

characterizes the beam resonance. The size of each port is characterized by a Q which is the ratio of the energy stored in the cavity to the energy per radian dissipated (or generated, in the case of the beam) via the mechanism being represented. For high gain the effective noise temperature, T_N , is given approximately by $T_N \cong (T_c/Q_0 + |T_B|/Q_B)(1/Q_B - 1/Q_0)^{-1}$. The value of T_B obtained from calculations involving the focuser voltages, geometry, and solid angles is $-1/4^\circ\text{K}$. The measured value of Q_0/Q_B is of the order of $5\frac{1}{2}$. Thus $|T_B|Q_0/(T_cQ_B) \ll 1$ and $T_N \cong T_c/[(Q_0/Q_B) - 1]$. In this case $T_c = 300^\circ\text{K}$, $Q_0/Q_B = 5.44$, and the predicted effective-noise temperature was $68 \pm 4^\circ\text{K}$, where the uncertainty results from the errors in the measurements leading to the value for Q_0/Q_B .

A second measurement yielded $72 \pm 15^\circ\text{K}$ for the effective noise temperature. The predicted value was $64 \pm 5^\circ\text{K}$.

The agreements between the experimental and theoretical values are good. The contribution to the noise from the beam was too small to be measured, but from these results an upper limit of about 20°K can be placed on the absolute value of the effective beam temperature.

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Angular Correlations between β Rays and Circularly Polarized γ Rays in Triple Cascade Transitions*

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THE angular correlations between beta rays and circularly polarized gamma rays for triple cascade transitions may furnish information concerning the relative magnitudes and phases of the ten coupling constants, C_i and C'_i , ($i=S, V, T, A, P$), of beta interactions. Therefore, we have calculated these angular correlations.

Since the various angular correlations using the old theory of beta decay, where the C'_i 's are zero, have already been given for triple cascade transitions,¹ the formulas of I can be extended, by slight modifications, to include the parity-nonconserving beta interactions and the circular polarization of the gamma rays.

Since the parity condition¹ is now dropped, the even integer $2n$ in the equations of I should be replaced by the integer n , and simultaneously, $p^{\delta+\delta'+L+L'+n}$, which gives the dependence on the circular polarization of the gamma rays, should be inserted as a multiplicative factor in some of the equations. Here, p is $+1$ (-1) for left (right) circularly polarized gamma rays; δ is equal to 0 ($+1$) for the magnetic (electric) 2^L -pole radiation. The other symbols in this letter are the same as those used in I. The decay scheme is assumed to be $j-\beta \rightarrow j_1 - \gamma_1 \rightarrow j_2 - \gamma_2 \rightarrow j_3$. We shall consider two cases.

(1) Angular correlation between β and circularly polarized γ_1 without observing γ_2 .²

$$W(\theta, p_1; \beta - \gamma_1) = \sum_n \left\{ \left[\sum_{L \leq L'} (-1)^{i_1 - i_b} b_{LL'}^{(n)} W(j_1 j_1 LL'; n j) (2j_1 + 1)^{\frac{1}{2}} \right] \times \left[\sum_{L_1 L_1'} (-1)^{L_1 + L_1'} p_1^{\delta_1 + \delta_1' + L_1 + L_1' + n} (j_1 \| L_1 \| j_2) \times (j_1 \| L_1' \| j_2) F_n(L_1 L_1' j_2 j_1) \right] \right\} P_n(\cos \theta),$$

with

$$F_n(LL' j_a j_b) = F_n(L' L j_a j_b) = (-1)^{i_b - i_a - 1} [(2j_b + 1)(2L + 1)(2L' + 1)]^{\frac{1}{2}} \times (LL' 1 - 1 | n 0) W(j_b j_b LL'; n j_a). \quad (1)$$

(2) Angular correlation between β and circularly polarized γ_2 without observing γ_1 .

$$W(\theta, p_2; \beta - \gamma_2) = \sum_n \left\{ \left[\sum_{L \leq L'} (-1)^{i_1 - i_b} b_{LL'}^{(n)} W(j_1 j_1 LL'; n j) (2j_1 + 1)^{\frac{1}{2}} \right] \times \left[\sum_{L_1} (j_1 \| L_1 \| j_2)^2 W(j_1 n L_1 j_2; j_1 j_2) \times ((2j_1 + 1)(2j_2 + 1))^{\frac{1}{2}} \left[\sum_{L_2 L_2'} (-1)^{L_2 + L_2'} \times p_2^{\delta_2 + \delta_2' + L_2 + L_2' + n} (j_2 \| L_2 \| j_3) (j_2 \| L_2' \| j_3) \times F_n(L_2 L_2' j_3 j_2) \right] \right] \right\} P_n(\cos \theta). \quad (2)$$

In Eqs. (1) and (2), we assumed that the strong interactions are invariant under time reversal.³ If this is not valid, the formulas should be modified as follows. For example,

$$2 \operatorname{Re}(C_T^* C_S') M_{GT}^* M_F (j_1 \| L_1 \| j_2) (j_1 \| L_1' \| j_2)$$

should be replaced by

$$C_T^* C_S' M_{GT}^* M_F (j_1 \| L_1 \| j_2)^* (j_1 \| L_1' \| j_2) + \text{c.c.}$$

for electron decay, and by

$$C_T^* C_S' M_{GT} M_F^* (j_1 \| L_1 \| j_2)^* (j_1 \| L_1' \| j_2) + \text{c.c.}$$

for positron decay, and so on. The $b_{LL'}^{(n)}$'s depend on

the beta interactions⁴ and are defined by Eqs. M(3) and M(4) in I. They are given explicitly in reference 2 for allowed transitions. Therefore we shall not rewrite them here. For forbidden transitions the $b_{LL', (n)}$'s can be derived by similar methods.^{5,6} The value of n is at most $2m+1$ for the m th forbidden transition. Other restrictions on n determined by the spin values of the nuclear levels and the multipolarities of gamma rays are explicitly shown in the Racah coefficients of Eqs. (1) and (2).

In the case of special decay schemes such as $j-2L-L-0$ with pure electric and/or magnetic 2^L -pole gamma rays (for example, $j-4-2-0$ with pure quadrupole gamma rays), the angular correlation between beta and circularly polarized γ_1 rays is equal to that between beta and circularly polarized γ_2 rays. This equality also holds in the case where the circular polarization of the gamma rays is not observed, and in the case of gamma-ray angular distributions from oriented nuclei in similar decay schemes.

The angular correlation functions of $\beta-\gamma_1-\gamma_2$ with simultaneous observation of three particles can be also derived from Eqs. (2)-(4) of I.

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³ Assume the invariance of strong interactions under time reversal. The reduced matrix elements of beta decay are divided into two classes: the reduced matrix elements belonging to the same class have the same phase (or a phase difference of π), and the phase difference between those belonging to different classes is $\pi/2$ (or $3\pi/2$) (see reference 6). For the allowed transitions,

$$\int \beta \sigma / \int \beta = \mathfrak{M}(\beta \sigma) / \mathfrak{M}(\beta) = M_{GT} / M_F = \text{real number,}$$

which may be plus or minus. The reality of the reduced matrix elements for gamma transitions was shown by S. P. Lloyd, Phys. Rev. **81**, 161 (1951).

⁴ The parameters $b_{LL', (n)}$ for beta rays are very convenient for expressing various formulas in beta decay. For example, the beta-ray angular distribution from oriented nuclei is simply expressed by

$W(\theta; \beta) = \sum_n \hat{f}_n(j) (-)^{i_1 - i_1 + L + L' + n} b_{LL', (n)} W(jjLL'; n j_1) P_n(\cos \theta)$, with $\hat{f}_n(j) = \sum_m (-)^{i - m} (jjm - m | n 0) a_m$, where the a_m 's are the relative populations of the initial magnetic substates. As is well known, its anisotropy can appear only from polarized nuclei in the case of allowed transitions, because the $\hat{f}_n(j)$ with odd n vanishes except for polarized nuclei, and the maximum value of n is one in allowed transitions.

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Relativistic Wave Equations for Spin-2 Particles with Unique Mass

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IT has been shown by several authors that relativistic wave equations for higher spin particles can be written as generalized Dirac equations,

$$(\alpha^k \partial_k + \chi) \psi = 0. \quad (1)$$

Such field equations have the advantage of making no use of subsidiary conditions, the handling of which is rather cumbersome, when interactions are introduced (see Bhabha¹).

If (1) is required to represent a particle with unique mass, all components of ψ must satisfy second-order wave equations,

$$(\partial^k \partial_k - K^2) \psi = 0, \quad (2)$$

with the same mass constant K . However, as shown by Bhabha¹ and Harish-Chandra,² it is impossible to fulfill this condition with Hermitian matrices for any spin higher than 1, if χ is a constant times the unit matrix. This form of χ is, however, not the most general one, and, for example, in the case of spin 1 it is too simple for the representation of the Maxwell equations.³

The present authors have investigated the case that is probably most interesting physically, namely, the irreducible spin-2 case, where the components of ψ consist of the components of a symmetrical tensor A_{kl} and of a vector A_k . The 14-row α 's of this case have been explicitly represented by means of the subspace method, given by Klein.^{4,5} The most general form of χ is proved to be an arbitrary linear combination of three commuting idempotents, thus containing three arbitrary constants, k_1 , k_2 , and k_3 . The components of (1) are found to be

$$\partial_k A_l + \partial_l A_k = ik_1 A_{kl} + \frac{1}{4} i (k_2 - k_1) A_m{}^m \delta_{kl}, \quad (3)$$

$$\partial_k A_m{}^m + \partial_m A_k{}^m = -ik_3 A_k. \quad (4)$$

Upon solving (3) with respect to A_{kl} , inserting in (4), and putting $k_2 = -5k_1$, it is seen that all components of A_k and thus of A_{kl} satisfy (2) with $K^2 = k_1 k_3$. Consequently it is *not* generally impossible to satisfy the requirement of unique mass for spin higher than 1.

If furthermore the equations are specialized to zero mass by putting $k_3 = 0$, we obtain

$$\partial^m \partial_m \gamma_{kl} = 0, \quad (5)$$

$$\partial^m \gamma_{mk} = 0, \quad (6)$$

where

$$\gamma_{kl} = A_{kl} + A_m{}^m \delta_{kl}. \quad (7)$$