

## Angular Distribution of Protons and Tritons from the Reactions $\text{Be}^9(d,p)\text{Be}^{10}$ and $\text{Be}^9(d,t)\text{Be}^8$ at Low Bombarding Energies\*

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The angular distributions of protons and tritons from the nuclear reactions  $\text{Be}^9(d,p)\text{Be}^{10}$  and  $\text{Be}^9(d,t)\text{Be}^8$  were measured for incident deuteron energies of 100 kev, 170 kev, and 200 kev. A nuclear plate camera was used in which the relative cross sections for protons and tritons could be measured at 36 different angles between  $15^\circ$  and  $165^\circ$ . The angular distributions of protons exhibit a minimum yield in the forward direction which increases to a maximum in the backward direction. The angular distributions of tritons also show a minimum cross section in the forward direction, but the maximum occurs between  $120^\circ$  and  $150^\circ$  and is quite broad. The maximum in the triton cross section increases in value relative to the cross section at  $90^\circ$  and moves toward the backward direction as the incident deuteron energy is decreased. This makes the angular distributions of protons and tritons much more alike in the case of 100-kev incident deuteron energy than for the 200-kev case. The angular distributions were analyzed in terms of the first three Legendre polynomials. Their possible relation to compound nucleus formation is discussed. The total cross section for the  $\text{Be}^9(d,p)\text{Be}^{10}$  reaction was found to be  $4.9 \pm 2$ ,  $16.0 \pm 4$ , and  $37.0 \pm 10 \mu\text{b}$  for the incident deuteron energies of 100, 170, and 200 kev, respectively. The ratio of the total cross section for tritons to the total cross section for protons was found to be 1.11, 0.97, and 0.91 for deuteron energies of 100, 170, and 200 kev, respectively.

### 1. INTRODUCTION

THE angular distributions of protons and tritons from the nuclear reactions  $\text{Be}^9(d,p)\text{Be}^{10}$  and  $\text{Be}^9(d,t)\text{Be}^8$  were measured by Resnick and Hanna<sup>1</sup> and

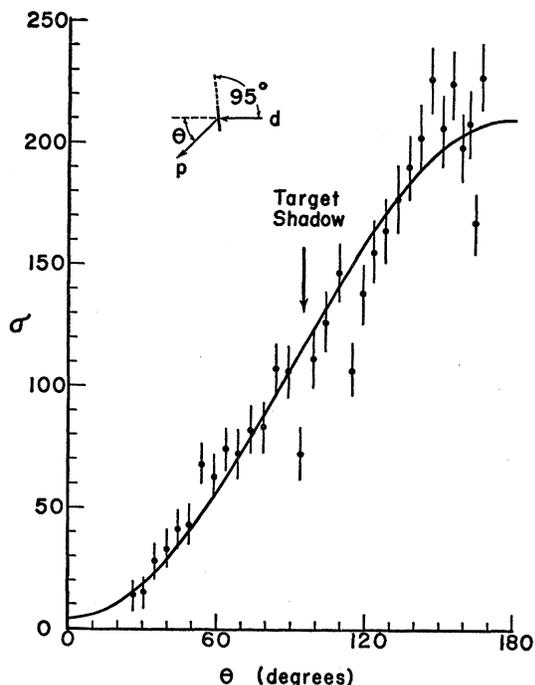


FIG. 1. Angular distribution of protons from the reaction  $\text{Be}^9(d,p)\text{Be}^{10}$  for an incident deuteron energy of 200 kev.

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<sup>1</sup> I. Resnick and S. S. Hanna, *Phys. Rev.* **82**, 463 (1951).

by DeJong, Endt, and Simons,<sup>2</sup> for incident deuteron energies between 300 kev and 880 kev. Although the curves fitted to their respective data by the two groups differ, especially in the case of the 300-kev incident deuteron energy, they are in fair agreement when one considers the relative magnitude of the errors involved. The angular distributions of both groups indicate a much larger yield of protons in the backward direction than in the forward direction at low incident energies. A similar but much less pronounced effect is observed for the tritons. The total cross sections for these two reactions was found to be approximately equal. Jurić<sup>3</sup> also investigated the angular distributions of these reactions for incident deuteron energies from 0.6 Mev to 1.2 Mev. His work also indicates this increased yield for protons in the backward direction. His angular distributions show much more fluctuation and variation with energy than do the corresponding angular distributions of DeJong, Endt, and Simons.<sup>2</sup>

In the present investigation, the above-mentioned angular distributions were studied at lower energies and in more detail. Incident deuteron energies of 100, 170, and 200 kev were used, and the relative cross sections were measured at 36 different angles between  $15^\circ$  and  $165^\circ$  (lab).

### 2. EXPERIMENTAL APPARATUS AND DATA

The accelerated deuteron beam was obtained from a small rf accelerator consisting of a single re-entrant cavity. The angular distributions were observed in a nuclear plate camera.<sup>4</sup> It consisted of nuclear emulsion photographic plates mounted on a plane just below the incoming deuteron beam and surrounding the beryllium

<sup>2</sup> DeJong, Endt, and Simons, *Physica* **18**, 407 (1952).

<sup>3</sup> M. K. Jurić, *Phys. Rev.* **98**, 85 (1955).

<sup>4</sup> The author is indebted to Dr. F. E. Steigert for the loan of the nuclear plate camera.

target on three sides. The protons and tritons produced by the nuclear reactions in the target entered the nuclear emulsions at angles between  $10^\circ$  and  $15^\circ$  with the surface of the emulsion. The angular resolution was  $2^\circ$  in the forward and backward directions, and  $4^\circ$  at  $90^\circ$  with respect to the incoming deuteron beam. The density of the nuclear tracks in the emulsions was measured with a projection system<sup>5</sup> which enlarged the tracks about a thousand times and projected them on a horizontal screen where they could be easily measured with a millimeter scale.

A spectrum of track lengths was taken at each of five different angles to insure the separation of the different groups from each other. Limits on acceptable track lengths were established as a function of angle. The

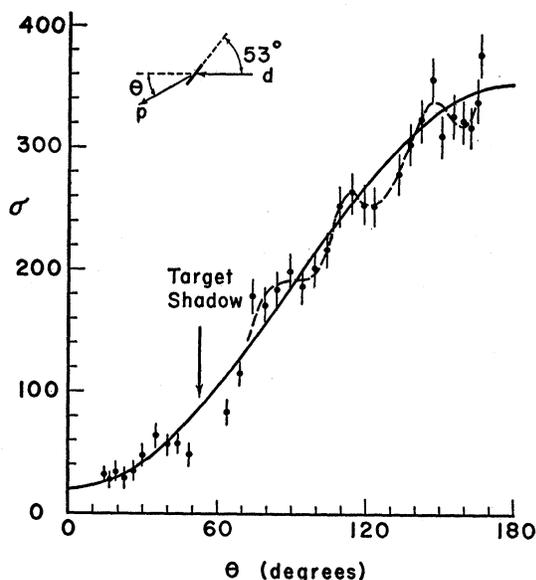


FIG. 2. Angular distribution of protons from the reaction  $\text{Be}^9(d,p)\text{Be}^{10}$  for an incident deuteron energy of 170 keV.

protons from the reaction  $\text{Be}^9(d,p)\text{Be}^{10}$  resulted in track lengths approximately three times as long as the triton tracks. The only competing reaction observed which gave rise to track lengths of comparable length was the  $\text{H}^2(d,p)\text{H}^3$  reaction resulting from the accumulation of deuterium on or in the beryllium target. Fortunately, at these low bombarding energies, the spectrum of track lengths of the protons resulting from the  $\text{H}^2(d,p)\text{H}^3$  reaction do not overlap either the proton or triton groups from the beryllium reactions. The angular distribution of the protons from the  $\text{H}^2(d,p)\text{H}^3$  reaction was measured and used as a check of the geometry as well as a check of the effective deuteron energy by comparing them with the previous work on this reaction.<sup>6-8</sup> The

<sup>5</sup> H. S. Plendl and F. E. Steigert, *Rev. Sci. Instr.* **27**, 239 (1956).

<sup>6</sup> Huntoon, Ellett, Bayley, and Van Allen, *Phys. Rev.* **58**, 97 (1940).

<sup>7</sup> Manning, Huntoon, Myers, and Young, *Phys. Rev.* **61**, 371 (1942).

<sup>8</sup> Bretscher, French, and Seidl, *Phys. Rev.* **73**, 815 (1948).

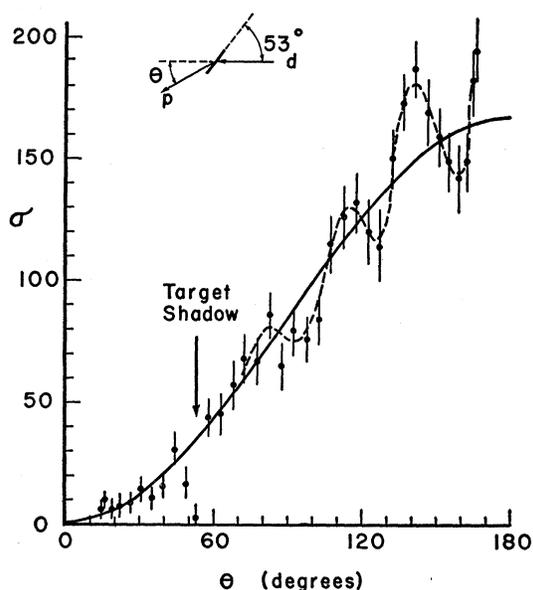


FIG. 3. Angular distribution of protons from the reaction  $\text{Be}^9(d,p)\text{Be}^{10}$  for an incident deuteron energy of 100 keV.

beryllium targets<sup>9</sup> used in this investigation were approximately 130 micrograms thick which corresponds to approximately 80-keV energy loss for both the 200-keV and 100-keV deuterons.<sup>10</sup> This plus the 10-keV energy spread in the accelerated deuterons results in a relatively large spread in the energy of the incident deuterons relative to the beryllium target nuclei. Total cross-section measurements indicated a decrease in yield by a factor of 7 as the beam energy decreased from 200 to 100 keV. This would weight the yield in favor of the high-energy component of the beam. Thus the 200-keV, 170-keV, and 100-keV incident deuteron beams might be considered to have an effective energy of 180 keV, 150 keV, and 90 keV, respectively. The 200-keV, 170-keV, and 100-keV incident deuteron energies mentioned above refer to the mean energy of the deuteron beam after acceleration and magnetic analysis.

The angular distribution of protons for incident deuteron energies of 200, 170, and 100 keV are shown in Figs. 1, 2, and 3 respectively. The angular distributions of tritons from the  $\text{Be}^9(d,t)\text{Be}^8$  reactions appear in Figs. 4, 5, and 6. The ordinate  $\sigma$  is a relative cross section per unit solid angle and has the same units in both the proton and triton data corresponding to the same incident deuteron energy. This allows the comparison of relative yields of protons and tritons at different angles.  $\theta$  is the angle between the direction of the incident deuteron beam and the out-going proton or triton. Both  $\sigma$  and  $\theta$  are in the center-of-mass system. A dip occurs in each of the angular distributions due to the interference of the edge of the target with the

<sup>9</sup> The beryllium foil for the targets was obtained from Dr. Hugh Bradner of the University of California at Berkeley.

<sup>10</sup> S. K. Allison and S. D. Warshaw, *Revs. Modern Phys.* **25**, 779 (1953).

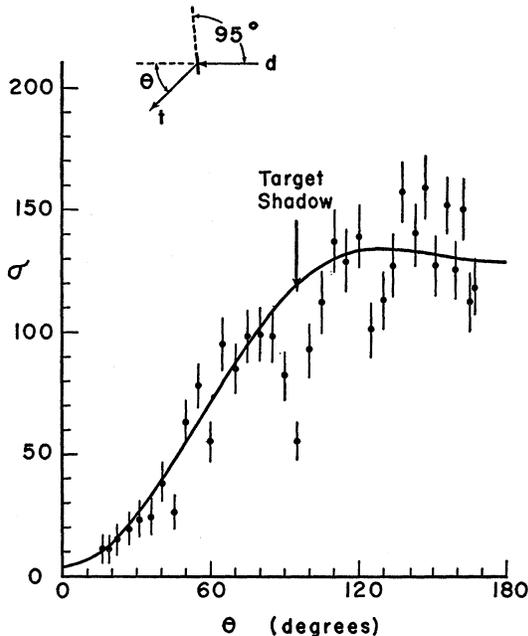


FIG. 4. Angular distribution of tritons from the reaction  $\text{Be}^9(d,t)\text{Be}^8$  for an incident deuteron energy of 200 keV.

protons or tritons emitted in that direction. This dip is indicated in the angular distributions by an arrow labeled "target shadow." The beryllium foil used as a target was supported on three sides only, the fourth side extending in the direction of the nuclear emulsions. This type of mounting reduced the width of the shadow. The effective width of the shadow is estimated to be approximately  $5^\circ$  to  $10^\circ$  wide, and therefore the points on the curve adjacent to the target shadow may be in considerable error. The vertical lines indicate the probable error derived from the square roots of the number of tracks observed plus an estimated error in the solid angle. The data have been corrected for small variations in the actual solid angle observed so that the probable error in the solid angle used in calculating  $\sigma$  is the order of 1%. The spectrum of track lengths of the tritons resulting from the  $\text{Be}^9(d,t)\text{Be}^8$  reaction has a low-energy tail, part of which was outside of the acceptance limits set for tritons. An estimated 2% of the tritons were lost in this manner and may cause some additional error in the triton angular distributions.

### 3. DISCUSSION

The proton angular distributions show a minimum yield in the forward direction and a maximum yield in the backward direction. The triton angular distributions exhibit a similar minimum in the forward direction, but the corresponding maximum in the backward direction is broader and much less pronounced. The three triton distributions show a tendency for the backward peaking to increase relative to the yield at  $90^\circ$  as

the deuteron bombarding energy is lowered. This trend is in good agreement with the work at higher energies.<sup>1,2</sup>

The angular distributions were analyzed in terms of the first three Legendre polynomials [ $P_0(\cos\theta)$ ,  $P_1(\cos\theta)$ ,  $P_2(\cos\theta)$ ] to obtain a smooth curve with the best fit to the experimental data. In the case of the proton angular distributions, less than 5% of the  $P_2$  polynomial was needed, so the proton data were reanalyzed using only the  $P_0$  and  $P_1$  polynomials. These fitted curves appear as solid lines superimposed on the experimental data. They are given by the relation:

$$\sigma(\theta) = A_0P_0(\cos\theta) + A_1P_1(\cos\theta) + A_2P_2(\cos\theta),$$

where the  $A$ 's are constants obtained in the fitting process. The ratios of the  $A$ 's to the appropriate  $A_0$ 's and the total cross sections for the two reactions are given in Table I. The relative values of the total cross sections are considerably more accurate than the absolute values. The ratio of the total cross section for tritons to the total cross section for protons is 0.91, 0.97, and 1.11 for the deuteron bombarding energies of 200, 170, and 100 keV, respectively. The approximate equality in total cross sections for the  $(d,p)$  and  $(d,t)$  reactions is not too surprising in light of their many similarities. The  $Q$  value for the  $\text{Be}^9(d,p)\text{Be}^{10}$  reaction is  $4.585 \pm 0.008$  MeV, and  $4.597 \pm 0.013$  MeV for the  $\text{Be}^9(d,t)\text{Be}^8$  reaction.<sup>11</sup> Both outgoing particle (protons and tritons) spins =  $\frac{1}{2}$  and both final nuclei ( $\text{Be}^{10}$  and  $\text{Be}^8$ ) have the same spin and parity ( $J=0^+$ ). The equality in cross section and similarity in angular distributions strongly suggests that similar processes are involved in both reactions.

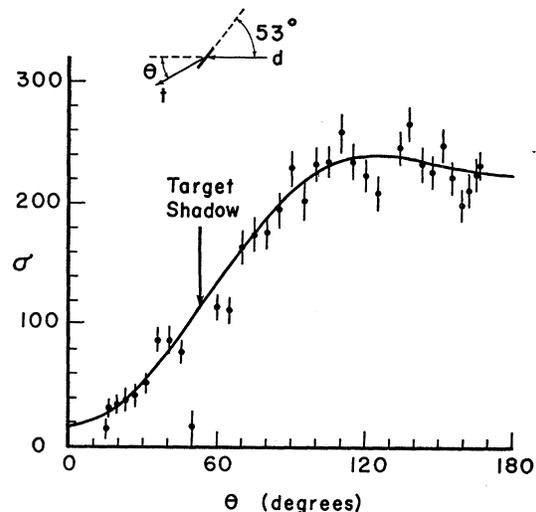


FIG. 5. Angular distribution of tritons from the reaction  $\text{Be}^9(d,p)\text{Be}^8$  for an incident deuteron energy of 170 keV.

<sup>11</sup> Strait, Van Patter, Buechner, and Speduto, Phys. Rev. **81**, 747 (1951); W. W. Buechner and E. N. Strait, Phys. Rev. **76**, 1547 (1949).

It is possible to explain these angular distributions in terms of compound nucleus formation. The angular dependence resulting from the formation of a single state in the compound nucleus will have the form<sup>12</sup>

$$\sigma(\theta) = \sum_{\nu} Z(JJ', S\nu) Z(J'J, S'\nu) P_{\nu}(\cos\theta), \quad (1)$$

where the  $Z$  coefficients are related to the Racah coefficients,<sup>13</sup> in which  $J$  is the spin of the compound state,  $l$  and  $l'$  are the orbital angular momentum while  $S$  and  $S'$  are the channel spins for the incoming particle and outgoing particle, respectively. The  $P_{\nu}(\cos\theta)$  are Legendre polynomials. If two resonances contribute to the cross sections at a particular energy, then the angular distribution will be the sum of the two individual angular distributions for the two states. The relative amplitudes of the two angular distributions will follow the same relation as their total cross sections. If the two resonances have the same entrance channel spin, then an interference term will occur. The angular dependence of the interference term is given by

$$\sigma_{12}(\theta) = \sum_{\nu} Z(l_1 J_1 l_2 J_2, S\nu) Z(l_1' J_1 l_2' J_2, S'\nu) P_{\nu}(\cos\theta), \quad (2)$$

where the subscripts denote the two resonances involved. The general form of the angular distribution is then given by

$$\sigma(\theta) = W_1(E)\sigma_1(\theta) + W_2(E)\sigma_2(\theta) \pm 2[W_1(E)W_2(E)]^{\frac{1}{2}} \cos\Psi \sigma_{12}(\theta),$$

where  $W_1(E)$  and  $W_2(E)$  are functions of the deuteron energy and contain the Breit-Wigner cross sections for the individual resonances,  $\sigma_1(\theta)$  and  $\sigma_2(\theta)$  are the angular dependence of the two resonances [Eq. (1)], and  $\sigma_{12}(\theta)$  is the angular dependence of their interference term [Eq. (2)].  $\Psi$  is given by

$$\Psi = \phi_1 - \phi_2 + \xi_1 - \xi_2 + \xi_1' - \xi_2',$$

where  $\phi = \tan^{-1}[\frac{1}{2}\Gamma/(E-E_0)]$ , and  $\xi$  and  $\xi'$  are the phase shifts of the incoming and outgoing channels. States with different parity produce odd Legendre

TABLE I. The total cross section and the ratio of the Legendre polynomial coefficients used in the series representation of the angular distributions.

Deuteron energy keV	Total cross section $\mu\text{b}$	$A_1/A_0$	$A_2/A_0$
<b>Be<sup>7</sup>(d, p)Be<sup>10</sup></b>			
100	4.9 ± 2	0.99 ± 0.04	0 ± 0.04
170	16 ± 4	0.89 ± 0.04	0 ± 0.04
200	37 ± 10	0.94 ± 0.04	0 ± 0.04
<b>Be<sup>9</sup>(d, t)Be<sup>10</sup></b>			
100	5.4 ± 2	0.70 ± 0.04	-0.19 ± 0.04
170	15 ± 4	0.58 ± 0.04	-0.33 ± 0.04
200	34 ± 10	0.63 ± 0.04	-0.31 ± 0.04

<sup>12</sup> J. M. Blatt and L. C. Biedenharn, Revs. Modern Phys. 24, 258 (1954).

<sup>13</sup> L. C. Biedenharn, Oak Ridge National Laboratory Report, ORNL-1501, 1953 (unpublished).

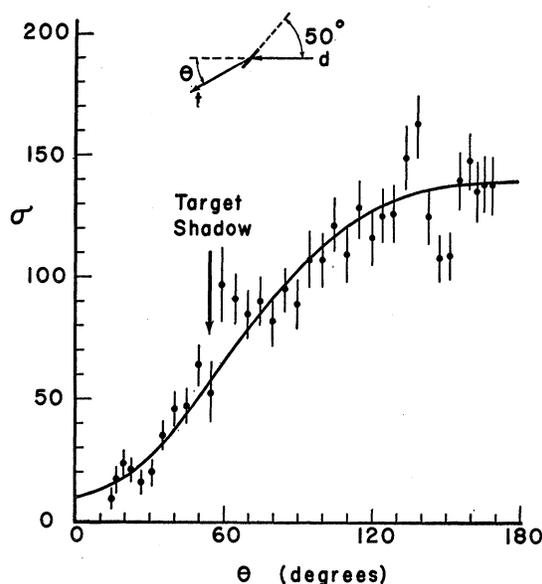


FIG. 6. Angular distribution of tritons from the reaction  $\text{Be}^9(d,t)\text{Be}^9$  for an incident deuteron energy of 100 keV.

polynomials while states with the same parity produce even Legendre polynomials in their interference terms.

The penetrability of 100-keV deuterons with orbital angular momentum of zero is 40 times larger than that for deuterons with  $l=1$  and 800 times larger than the penetrability of 100-keV deuterons with  $l=2$ .<sup>14</sup> Thus the penetrability factor strongly favors the formation of compound states resulting from deuterons with small orbital angular momentum. This and the fact that the two states must be of different parity in order to produce a  $P_1(\cos\theta)$  polynomial in the interference term suggest that  $l=0$  and  $l=1$  for the orbital momentum of the incoming deuterons. The penetration of a deuteron with  $l=0$  into the  $\text{Be}^9$  nucleus is ten times less probable at 100 keV and at 200 keV.<sup>14,15</sup> The seven-to-one ratio found in the total cross sections is therefore quite compatible with the suggestion of compound nucleus formation.

An analysis was made of the angular distributions obtained from all possible pairs of compound states formed by  $l=0$  and  $l=1$  deuterons. Two pairs were found that would fit the requirements,  $A_1/A_0 \approx 1$  and  $A_2/A_0 \approx 0$ , of the proton angular distributions. They were the combination of a  $\frac{1}{2}^-$  state with a  $\frac{1}{2}^+$  state and the combination of a  $\frac{3}{2}^-$  state with a  $\frac{1}{2}^+$  state formed with  $l=0$  and  $l=1$  deuterons, respectively.

Only one pair of compound states was found which would fulfill the requirements of  $A_1/A_0 \approx 0.7$  and  $A_2/A_0 \approx -0.2$  for the triton angular distributions. This pair was the combination of a  $\frac{3}{2}^-$  state and a  $\frac{3}{2}^+$  state formed with  $l=0$  and  $l=1$  deuterons, respectively. If one assumes that a small amount of  $P_2(\cos\theta)$  is also

<sup>14</sup> H. A. Bethe, Revs. Modern Phys. 9, 79 (1937).

present in the proton angular,  $A_2/A_0 \simeq -0.06$ , then it is possible to explain both reactions in terms of this pair of states in the compound nucleus. A similar analysis of all pairs of states formed from  $l=1$  and  $l=2$  deuterons lead to the same possible combinations of states in the compound nucleus.

*Note added in proof.*—The above discussion does not mean to preclude processes other than compound nucleus formations. It may be possible to explain these angular distributions in terms of other processes such as the direct interaction mechanism discussed by Owen and Madansky<sup>15</sup> for the reaction  $B^{11}(d,n)C^{12}$ .

The proton angular distribution at 100 keV suggests

<sup>15</sup> G. E. Owen and L. Madansky, *Phys. Rev.* **105**, 1766 (1957).

some fine structure superimposed on the smooth curve (see dashed curve in Fig. 3). Both the experimental data and the solid angle corrections were rechecked but no source of systematic error could be found. A similar effect is present in the proton angular distribution at 170 keV but there the amplitude is much smaller and not too significant by itself.

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## Deuteron-Proton Scattering at 11.7 Mev\*†

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Absolute differential scattering cross sections for deuteron-proton scattering at an incident deuteron energy of  $11.7 \pm 0.15$  Mev have been obtained by using Ilford C-2 nuclear plates as detectors, with an accuracy of about 1% at a number of angles from  $70^\circ$  to  $150^\circ$  in the center-of-mass system. The results appear to be in good agreement with the cross sections obtained by other experiments in this energy range.

**I**N the low-energy range (0–20 Mev), deuteron-proton scattering has been investigated by a number of investigators,<sup>1–6</sup> and a theoretical analysis in this energy region has been given by Christian and Gammel.<sup>7</sup> The present paper adds to the previous low-energy results by giving the experimental cross sections obtained from deuteron-proton scattering data recorded photographically at an incident deuteron energy of  $11.7 \pm 0.15$  Mev.

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<sup>1</sup> Sherr, Blair, Kratz, Bailey, and Taschek, *Phys. Rev.* **72**, 662 (1947).

<sup>2</sup> Rodgers, Leiter, and Kruger, *Phys. Rev.* **78**, 656 (1950).

<sup>3</sup> Karr, Bondelid, and Mather, *Phys. Rev.* **81**, 37 (1951).

<sup>4</sup> L. Rosen and J. C. Allred, *Phys. Rev.* **82**, 777 (1951).

<sup>5</sup> Brown, Freier, Holmgren, Stratton, and Yarnell, *Phys. Rev.* **88**, 253 (1952).

<sup>6</sup> Allred, Armstrong, Bondelid, and Rosen, *Phys. Rev.* **88**, 433 (1952).

<sup>7</sup> R. S. Christian and J. L. Gammel, *Phys. Rev.* **91**, 100 (1953).

#### APPARATUS AND PROCEDURE

The photographic scattering chamber, originally constructed by Rodgers,<sup>2</sup> Leiter,<sup>8</sup> and Meagher,<sup>9</sup> and modified by Kreger<sup>10</sup> and Kerman,<sup>11</sup> was employed. The construction and operation of the chamber and its vacuum and gas filling system, the details of the three annular slits used, and the method of measuring total charge are all described by Kreger.<sup>10</sup> Hydrogen gas was admitted to the chamber through a heated palladium thimble to a pressure of about five cm of Hg. Liquid nitrogen traps were not used on the system. Six 50-micron Ilford C-2 nuclear plates (mounted around the periphery of the scattering chamber) recorded recoil protons and scattered deuterons which were able to reach the plates by passing through a narrow annular slit. The 30 nuclear plates resulting from five different runs were analyzed using a Spencer binocular micro-

<sup>8</sup> Leiter, Rodgers, and Kruger, *Phys. Rev.* **78**, 663 (1950).

<sup>9</sup> R. E. Meagher, *Phys. Rev.* **78**, 667 (1950).

<sup>10</sup> Kreger, Jentschke, and Kruger, *Phys. Rev.* **93**, 837 (1954).

<sup>11</sup> Zimmerman, Kerman, Singer, Kruger, and Jentschke, *Phys. Rev.* **96**, 1322 (1954).