$

with

$$F = \frac{1 + (1-x)^2}{1-x} + C \frac{(2-x)^2 + 2x^2}{2(1-x)},$$

$$C = 2 \left[\ln \left(\frac{(1+U^2)^2}{\alpha} \right) + (1+B)e^B \text{Ei}(-B) - 3 + M(\varphi) \right]^{-1},$$

which goes over into May's expression if we substitute

$$C = 2 \left[\ln \left(\frac{(1+U^2)^2}{\alpha} \right) - 4 \right]^{-1}.$$

Now M is proportional to $\delta^{-1} = 2\epsilon_1(1-x)/x$, so for high energies, C can become much smaller than the amorphous-target value, and P larger, if x and the crystal orientation θ are chosen appropriately. To give an example, at 600 Mev, $\theta = 6.7$, and $x = 0.2$, the value of F would be 2.932 in the crystal as compared to 2.365 for one-atom bremsstrahlung. At $U = 1$, this corresponds to a polarization $P = 0.75$, and to an increase of a factor 1.5 above May's polarization value of $P = 0.52$. An average over ψ_1 gives the same result as $\psi_1 = (2n+1) \times (\pi/4)$, so that P must have alternately maxima and minima at $\psi_1 = \frac{1}{2}n\pi$, and the maximum polarization may be much larger than the one determined by C .

The situation of most experimental interest arises, however, if we consider the total emitted bremsstrahlung cone. With amorphous targets, the problem is

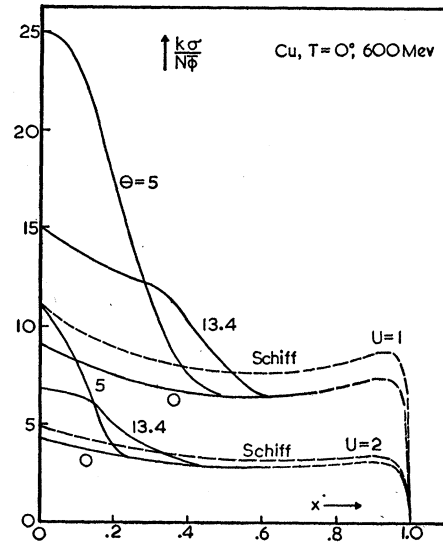


FIG. 2. Plots of $4U$ times the expression inside the curly brackets in Eq. (1) for Cu, $T = 0^\circ$, $\epsilon_1 = 600$ Mev, $\psi_1 = \pi/4$ and various values of θ and U . Schiff's differential spectrum is shown for comparison.

symmetrical around the direction of incidence, and no net polarization results. A crystal target determines a reference plane ($\mathbf{p}_1, \mathbf{a}_1$), and it turns out that with respect to it, a net polarization actually remains. This gives us a considerable experimental advantage. Before, polarized radiation could be obtained⁹ only by singling out a section of the bremsstrahlung cone of opening $U \sim 1$, which at 600 Mev corresponds to a very small Θ_1 of order 10^{-3} radian, hard to realize experimentally; moreover, very thin targets must be used such that the angular spread due to multiple electron scattering should not be of the same order as Θ_1 (see Sec. IV). All these restrictions result in a very small intensity of the obtained radiation. With a crystal, the use of the entire bremsstrahlung cone together with a larger permissible target thickness (multiple scattering competes now with $\theta \gtrsim 10\Theta_1$, rather than with Θ_1) gives a considerable gain in intensity; also, the collimation of the incident beam need not be so high, as the standard is now again θ rather than Θ_1 .

We define polarization with respect to the ($\mathbf{p}_1, \mathbf{a}_1$) plane, and use the integrated cross sections:

$$\begin{aligned} \sigma &= 2N \bar{\phi} \frac{dk}{k} \left\{ [1 + (1-x)^2] (2 \ln \beta + S + 2 + \frac{1}{2} \Psi_1^0) \right. \\ &\quad \left. - \frac{2}{3} (1-x) (2 \ln \beta + S + 5/3 + \frac{1}{2} \Psi_2^0) \right\}, \quad (5) \end{aligned}$$

$$\begin{aligned} \sigma_{\perp} &= N \bar{\phi} \frac{dk}{k} \left\{ [1 + (1-x)^2] \frac{1}{2} \Psi_1^0 \right. \\ &\quad \left. - \frac{2}{3} (1-x) \frac{1}{2} \Psi_2^0 + 2(1-x) \frac{1}{2} \Psi_3^0 \right\}, \end{aligned}$$

⁹ R. E. Taylor and R. F. Mozley, Bull. Am. Phys. Soc. Ser. II, 1, 375 (1956).

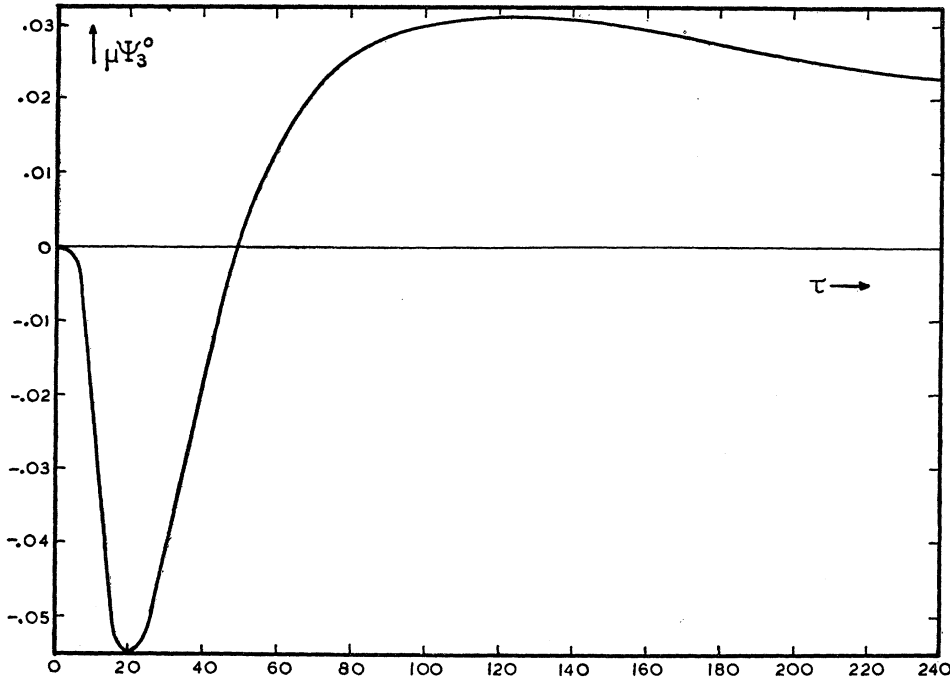


FIG. 3. Plot of $\Psi_3^0(\tau)$ for Cu, $T=0^\circ$.

where σ^i = interference part of cross section only,

$$S = (1+B)e^B \text{Ei}(-B),$$

$\Psi_{12}^0(\tau)$ is defined in I,

$$\Psi_3^0(\tau) = \frac{4}{\pi\mu} \int_{1/\tau}^{\infty} \frac{\exp(-Aq^2) q^3 dq}{(\beta^2 + q^2)^2} \frac{\tau^2 q^2 - 4}{3\tau^4 q^4} (\tau^2 q^2 - 1)^{\frac{1}{2}},$$

$\tau = \theta/\delta$ and $\mu = \delta/(2\pi/a)$. We then obtain an average polarization

$$\langle P \rangle = (2\sigma_1^i - \sigma^i)/\sigma,$$

or

$$\langle P \rangle = \Psi_3^0 \left[\frac{1 + (1-x)^2}{1-x} (2 \ln \beta + S + 2 + \frac{1}{2} \Psi_1^0) - \frac{5}{3} (2 \ln \beta + S + \frac{1}{2} \Psi_2^0) \right]^{-1}. \quad (6)$$

The quantity $\mu\Psi_3^0(\tau)$ is plotted vs τ in Fig. 3 (for Cu at $T=0^\circ$): it becomes zero exponentially for $\tau \rightarrow 0$ and goes as τ^{-1} for $\tau \rightarrow \infty$. The polarization is then shown in Fig. 4, for 600 Mev, for several values of the angle θ .

The "crystal screening functions" Ψ_{123}^0 can be partly evaluated in terms of the error function, with the result:

$$\begin{aligned} \mu\Psi_1^0(b) &= \frac{e^D}{(b+1)^{\frac{1}{2}}} \left(1 + 2D + \frac{2D}{b} \right) \\ &\quad \times [1 - \Phi((b+1)^{\frac{1}{2}} D^{\frac{1}{2}} b^{-\frac{1}{2}})] - 2 \left(\frac{D}{\pi b} \right)^{\frac{1}{2}} e^{-D/b}, \\ \mu\Psi_2^0(b) &= 3\mu\Psi_3^0(b) \\ &\quad + \frac{12}{\pi} \int_1^{\infty} \frac{\exp(-Dx^2/b)}{(b+x^2)^2} \ln[x + (x^2-1)^{\frac{1}{2}}] dx, \end{aligned}$$

$$\begin{aligned} \mu\Psi_3^0(b) &= \frac{8}{3b^2} [1 - \Phi(D^{\frac{1}{2}} b^{-\frac{1}{2}})] \\ &\quad + \frac{1}{3} \frac{e^D}{(b+1)^{\frac{1}{2}}} \left(1 + 2D - 2 \frac{2-5D}{b} - 8 \frac{1-D}{b^2} \right) \\ &\quad \times [1 - \Phi((b+1)^{\frac{1}{2}} D^{\frac{1}{2}} b^{-\frac{1}{2}})] \\ &\quad - \frac{2}{3} \left(\frac{D}{\pi b} \right)^{\frac{1}{2}} \frac{b+4}{b} e^{-D/b}, \end{aligned}$$

with $b = \tau^2/\beta^2$ and $D = \beta^{-2}A$.

IV. INFLUENCE OF ELECTRON SCATTERING

In amorphous thick targets the angular distribution of bremsstrahlung is broadened by multiple elastic scattering of electrons, and the polarization is reduced. In monocrystals, electron scattering will be strongly modified by diffraction effects. The differential cross section has a diffraction factor (notation as in I):

$$\frac{(2\pi)^3}{\Delta} \sum_{\mathbf{g}} \delta^3(\mathbf{q} - 2\pi\mathbf{g}), \quad (7)$$

which is derived most easily from the Born-approximation matrix element for scattering, containing a factor $\int V \exp(i\mathbf{q}\mathbf{r}) d\mathbf{r}$, but is of general validity. Here we introduced a momentum transfer $\mathbf{q} = \mathbf{k}_0 - \mathbf{k}$, $q = 2k_0 \sin \frac{1}{2}\chi$, with primary momentum \mathbf{k}_0 and a scattering angle χ . Momentum transfers are thus restricted to reciprocal-lattice points, with spacing $2\pi/a$ in a cubic crystal. The scattering is mostly forward, with a width of $\chi \lesssim \chi_0 = (a_0 k_0)^{-1} \equiv Z^{\frac{1}{2}}/137k_0$, (a_0 = Thomas-Fermi radius), or $q \lesssim q_0 = a_0^{-1}$; for Cu, q_0 is almost four times larger than

$2\pi/a \sim 7 \times 10^{-3}$. The momentum transfer \mathbf{q} is mostly transverse:

$$q_{\perp} = q(1 - q^2/4k_0^2)^{1/2}, \quad q_{\parallel} = q^2/2k_0.$$

If we consider small angles $\theta = \angle(\mathbf{k}_0, \mathbf{a}_1)$, we can approximate the planes of reciprocal-lattice points perpendicular to \mathbf{a}_1 by smooth planes. This will not be so good as in the bremsstrahlung case, as $\sigma_{\text{sc}} q dq \sim q^{-3} dq$, whereas $\sigma_{\text{br}} q dq \sim q^{-1} dq$, ($q \lesssim 1$) asymptotically. However, it will suffice for an order of magnitude estimate, as well as the use of the Born approximation cross section

$$\sigma(\chi) \sin \chi d\chi = \frac{8\pi Z^2 r_0^2}{(q^2 + a_0^{-2})^2} \sin \chi d\chi, \quad r_0 = \frac{e^2}{mc^2}; \quad (8)$$

in the crystal, this is to be multiplied by

$$\frac{2\pi}{a} \sum_h \delta\left(\mathbf{q} \cdot \frac{\mathbf{a}_1}{a} - \frac{2\pi h}{a}\right) \quad (7')$$

instead of (7). Denoting the angle between the $\mathbf{k}_0, \mathbf{a}_1$ plane and the \mathbf{k}_0, \mathbf{k} plane by φ , we find

$$\mathbf{q} \cdot \mathbf{a}_1 / a = q_{\parallel} \cos \theta - q_{\perp} \sin \theta \cos \varphi,$$

and obtain an integrated cross section

$$\begin{aligned} \sigma &= 16\pi Z^2 r_0^2 a_0^4 \frac{\pi}{ak_0} \sum_h \left[1 + 2a_0^2 k_0^2 \left(\sin^2 \theta + \frac{2\pi h}{ak_0} \cos \theta \right) \right] \\ &\times \left[1 + 4a_0^2 k_0^2 \left(\sin^2 \theta + \frac{2\pi h}{ak_0} \cos \theta \right) \right. \\ &\left. + \left(4a_0^2 k_0^2 \frac{\pi h}{ak_0} \right)^2 \right]^{-1/2}, \quad (9) \end{aligned}$$

as compared to the cross section in amorphous material,

$$\sigma_0 = 16\pi Z^2 r_0^2 a_0^4 (1 + 4a_0^2 k_0^2)^{-1}. \quad (9')$$

As a function of θ , the cross section given in (9) has a principal maximum ($h=0$) at $\theta=0$, with a width of $\sim \theta_0 = (a_0 k_0)^{-1} \equiv \chi_0$, and a height of $\sigma(0)/\sigma_0 = 2a_0 k_0 \times (2\pi a_0/a)$. For primary energies of 600 Mev, $k_0 = 1.2 \times 10^3$, this is of order $\theta_0 \sim 2 \times 10^{-5}$, $\sigma(0)/\sigma_0 \sim 3 \times 10^4$. The first side maxima lie at $\sin \theta \sim (2\pi |h| a_0/a) \sim 0.3 |h|$, and are of order $\sigma_{h \text{ max}}/\sigma_0 \sim 0.3 |h|$. For an angle $\theta' \sim a_0/k_0$ (maximum of interference bremsstrahlung), we find $\sigma/\sigma_0 \sim (2\pi a_0/a)(k_0/a_0) \sim 8$. If we disregard the anisotropy of the elementary scattering in the crystal and use Molière's theory¹⁰ to estimate the root mean square angular spread of the electrons, we recognize that an increase of the scattering cross section by a factor of σ/σ_0 has to accompany a reduction of target thickness t by a factor of σ_0/σ in order to keep the angular spread constant. If we want this spread (see also reference 3)

¹⁰ H. A. Bethe, Phys. Rev. **89**, 1256 (1953). For an accurate treatment of the physical situation, the multiple scattering of electrons in a crystal ought to be worked out.

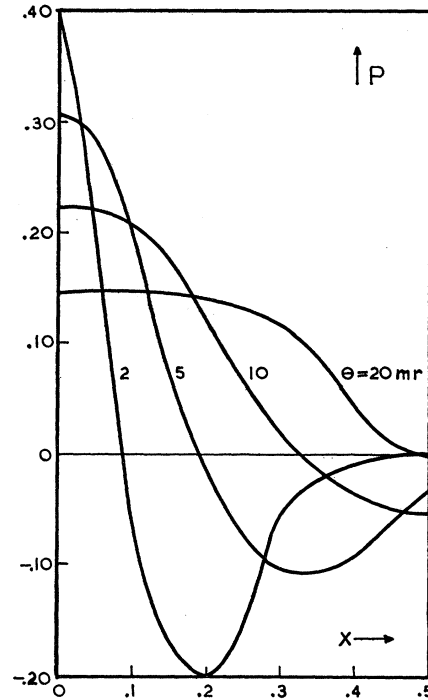


FIG. 4. Polarization of the entire bremsstrahlung cone at $\epsilon_1 = 600$ Mev from a Cu crystal at $T = 0^\circ$, as a function of $x = k/\epsilon_1$; $\theta = \angle$ (primary direction, crystal axis) in milliradians.

to be not larger than θ' , we have to use targets of thickness less than t' defined by

$$z \ln z \leq \frac{1}{2} a_0^2 (k_0 \theta')^2, \quad z = 137\pi^{1/2} a_0^2 (\ln 183Z^{-1/3})^{-1} t' (\sigma/\sigma_0).$$

Here t is measured in radiation lengths. For Cu, we find $t' \leq 1.6(\sigma_0/\sigma)$ radiation lengths. So there is no danger that multiple scattering will decollimate our incident beam; its qualities need not be very high anyway, as θ' is fairly large. If, however, we want to single out an emission angle of order $\theta'' = 1/k_0$ corresponding to $U = 1$, much thinner targets are required, with thickness $t'' \leq 2 \times 10^{-3}(\sigma_0/\sigma)$ for Cu; but at such an angle, $\sigma/\sigma_0 \sim (2\pi a_0/a) k_0 \sim 4 \times 10^2$. This result is not very reliable, however, as at this small angle, the anisotropy of the elementary scattering law must have considerably modified the multiple scattering. Altogether, we see that electron scattering in the crystal is increased in the angular range of interest, but that an experiment using the entire bremsstrahlung cone is hardly affected.

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