TABLE I. Percent difference in transmission through magnetized iron for annihilation quanta above 2 Mev.

	Calc	Ga^{66} $E_\beta=3$ Mev	Ga^{66} $E_\beta=2.6$ Mev	Cl^{34} $E_\beta=3$ Mev
Thick analyzer	8.8±1.0	8.4±0.5	8.8±1.0	5.6±2.1
Thin analyzer	4.4±1.5	4.9±0.9	···	5.4±1.3

available source strength was smaller by about a factor five. The short half-life prevented a complete cycle of check runs with each source. The gamma-ray background was relatively higher, especially for smaller pulse heights. The results are summarized in Table I, which presents the average values of δ for counter pulses above 2 Mev. The uncertainty in the theoretical value reflects our uncertainty concerning the magnetic flux distribution in the analyzer.

Although first results of a new method should be accepted with some caution, our experiment shows convincingly that the positrons from Ga^{66} are highly polarized with positive $\boldsymbol{\sigma}\cdot\mathbf{p}$. The results suggest complete polarization. They contradict a recent scattering experiment.[5] While they remove the only strong argument for a basic difference in the behavior of Gamow-Teller and Fermi transitions, no affirmative conclusions can be drawn from the Ga^{66} data until the parity of this nuclide is established with certainty. The results for Cl^{34} indicate that positrons from a pure Fermi transition are also predominantly polarized with positive $\boldsymbol{\sigma}\cdot\mathbf{p}$. The degree of polarization remains uncertain. Our data are compatible with complete polarization and make it quite improbable that the value is near zero.

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Coulomb Scattering of Polarized Electrons*

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EXPERIMENTS carried out with longitudinally polarized β beams have focused attention on the Coulomb scattering of polarized electrons. The final-state density matrix can be calculated by means of the scattering matrix M which in turn depends on two complex functions F and G (first introduced by Mott) that have been tabulated for a number of values of $\alpha=Z/137$, $\beta=v/c$, and θ (scattering angle) by Bartlett and Watson,[1] McKinley and Feshbach,[2] Sherman,[3] and others. In this note, the amount of longitudinal depolarization L and the asymmetry ratio A for a β beam are expressed exactly in terms of F and G. Approximate analytical expressions for L and A (corresponding to the second-order Born approximation) are also derived. The results are applied to a double-scattering experiment performed by de-Shalit *et al.*[4]

Denoting the unit vectors of the momenta of the initial and scattered electrons by $\mathbf{n}_1$ and $\mathbf{n}_2$ and the unit vector normal to the plane of scattering by $\mathbf{n}$, we can write the scattering matrix in the form

$$M=f-ig\mathbf{n}\cdot\boldsymbol{\sigma}=\lambda[G\sec(\tfrac{1}{2}\theta)-i\mathbf{n}\cdot\boldsymbol{\sigma}F'\csc(\tfrac{1}{2}\theta)]\times\exp(-\tfrac{1}{2}i\mathbf{n}\cdot\boldsymbol{\sigma}\theta),\quad(1)$$

where

$$F'=iq(1-\beta^2)^{\frac{1}{2}}F,\quad\text{with}\quad q=\alpha/\beta.$$

If $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ are the expectation values of the initial and final spin vectors of the electron in its rest frame (see Tolhoek[5] whose notations we follow closely), the final state density matrix is given by

$$\rho_2=\tfrac{1}{2}I(1+\boldsymbol{\zeta}_2\cdot\boldsymbol{\sigma})=\tfrac{1}{2}M(1+\boldsymbol{\zeta}_1\cdot\boldsymbol{\sigma})M^\dagger,\quad(2)$$

where I is the differential cross section $d\sigma/d\Omega$. In the case of an initial beam with longitudinal polarization of degree $P_1=|\boldsymbol{\zeta}_1|$, we have $\boldsymbol{\zeta}_1=P_1\mathbf{n}_1$, and, using (1) and (2),

$$\boldsymbol{\zeta}_2=S(\theta)\mathbf{n}+P_1U(\theta)\mathbf{n}_2+P_1T(\theta)\mathbf{n}\times\mathbf{n}_2,\quad(3)$$

where

$$\bar{I}(\theta)S(\theta)=-2i(F'G^*-GF'^*)\csc\theta,\quad(4a)$$

$$\bar{I}(\theta)T(\theta)=2(F'G^*+GF'^*)\csc\theta,\quad(4b)$$

$$\bar{I}(\theta)U(\theta)=|G|^2\sec^2(\tfrac{1}{2}\theta)-|F'|^2\csc^2(\tfrac{1}{2}\theta),\quad(4c)$$

$$\bar{I}(\theta)=|G|^2\sec^2(\tfrac{1}{2}\theta)+|F'|^2\csc^2(\tfrac{1}{2}\theta).\quad(4d)$$

We note the indentity $S^2+T^2+U^2=1$. For arbitrary initial polarization $\boldsymbol{\zeta}_1$, the scattered intensity is

$$I(\theta)=\mathrm{Tr}\rho_2=\lambda^2\bar{I}(\theta)[1-\boldsymbol{\zeta}_1\cdot\mathbf{n}S(\theta)].\quad(5)$$

$S(\theta)$ is the asymmetry function tabulated by Sherman.[3] $U(\theta)$ measures the longitudinal polarization after scattering since $P_1U(\theta)=\boldsymbol{\zeta}_2\cdot\mathbf{n}_2$. The longitudinal depolarization is therefore $L=P_1(1-U)$. Since $\mathbf{n}\times\mathbf{n}_2=(\mathbf{n}_2\cos\theta-\mathbf{n}_1)\csc\theta$, we also find

$$\boldsymbol{\zeta}_2\cdot\mathbf{n}_1=P_1[U(\theta)\cos\theta-T(\theta)\sin\theta].\quad(6)$$

for the remaining initial polarization in the direction $\mathbf{n}_1$. If the emerging beam (with polarization $\boldsymbol{\zeta}_1'=\boldsymbol{\zeta}_2$) is further scattered through an angle θ' and plane of scattering $\mathbf{n}'$, Eq. (5) gives

$$I(\theta')=\bar{I}(\theta')[1-\boldsymbol{\zeta}_1'\cdot\mathbf{n}'S(\theta')],$$

TABLE I. Asymmetry ratios, $-200A/P_1$. (Right-angle scattering on Al, followed by scattering through θ' on Au.)

E_K (Mev)	β	$\theta'=60°$	$\theta'=75°$	$\theta'=90°$
0.975	0.94	4.9	10.2	16.5
1.15	0.95	4.6	8.2	13.4
1.30	0.96	3.8	6.9	11.8
1.57	0.97	2.8	5.0	8.8

leading to the usual right-left asymmetry ratio

$$A=(I_R-I_L)/(I_R+I_L)=\boldsymbol{\zeta}_2\cdot\mathbf{n}'S(\theta'), \qquad (7)$$

where I_R and I_L are intensities in opposite final directions $\mp\mathbf{n}_2'$, and $\boldsymbol{\zeta}_2$ is given by (3), so that we find

$$A=S(\theta)S(\theta')\mathbf{n}\cdot\mathbf{n}'+P_1T(\theta)S(\theta')(\mathbf{n}\times\mathbf{n}_2)\cdot\mathbf{n}'. \qquad (8)$$

In the special case of successive right-angle scatterings with $\mathbf{n}_1\times\mathbf{n}_2=\mathbf{n}_2'$, Eq. (8) reduces to

$$A=-P_1T(\tfrac{1}{2}\pi)S(\tfrac{1}{2}\pi).$$

We now give approximate formulas in the case of small Z and large v. From reference 2 and Curr,[6] we find

$$F'=-\tfrac{1}{2}q(1-\beta^2)^{\frac{1}{2}}[1+2i\alpha\beta^{-1}(\gamma+\ln\sin\tfrac{1}{2}\theta)], \qquad (9)$$

$$G=\tfrac{1}{2}q\,\{\cot^2(\tfrac{1}{2}\theta)+\tfrac{1}{2}\alpha\beta\pi[\csc(\tfrac{1}{2}\theta)-1]+i\alpha[2\beta^{-1}\times\cot^2(\tfrac{1}{2}\theta)(\gamma+\ln\sin\tfrac{1}{2}\theta)-\tfrac{1}{2}\beta\ln\csc^2(\tfrac{1}{2}\theta)]\}, \qquad (10)$$

where γ is the Euler constant. To the first order in α, we obtain

$$S(\theta)=-[R(\theta)]^{-1}\alpha\beta(1-\beta^2)^{\frac{1}{2}}\tan\tfrac{1}{2}\theta\times\sin^2(\tfrac{1}{2}\theta)\ln\csc^2(\tfrac{1}{2}\theta), \qquad (11a)$$

$$T(\theta)=-[R(\theta)]^{-1}(1-\beta^2)^{\frac{1}{2}}\tan\tfrac{1}{2}\theta[1+\cos\theta+\alpha\beta\pi\sin\tfrac{1}{2}\theta(1-\sin\tfrac{1}{2}\theta)], \qquad (11b)$$

$$1-L(\theta)/P_1=U(\theta)=[R(\theta)]^{-1}[\cos\theta+\beta^2\sin^2(\tfrac{1}{2}\theta)+\alpha\beta\pi\sin\tfrac{1}{2}\theta(1-\sin\tfrac{1}{2}\theta)], \qquad (11c)$$

$$R(\theta)=1-\beta^2\sin^2(\tfrac{1}{2}\theta)+\alpha\beta\pi\sin(\tfrac{1}{2}\theta)[1-\sin(\tfrac{1}{2}\theta)]. \qquad (11d)$$

$R(\theta)$ is the ratio of relativistic to Rutherford cross section first calculated correctly in reference 2. The expression for $U(\theta)$ shows that the longitudinal polarization of the electron is not changed in forward scattering, that it is reversed in backward scattering ($\theta=\pi$), and that for right-angle scattering $\boldsymbol{\zeta}_2\cdot\mathbf{n}_2$ decreases from P_1 to 0 as β decreases from 1 to 0.

In a recent experiment performed by de-Shalit *et al.*,[4] an initially longitudinally polarized electron beam was first scattered through a right angle from aluminum and then through an angle θ' from gold ($65°<\theta'<90°$). The intensities were measured in directions symmetrical with respect to the first scattering plane such that $\mathbf{n}_2'\cdot\mathbf{n}=\pm\sin 65°$. In this case the first term in (8) is negligible and we find approximately

$$2A=2(I_R-I_L)/(I_R+I_L)=-2P_1T(\tfrac{1}{2}\pi)S(\theta')\times\sin 65°/\sin\theta'. \qquad (12)$$

Since the kinetic energy of the electrons accepted in this experiment varies between 0.9 and 1.7 Mev, some values for $2A/P_1$ are shown in Table I. A weighted average kinetic energy is about $\bar{E}_K=1.15$ Mev. If we choose $\bar{\theta}'=77.5°$ and interpolate, we find $2A=9\%$ (taking $P_1\simeq\beta\simeq1$) instead of the experimental $(5.1\pm0.6)\%$. By considering plural scattering in each of the scatterers, one finds a strong over-all reduction in asymmetry, giving a probable explanation of the discrepancy.

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Note added in proof.—Since completion of this work the attention of the author has been drawn to the fact that a similar calculation for the asymmetry ratio A in double scattering was also made by L. J. Tassie.[7]

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