tainly does not attain this value, but our attempts to derive a theory based on maximizing (Φ_0,Ψ_0) have led to intractable equations. The φ_μ should also approximate, for similar reasons, the single-particle orbitals of maximum convergence for one-particle operators. In fact they satisfy an equation not very different from these orbitals, as given by Löwdin in his theory of "natural orbitals." Slater[5] has developed a self-consistent field theory based on the intuitively attractive form for the interaction energy, $\frac{1}{2}\int \rho V d^3\mathbf{r}$; this leads to a Hamiltonian differing from $\mathfrak{F}$ by a factor of one-half in front of the multiparticle terms. The sum of the one-particle eigenvalues is E_0, but they are not (individually) estimates of ionization energies. The one-particle wave functions, however, have an obvious physical connection with the problem.

On the other hand, Löwdin has shown that it is possible to obtain the "natural orbitals" by the solution of a linear eigenvalue problem once the E_0 and A_{i0} (or G_{i0}) are known. The procedure is to diagonalize the representation of the (first-order) density matrix in the φ_μ:

$$\gamma^{(0)}(\mathbf{r},\mathbf{r}')=\textstyle\sum_\mu N_\mu^{(0)}\varphi_j^{(0)*}(\mathbf{r})\varphi_\mu^{(0)}(\mathbf{r}'),$$

where

$$\gamma^{(0)}(\mathbf{r},\mathbf{r}')=\int d^{3(N-1)}\mathbf{r}''\Psi_0^*(\mathbf{r}_1,\mathbf{r}_2'',\mathbf{r}_3'',\cdots\mathbf{r}_N'')\times\Psi_0(\mathbf{r}_1',\mathbf{r}_2'',\cdots\mathbf{r}_N'').$$

The solution of the problem for the energies E_n, the coefficients A_{in}, the occupation numbers $N_\mu^{(n)}$, and the one-particle orbitals $\varphi_\mu^{(n)}$, might be little more tedious than the self-consistent field problem, and might lead to results of greater usefulness, although the physical interpretation of the changes in the $\varphi_\mu^{(n)}$ with n might be more complex than one would desire. Only experience can answer the question: which set of one-particle wave functions is most desirable?

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Reactions O^{16} (d,p), (d,α), and (d,d) in the Energy Range 3.4 to 4.2 Mev

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Cross sections and angular distributions are given for the (d,p), (d,α), and (d,d) reactions on O^{16} in the deuteron energy range 3.4 to 4.2 Mev.

I. INTRODUCTION

THE reaction $O^{16}(d,p)$ has previously been studied by several groups of investigators,[1–5] using deuterons of energies up to 3.7 Mev. The angular distributions found for the two proton groups corresponding to the formation of O^{17} in its ground state and in its first excited state showed forward maxima suggesting stripping reactions with $l_n=2$ and 0, respectively. However the differential cross sections were found to vary strongly with energy,[6] so it seems reasonable to describe the reactions as proceeding partly through compound nucleus formation and partly through stripping, the two processes being coherent. If this assumption is correct, one should be able to account for the observed angular distributions by means of a calculation including interference effects, providing that the l_n values for the stripping mode, and the spins and parities for the intermediate states in the compound nucleus mode are known.[7] The experiments reported here were undertaken in an effort to obtain the required information so that such calculations could be made and the results could be compared with the distributions found experimentally. Unfortunately, we were unable to obtain all the required properties of the compound nucleus, which seems to involve a number of overlapping levels.

II. METHOD

A deuteron beam[8] ranging from 3.4 to 4.2 Mev with a measured resolution of better than 0.3% was used.[9]

* On leave from the University of Basel, Basel, Switzerland.
[1] N. P. Heydenburg and D. R. Inglis, Phys. Rev. **73**, 230 (1948).
[2] Berthelot, Cohen, Cotton, Faraggi, Grjebrine, Leveque, Naggiar, Roclawski-Conjeaud, and Szteinsznaider, Compt. rend. **238**, 1312 (1954).
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[4] M. K. Juric and M. M. Petrovic, Bull. Inst. Nuclear Sci., Boris Kidrich **5**, 1 (1955).
[5] J. C. Grosskreutz, Phys. Rev. **101**, 706 (1956).
[6] Similar effects have been reported for the $C^{12}(d,p)$ reaction and the $C^{13}(d,p)$ reaction. See J. B. Marion and G. Weber, Phys. Rev. **103**, 167 (1956); Bonner, Eisinger, Kraus, and Marion, Phys. Rev. **101**, 209 (1956); M. T. McEllistrem, Phys. Rev. **100**, 167 (1956); McEllistrem, Jones, Chiba, Douglas, Herring, and Silverstein, Phys. Rev. **104**, 1008 (1956).
[7] Theories taking into account both modes are given by I. P. Grant, Proc. Phys. Soc. (London) **A67**, 981 (1954) and R. G. Thomas, Phys. Rev. **100**, 25 (1955).
[8] Produced by the variable-energy cyclotron at the University of Rochester. Fulbright, Bromley, Bruner, Hamann, and Hawrylak, U. S. Atomic Energy Commission Report NYO-6541, 1954 (unpublished).
[9] Determined by observations on the narrow 3.47-Mev resonance in the elastic scattering of protons by O^{16}.

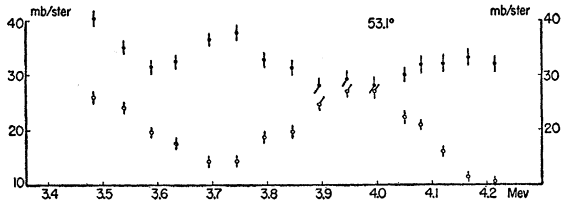

Fig. 1. Differential cross section in the center-of-mass system, $(d\sigma/d\Omega)_{53.1°}$, *vs* lab energy of $O^{16}(d,p)O^{17}$. Solid points, ground state; open circles, 0.872-Mev state.

A thin silver foil covering the first slit of the beam analyzer system served to break up molecular hydrogen ions which might have interfered with the measurements. The cross section of the beam at the target was about 3 mm wide and 1 cm high. The angular spread of the beam in the (horizontal) experimental plane was less than 0.3 degree.

The protons, alpha particles, and deuterons from the nuclear reactions were detected by means of an argon-filled ionization chamber having a thin aluminum window with an aperture about 1 cm wide located at a radius 25 cm from the target foil. Ion chamber conditions used in a particular run were chosen for greatest convenience in analyzing the ion chamber pulses. For example, a 4-mil window used when protons were being observed was thick enough to stop scattered deuterons as well as alpha particles from $O^{16}(d,\alpha)N^{14}$. A 1-mil foil used when deuterons were being observed stopped alpha particles. A 0.35-mil foil transmitted all three types of particles, but the alpha-particle pulses stood out clearly when the pressure of the argon was reduced so that the range of the alpha particles was approximately the effective length of the ion chamber and the ranges of the deuterons and protons were therefore much greater. Protons from the formation of O^{17} in its higher excited states caused no trouble.

A 30-channel pulse-height analyzer was used. Groups of pulses were identified as being due to particles of particular types by comparing the observed energies with those calculated from reaction kinetics and known Q values, due allowances being made for energy losses in the ion chamber window.

The targets were of three kinds. WO_3 smoke deposited on gold leaf in the process of burning wolfram in oxygen, and self-supporting foils of SiO, made by the evaporation procedure of Sawyer,[10] and of Al_2O_3, made by the electrolytic method of Strohmaier,[11] all proved satisfactory.

Cross sections were calculated from the experimental results by a method based on the assumption that the small-angle scattering of 4-Mev deuterons by W, Si, and Al is predominantly Rutherford scattering. The chemical compositions of the targets were assumed to be exactly as stated above. No variation with time in the relative amount of scattering from oxygen and the heavier nucleus was observed in any experiment. All three targets gave essentially the same values for the cross sections. At least six check points were involved in each case, and in one case there were thirty. The greatest discrepancy was 20%. We therefore believe that the absolute cross sections quoted later are probably accurate within about 15%. The relative cross sections should be accurate within about 10%.

The energy scale was fixed at one point by calibrating the analyzer magnet with the narrow 3.47-Mev reso-

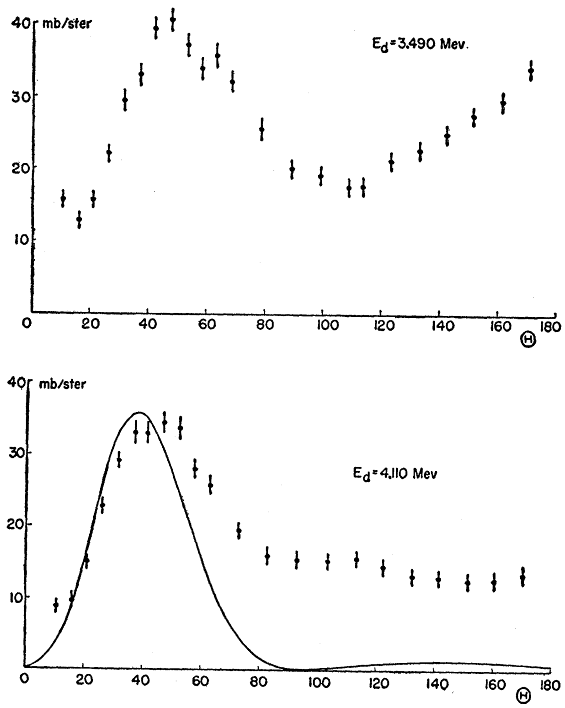

Fig. 2. Differential cross section in the center-of-mass system, $d\sigma/d\Omega$, of $O^{16}(d,p)O^{17}$ *vs* c.m. angle for E_d=3.490 and 4.110 Mev (lab). Solid curve, Butler distribution for E_d=3.62 Mev. (For other constants see text.)

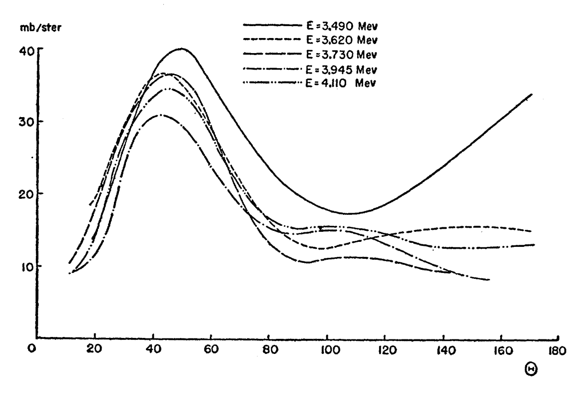

Fig. 3. Ground-state angular distribution of $O^{16}(d,p)O^{17}$ for five different energies. c.m. angles and cross sections, lab energies.

[10] G. A. Sawyer, Rev. Sci. Instr. 23, 604 (1952).
[11] K. Strohmaier, Z. Naturforsch. A6, 508 (1951).

nance in the elastic scattering of protons from O^{16}.[12] The analyzer magnet is of the wedge-shaped, uniform-field type, operating with B less than 8000 gauss, so one can safely assume a closely linear relationship between the momentum of the particles in the analyzed beam and the field strength at the position of the proton resonance probe used for field measurements. The energy scale is probably accurate to better than 1%.

III. RESULTS

(A) $O^{16}(d,p)$

Figure 1 shows $(d\sigma/d\Omega)$ in the center-of-mass system at 53.1° (center-of-mass angle) for the ground state and first excited state groups from the reaction $O^{16}(d,p)O^{17}$. The agreement with the results of Stratton *et al.*[3] in the overlap region, 3.48 to 3.85 Mev, is within the range of possible experimental errors quoted. At higher energies our curves show an additional maximum at 3.74 Mev for the ground-state group as well as a minimum at the same energy for the excited state group.

Figure 2 shows the angular distributions in the center-of-mass system of the ground-state protons for two deuteron energies, along with the prediction of Butler's stripping theory for parameters: $l_n=2$, $E_d=3.62$ Mev, $r_0=6.3\times10^{-13}$ cm. The peak in the stripping curve occurs at 42°. The experimental values for five energies (see Fig. 3) range from 42° to 48°. The peak cross sections are also fairly constant, ranging from 31 to 40 mb/sterad. The most obvious effect a change of energy has on the angular distribution is seen at the larger angles. The especially pronounced backward peak for $E_d=3.490$ Mev is similar to that reported by Stratton *et al.*[3] for $E_d=3.43$ Mev.

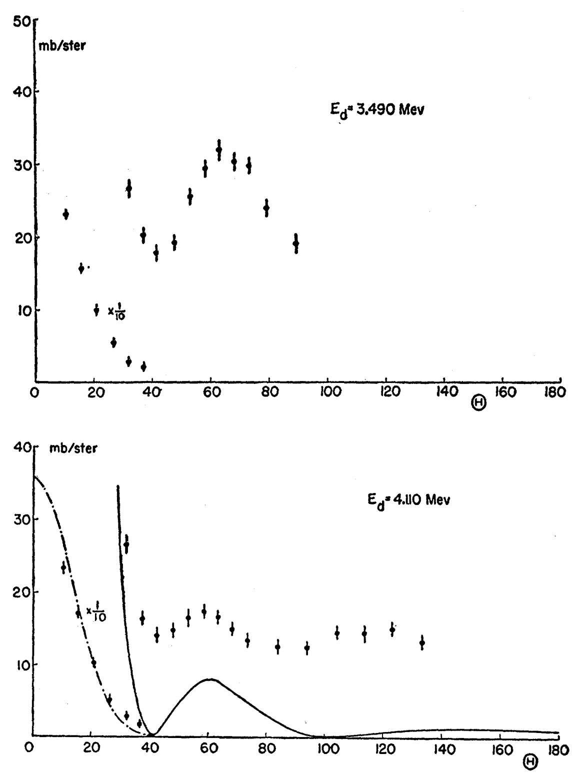

FIG. 4. Differential cross section in the center-of-mass system, $d\sigma/d\Omega$, of $O^{16}(d,p)O^{17*}$ *vs* c.m. angle for $E_d=3.490$ and 4.110 Mev (lab). Solid curve, Butler distribution for $E_d=3.62$ Mev.

[12] Laubenstein, Laubenstein, Koester, and Mobley, Phys. Rev. 84, 12 (1951).

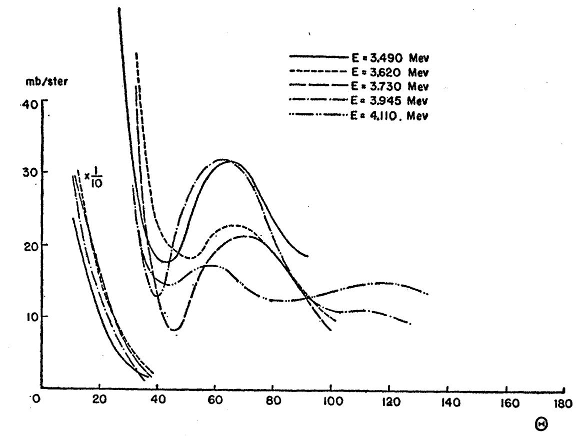

FIG. 5. First excited state angular distributions for five different energies. c.m. cross sections and angles, lab energies.

Figure 4 shows the center-of-mass angular distributions of the first excited state protons for two deuteron energies, along with the corresponding stripping theory curve ($l_n=0$, $E_d=3.62$ Mev, $r_0=6.3\times10^{-13}$ cm). The curves show the strong forward peak expected. The first minimum in the stripping curve appears at 42°. The experimental values for five energies (see Fig. 5) range from 40° to 50°. The first secondary maximum in the stripping curve appears at 66°. The experimental values range from 58° to 70°. The magnitude of the first secondary maximum is found to vary from 33 to 17.6 mb/sterad as E_d is changed from 3.945 to 4.110 Mev. Even at small angles, where stripping might be expected to be dominant, the cross section varies considerably with energy, changing at 10° from 320 mb/sterad for the middle three energies to 240 mb/sterad for the two extreme energies.

(B) $O^{16}(d,\alpha)$ Reaction

Figure 6 shows the variation in center-of-mass cross section of the (d,α) reaction (leading to the ground state of N^{14}) with deuteron energy for four angles. There is clear evidence for at least two resonances, one near 4.0 Mev having a width of 35 kev, the other near 3.85 Mev having a width of 100 kev, very roughly speaking. The curves also indicate that there are other, overlapping states involved. A satisfactory phase-shift analysis of the results using the single-level formula was clearly not possible, but an attempt was made in that

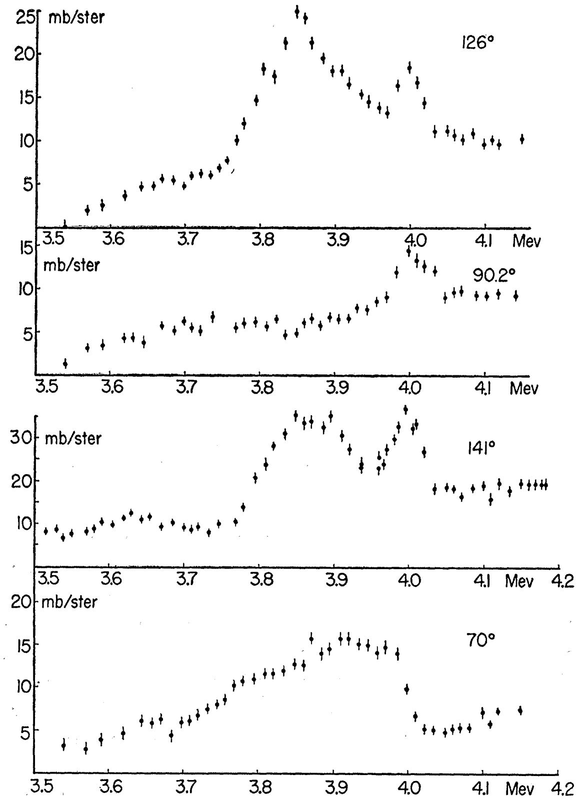

FIG. 6. Differential cross section in the center-of-mass system, $d\sigma/d\Omega$, for $O^{16}(d,\alpha)N^{14}$ *vs* lab energy for four different c.m. angles.

way to derive some information concerning the possible spins and parities for the levels near 3.85 and 4.0 Mev. The results were inconclusive.

(C) $O^{16}(d,d)$ Reaction

In Fig. 7 cross sections in the center-of-mass system are shown plotted against energy for five angles. Again a strong energy dependence is seen, but in this case a phase-shift analysis was even more difficult to carry out because of the additional potential phase shifts involved. An effort in that direction produced no worthwhile results.

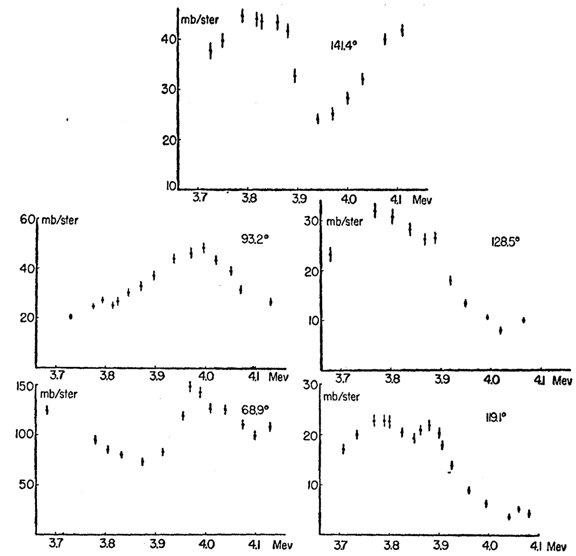

FIG. 7. Differential cross section in the center-of-mass system, $d\sigma/d\Omega$, for $O^{16}(d,d)O^{16}$ *vs* lab energy for five different c.m. angles.

IV. CONCLUSIONS

The peaks in the (d,α) reaction at energies of about 3.85 and 4.00 Mev indicate resonance levels in F^{18} at about 10.95 and 11.09 Mev. The broad underlying structure of the curves, especially at 70°, suggests that there are other, overlapping states involved. The (d,p) reactions deviate from stripping and show broad resonance effects with larger half widths than those of the (d,α) reaction. This may be due to the existence of overlapping states or to a mechanism different from compound nucleus formation.

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