PHYSICAL REVIEW VOLUME 107, NUMBER 6 SEPTEMBER 15, 1957

Pion Contribution to Hyperon-Nucleon Forces*

DON B. LICHTENBERG† AND MARC H. ROSS
Indiana University, Bloomington, Indiana
(Received May 24, 1957)

A calculation is made of the forces between hyperons and nucleons under the assumption that the forces are due to the exchange of pions. The Λ and Σ are assumed to have spin $\frac{1}{2}$ and the same parity. The Hamiltonian is taken to be the same as the static-model pion-nucleon Hamiltonian except for differences in isotopic spin and possibly in the strength of the coupling. The same approximations are made as those ordinarily made in evaluating the nucleon-nucleon potential, although it is noted that in case all the pion-baryon coupling constants are equal, the hyperon-nucleon potentials are just linear combinations of the observed nucleon-nucleon potentials. The pion-hyperon coupling constant is determined by comparison with potentials obtained from analysis of hyperfragment data. The resulting value is about the same as the pion-nucleon constant. The elastic hyperon-nucleon and $\Lambda+N\leftrightarrow\Sigma+N$ scattering cross sections are evaluated. Results are consistent with experiment.

I. INTRODUCTION

THE fact that the Λ hyperon can be bound in nuclear matter is evidence that there are strong forces between the Λ and the nucleon. It is plausible that these forces may be due in large part to the exchange of pions. In this paper we adopt the hypothesis that Λ and Σ hyperons interact strongly with nucleons via the pion field. With certain simplifying assumptions, we are able to evaluate Λ-nucleon and Σ-nucleon potentials and to calculate the resulting Λ-nucleon and Σ-nucleon elastic and inelastic scattering cross sections. We briefly consider the experimental data on the hyperon-nucleon interaction such as hyperfragment binding energies, the scattering of charged hyperons in bubble chambers and emulsions, and the scattering of hyperons in the nuclear matter in which they are produced.

We need to make certain assumptions about spins, parities, and couplings, in order to perform the calculation. In a previous work[1] (to be denoted by I) we assumed a simple model to describe the interaction between pions and hyperons.[2] In constructing this model we assumed that hyperons react with pions essentially as nucleons do except for the necessary differences in isotopic spin. We assumed that the elementary pion-hyperon interactions are

$$\Lambda\leftrightarrow\Sigma+\pi, \tag{1}$$

and

$$\Sigma\leftrightarrow\Lambda+\pi, \tag{2}$$

the process

$$\Lambda\leftrightarrow\Lambda+\pi$$

being forbidden because it does not conserve isotopic spin. Subsequently[3] we postulated the existence of the interaction

$$\Sigma\leftrightarrow\Sigma+\pi, \tag{3}$$

assuming it to be governed by the same coupling constant as processes (1) and (2). Since we wish the theory to be charge-independent, we require that the Hamiltonian be invariant under rotations in isotopic spin space. The form of the Hamiltonian in isotopic spin space is then unique. Since the pion field $\mathbf{\Phi}_\pi$ and the Σ field $\mathbf{\Psi}_\Sigma$ are vectors in isotopic spin space, while the Λ field ψ_Λ is a scalar, the interactions (1) and (2) must have the following form in isotopic spin space:

$$\Psi_\Lambda{}^*\mathbf{\Psi}_\Sigma\cdot\mathbf{\Phi}_\pi. \tag{4}$$

Similarly, for interaction (3), the Hamiltonian must be of the form

$$\mathbf{\Psi}_\Sigma{}^*\times\mathbf{\Psi}_\Sigma\cdot\mathbf{\Phi}_\pi. \tag{5}$$

We also assumed in I that the spins of the Λ and Σ hyperons are $\frac{1}{2}$. The experimental results seem consistent with spin $\frac{1}{2}$ but are by no means conclusive especially for the Σ.[4] We prefer to retain the assumption of spin $\frac{1}{2}$ as being the simplest. We assume that the hyperons remain fixed during their interaction with pions (static baryon model), and that the interaction proceeds via gradient coupling. We do not require knowledge of the parity of hyperons relative to nucleons but do require the assumption that the parity of the Λ and Σ be the same.

We can find a reasonable relation between the coupling constants of the Hamiltonians for processes (1), (2), and (3), thereby reducing the ambiguity in the interaction to a single parameter. In I, we did this by resolving the hyperon fields in the Hamiltonian into a spin $\frac{1}{2}$, isotopic spin $\frac{1}{2}$ "nucleon" field with which the pion interacts, and another field carrying spin zero and isotopic spin $\frac{1}{2}$ (and strangeness -1) with which the pion does not interact. The field with which the pion does not interact does not need to be thought of as the field of a K meson. The

* Supported in part by the National Science Foundation.
† Address during 1957–1958 academic year, Physikalisches Staatsinstitut, University of Hamburg, Hamburg, Germany.
[1] D. B. Lichtenberg and M. Ross, Phys. Rev. **103**, 1131 (1956).
[2] N. Dallaporta and F. Ferrari, Nuovo cimento **5**, 111 (1957), have independently made a calculation of the Λ-nucleon force with this model.
[3] D. B. Lichtenberg and Marc Ross, Bull. Am. Phys. Soc. Ser. II, **1**, 337 (1956).
[4] D. Glaser, L. Alvarez *et al.*, *Proceedings of The Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957), Sessions V and VI.

procedure is quite general and may be considered as a formal device for writing down the interactions (4) and (5) and for fixing relative coupling constants by assuming that the pion interacts with the "nucleon" part of the hyperon field with a single coupling strength h.[5] It is by using this formal device, considering the Λ as a singlet state of two fields of isotopic spin $\frac{1}{2}$ and the Σ as a triplet state, that the calculations are carried out most readily.[6] The sign of h relative to the sign of the pion-nucleon coupling constant f has physical consequences. Since it is our philosophy that the hyperon-pion interaction is essentially the same as the nucleon-pion interaction, we choose the plus sign. The Hamiltonian then has the same form as the fixed-source pion-nucleon Hamiltonian, except possibly for the magnitude of h. If we regard h as a parameter to be determined by experiment (for example, by the data on the binding energies in hyperfragments), we find that h is of the same order of magnitude as f. This result lends support to the proposal of Wigner[7] that all baryons may have the same strength of interaction with the pion field.

Thus far, we have not considered the Ξ hyperon since it does not directly contribute to Λ and Σ interactions with single nucleons. Furthermore, there is little probability of much experimental information about the Ξ being obtained in the near future. However, since the Ξ is probably a particle of isotopic spin $\frac{1}{2}$ just as the nucleon is, its interaction with nucleons may be similar to the nucleon-nucleon interaction.

It is also possible that K mesons may contribute appreciably to the hyperon-nucleon interaction. In a second paper we shall consider forces due to K-particle exchange alone. A comparison with experiment will be made to see to what extent we can distinguish between K-particle and pion theories of hyperon-nucleon interaction. At the present state of theory and experiment it does not seem fruitful to consider strong K and π effects simultaneously.

In Sec. II of this paper, we calculate, to fourth order in the interaction Hamiltonian, the hyperon-nucleon potentials arising from the exchange of pions. We obtain not only Λ-nucleon and Σ-nucleon potentials, but a nondiagonal potential which leads to the conversion of a Λ to a Σ (and vice versa) in a scattering process. We call this hyperon-exchange scattering. In Sec. III we evaluate the various hyperon-nucleon cross sections using the nondiagonal as well as the diagonal potentials. In Sec. IV, we discuss the validity of our results and briefly compare with experiment.

[5] M. Gell-Mann [Phys. Rev. **106**, 1296 (1957)] uses a different formal device to obtain essentially the same Hamiltonian.

[6] The interaction Hamiltonian obtained in this way differs from the Hamiltonian of Eqs. (4) and (5) (and the Hamiltonian of Gell-Mann, reference 5) in the relative signs of certain terms. These sign differences have no physical consequences.

[7] E. P. Wigner, Proc. Natl. Acad. Sci. U. S. **38**, 449 (1952). See also J. Schwinger, *Proceedings of The Seventh Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1957), Session IX.

II. CALCULATION OF THE POTENTIALS

To obtain the pion contribution to the potentials describing the interaction of hyperons and nucleons, we use the fixed-source pion-nucleon Hamiltonian combined with the hyperon-pion Hamiltonian as discussed in I and in the previous section. The static-model Hamiltonian for the interaction of a pion with a hyperon has the form ($\hbar=c=1$):

$$H=\sum_{k\alpha}(a_{k\alpha}V_{k\alpha}+a_{k\alpha}^*V_{k\alpha}^*),$$
$$V_{k\alpha}=(ih/\mu)\tau_\alpha\boldsymbol{\sigma}\cdot\mathbf{k}(2\omega_k)^{-\frac{1}{2}} \tag{6}$$

where $a_{k\alpha}$ annihilates a pion of momentum $\mathbf{k}$ and isotopic spin component α, ω_k is the energy of the pion of mass μ, and $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the usual Pauli spin and isotopic spin operators. The operator $\boldsymbol{\tau}$ operates only on the "nucleon" part of the hyperon field. If the hyperon is not at the origin, $V_{k\alpha}$ contains the additional factor $\exp(i\mathbf{k}\cdot\mathbf{r})$ where $\mathbf{r}$ is the position of the hyperon.

In obtaining the potentials from this Hamiltonian we follow the procedure used by Brueckner and Watson in their calculation of the two-nucleon potential.[8] Thus, to obtain the Σ-nucleon potential we consider second-order diagrams (exchange of one pion) and fourth-order diagrams (exchange of two pions) in which at least one pion appears in all intermediate states. We shall refer to these diagrams as "proper" graphs. Diagrams in which there are intermediate states with no pions present, we refer to as "improper" graphs. The reason improper fourth-order graphs are not included in the calculation of the Σ-nucleon potential (or the nucleon-nucleon potential) is that these graphs are included as iterations of the second-order potential when the Schrödinger equation is solved. This statement is not exact,[9] but is a good approximation for the static model at low energies. Thus, if we include the contribution from improper graphs to the potential, we are in effect counting them twice in their contribution to Σ-nucleon scattering.

However, these arguments for the omission of improper fourth-order diagrams do not apply directly to the Λ-nucleon potential, since second-order diagrams are absent. Therefore, in I, we included the contribution from improper graphs in evaluating the Λ-nucleon fourth-order potential. In doing so, we considerably overestimated their importance because of an approximation in which we neglected baryon kinetic energies in intermediate states. We can see this as follows. An order-of-magnitude estimate of the contribution to the potential from any diagram can be obtained by estimating the average magnitude of the intermediate-state energy denominators. If we neglect the mass difference Δ between the Σ and Λ compared to the energy of a pion, then the energy denominator of any proper fourth-order diagram contains (in addition to other

[8] K. A. Brueckner and K. M. Watson, Phys. Rev. **92**, 1023 (1953).

[9] Fukuda, Sawada, and Taketani, Progr. Theoret. Phys. Japan **12**, 156 (1954).

quantities) the factor

$$\omega_k+\omega_{k'}+T \qquad (7)$$

where ω_k and $\omega_{k'}$ are the energies of two intermediate state pions and T is the intermediate-state kinetic energy of the baryons (we neglect the total energy E). The improper fourth-order graphs have the same energy denominators as the proper ones, except that instead of containing the factor shown in (7), they contain the factor $\Delta+T$. If we neglect T, the ratio of the contribution to the potential from improper graphs to the contribution from proper graphs is $(\omega_k+\omega_{k'})/\Delta$. Since Δ is small ($\Delta\simeq\frac{1}{2}\mu$), we found in I that improper graphs gave a contribution to the potential about five times as great as the contribution from proper graphs. The baryon kinetic energy T depends on the distance between the hyperon and nucleon. To get an estimate of the validity of the approximation of neglecting T, we look at the magnitude of the potential as a function of the distance between the two baryons. Since the total energy of the system is near zero, T will be approximately equal to minus the potential. (We are neglecting an extra contribution to T from the recoil of a baryon when it emits a pion.) The triplet central Λ-nucleon potential calculated in I is 80 Mev deep for $\mu r=0.65$, so that for distances smaller than this, T is greater than Δ (80 Mev), and the approximation breaks down. The situation with respect to proper diagrams is a little better. In this case the potential should be compared with the energy of an intermediate-state pion. The relevant momentum of a pion emitted between two baryons when they are a distance r apart is $k\simeq 1/r$. Then the requirement that T be less than the energy of the pion is equivalent to the requirement that the potential $V(r)$ satisfy the inequality

$$-V(r)<(\mu^2+1/r^2)^{\frac{1}{2}}.$$

For the singlet even-state two-nucleon potential of Brueckner and Watson,[8] this inequality breaks down for $\mu r\lesssim 0.5$. Therefore, even when only proper diagrams are considered, at small distances the static two-baryon potentials must be treated phenomenologically. We shall replace the field-theoretical potentials at small distances by repulsive cores.

In this paper, rather than attempt to improve the evaluation of the contribution from improper graphs to the Λ-nucleon potential, we shall omit them entirely in the potential. Instead, we shall consider a nondiagonal potential which converts a Λ hyperon to a Σ hyperon and arises from proper diagrams such as those shown in Fig. 1. When the Schrödinger equation is solved with this nondiagonal potential, iterations of the potential

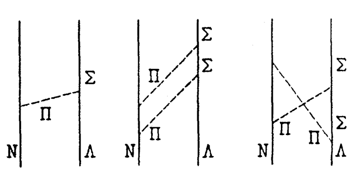

FIG. 1. Some proper graphs contributing to a nondiagonal potential which converts a Λ hyperon into a Σ hyperon.

will lead to the inclusion in the Λ-nucleon scattering of the effects of the improper graphs which we have omitted. In addition, since we are considering the nondiagonal potentials to fourth order, we are including the effects of many graphs which we did not consider in I. Using this approach, we are able to calculate the cross sections for hyperon-exchange scattering as well as the Λ-nucleon and Σ-nucleon elastic scattering cross sections.

If we neglect Δ compared to the energy of a pion in intermediate states, we can readily evaluate the potentials. The sum over intermediate hyperon states becomes equivalent to a sum over a complete set of (equal energy) spin and isotopic-spin states, just as in the two-nucleon problem. Then we need merely use the two-nucleon potentials given by Brueckner and Watson with one modification. The isotopic spin operator $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ (where the subscripts 1 and 2 refer to the two baryons) appearing in the potential does not have the same expectation value in the hyperon-nucleon problem as it does in the two-nucleon case. Since the isotopic spin of the Σ is unity, the Σ-nucleon system can be either in a $T=\frac{3}{2}$ or $T=\frac{1}{2}$ isotopic spin state. The Λ-nucleon system can be only in a $T=\frac{1}{2}$ state, since the Λ has isotopic spin zero. If we denote the Σ-nucleon $T=\frac{3}{2}$ and $T=\frac{1}{2}$ isotopic-spin wave functions by η_3 and η_1, respectively, and the Λ-nucleon isotopic-spin wave function by η_Λ, we find that

$$\begin{aligned}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\eta_3&=\eta_3,\\ \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\eta_1&=-2\eta_1-\sqrt{3}\eta_\Lambda,\\ \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\eta_\Lambda&=-\sqrt{3}\eta_1.\end{aligned} \qquad (8)$$

We see that η_1 and η_Λ are not eigenfunctions of $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$, leading to hyperon exchange scattering in the $T=\frac{1}{2}$ state. Making use of Eq. (8), we can easily evaluate the potentials from the expressions of Brueckner and Watson. The strength of the coupling constant and the radii of the phenomenological repulsive cores remain as variables. We can regard them as parameters to be determined by experiment. However, at the present time the experimental data are insufficient to enable us to fix the coupling constant plus the core radii in the various spin and isotopic spin states. If we assume that the hyperon-nucleon coupling constant h is equal to the nucleon-nucleon coupling constant f, we can use the data on hyperfragment binding energies to fix the Λ-nucleon repulsive cores. For mathematical convenience we actually reversed this procedure, letting the repulsive core radii be the same as the radii given by Brueckner for the two-nucleon problem.[10] The triplet-state core radius is $r_t=0.3\mu^{-1}$; the singlet radius is $r_s=0.384\mu^{-1}$. We then have the single parameter h with which to fit the data on the binding energies of the Λ in hyperfragments. Our procedure for doing this is described in Sec. IV. We find a satisfactory fit with approximately $fh/4\pi=0.095$, a value slightly higher than the $f^2/4\pi=0.085$ used by Brueckner and Watson in fitting the

[10] Brueckner, Levinson, and Mahmoud, Phys. Rev. 95, 217 (1954).

low-energy two-nucleon data. We can regard this result as indicating that h equals f. Certainly, if $h=f$ one notes that the associated Λ-nucleon core radii are slightly smaller than the corresponding two-nucleon radii.

In the $T=\frac{3}{2}$ state, the Σ-nucleon potentials are the same as the nucleon-nucleon potentials in the $T=1$ state, except for a possible difference in coupling constants. If the coupling constants are unequal, the shape, as well as the magnitude, of the potentials will be different in the two cases because the ratio of the contributions from second- and fourth-order diagrams is changed. It should be noted that in the two-nucleon case, certain states are forbidden by the Pauli principle, a fact that does not occur here. For a given spin and isotopic spin, the hyperon-nucleon potentials are the same for even and odd states of angular momentum. For $h=f$, the singlet $T=\frac{3}{2}$ Σ-nucleon potential is the same as the proton-proton singlet potential in even orbital angular momentum states; the triplet $T=\frac{3}{2}$ potential is the same as the proton-proton triplet potential for odd orbital angular momentum.

In the $T=\frac{1}{2}$ state, the hyperon-nucleon potentials are different from the two-nucleon potentials. However, if the coupling constants are equal, the hyperon-nucleon potentials can be formed by taking appropriate linear combinations of the two-nucleon potentials. Let us denote the two-nucleon potentials in the isotopic-spin triplet and singlet states by V_1 and V_0. If the Σ-nucleon $T=\frac{3}{2}$ potential is denoted by $V_{\frac{3}{2}}$ and the hyperon-nucleon $T=\frac{1}{2}$ potentials by V_Σ (diagonal Σ-nucleon), V_Λ (diagonal Λ-nucleon), and $V_{\Lambda\Sigma}$ (nondiagonal), then, making use of Eqs. (8) and the fact that the expectation values of $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ in the two-nucleon case are 1 and -3 for isotopic spin one and zero, respectively, we have

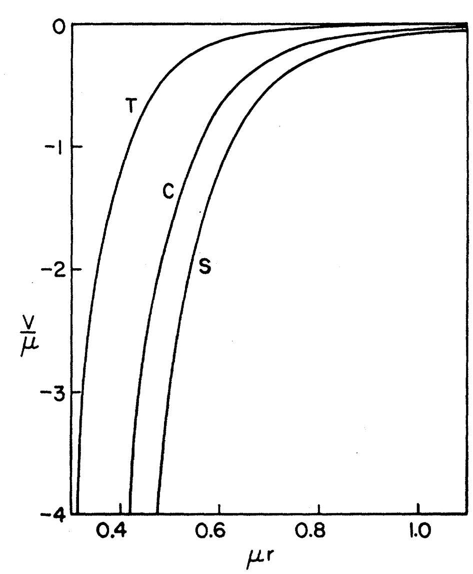

FIG. 2. The Λ-nucleon triplet central potential (C), triplet tensor potential (T), and singlet potential (S) with coupling constant $f^2/4\pi=0.095$. The distance between the baryons (abscissa) is in units of 1.4×10^{-13} cm and the potentials in units of 140 Mev.

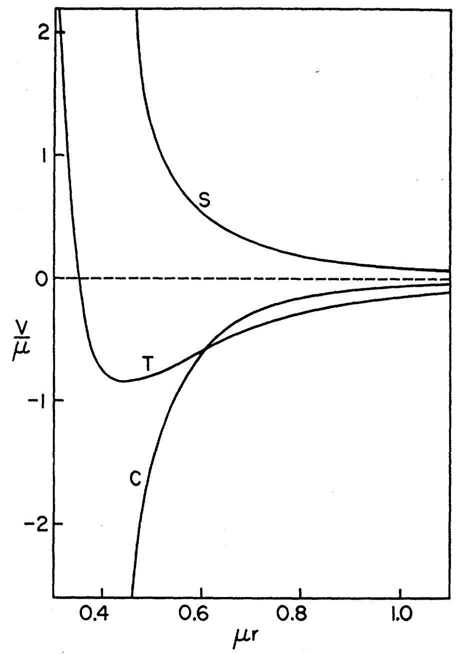

FIG. 3. The Σ-nucleon triplet central potential (C), triplet tensor potential (T), and singlet potential (S) in the isotopic-spin $\frac{1}{2}$ state. The units are the same as in Fig. 2.

$$
\begin{aligned}
V_{\frac{3}{2}}&=V_1,\\
V_\Sigma&=\tfrac{1}{4}(V_1+3V_0),\\
V_\Lambda&=\tfrac{1}{4}(3V_1+V_0),\\
V_{\Lambda\Sigma}&=\tfrac{1}{4}\sqrt{3}(V_0-V_1).
\end{aligned}
\tag{9}
$$

The $T=\frac{1}{2}$ potentials are plotted in Figs. 2, 3, and 4 for the coupling constant $fh/4\pi=0.095$. The $T=\frac{3}{2}$ potentials are the same as the $T=1$ potentials plotted in reference 10.

The nondiagonal triplet central potential nearly vanishes because of cancellation of the contributions from second- and fourth-order diagrams. This result is in qualitative disagreement with that found in I, where we neglected diagrams corresponding to the fourth-order nondiagonal potential. The Λ-nucleon potential (including the effect of the nondiagonal potentials) is now more attractive in the singlet state than in the triplet, a result opposite to that found in I.

III. SOLUTION OF THE SCHRÖDINGER EQUATION

In the $T=\frac{3}{2}$ state, the Schrödinger equation is diagonal in the potentials (except for tensor forces). It can be solved, therefore, without any special difficulties.

In the $T=\frac{1}{2}$ state, as we have seen, the terms in the potentials containing $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ mix Λ-nucleon and Σ-nucleon wave functions. The Schrödinger equation can be written

$$(T+V)\Psi_E=E\Psi_E, \tag{10}$$

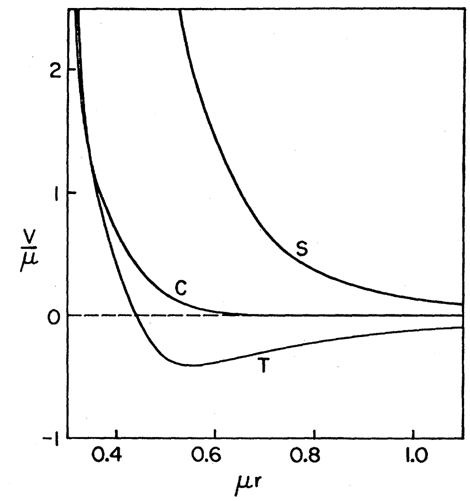

FIG. 4. The hyperon-nucleon nondiagonal triplet central potential (C), triplet tensor potential (T), and singlet potential (S). This potential converts a Λ hyperon to a Σ and vice versa. The units are the same as in Fig. 2.

where the eigenfunctions Ψ_E contain a mixture of the wave functions of a Λ-nucleon system and a Σ-nucleon system. The kinetic energy operator T contains only diagonal terms which operate on the Λ-nucleon and Σ-nucleon parts of the wave function separately, while the potential V contains the term $V_{\Lambda\Sigma}$ which changes a Λ into a Σ and vice versa in addition to the diagonal terms V_Λ and V_Σ. Equation (10) can be separated into two simultaneous equations which have the form

$$-[1/(2\mu_\Sigma)]\nabla^2\psi_\Sigma+(V_\Sigma-E_\Sigma)\psi_\Sigma=-V_{\Lambda\Sigma}\psi_\Lambda, \quad (11a)$$

$$-[1/(2\mu_\Lambda)]\nabla^2\psi_\Lambda+(V_\Lambda-E_\Lambda)\psi_\Lambda=-V_{\Lambda\Sigma}\psi_\Sigma, \quad (11b)$$

where μ_Σ and μ_Λ are the reduced masses of the Σ-nucleon and Λ-nucleon systems, respectively. The energy of the Σ-nucleon system E_Σ and of the Λ-nucleon system E_Λ are related by

$$E_\Sigma-E_\Lambda=M_\Sigma-M_\Lambda=\Delta.$$

We now expand ψ_Σ and ψ_Λ in partial waves, noting that in the triplet state tensor forces are present. Restricting ourselves to states of lowest angular momentum, we obtain from Eq. (11a)

$$\begin{aligned} -\frac{1}{2\mu_\Sigma}\frac{d^2u_\Sigma}{dr^2}+(E_\Sigma-V_\Sigma)u_\Sigma &= V_{\Lambda\Sigma c}u_\Lambda+2\sqrt{2}V_{\Sigma t}w_\Sigma+2\sqrt{2}V_{\Lambda\Sigma t}w_\Lambda, \\ -\frac{1}{2\mu_\Sigma}\left(\frac{d^2u_\Sigma}{dr^2}-\frac{6w_\Sigma}{r^2}\right)+(E_\Sigma+2V_{\Sigma t}-V_{\Sigma c})w_\Sigma &= (V_{\Lambda\Sigma c}-V_{\Lambda\Sigma t})w_\Lambda+2\sqrt{2}V_{\Sigma t}u_\Sigma+2\sqrt{2}V_{\Lambda\Sigma t}u_\Lambda, \end{aligned} \quad (12)$$

where u_Σ/r and w_Σ/r are the radial S- and D-state wave functions of the Σ and nucleon. The subscripts c and t refer to central and tensor, respectively. There are two more equations which can be obtained from Eq. (11b), or, more simply, by permuting the subscripts Λ and Σ in Eqs. (12). Of course $V_{\Lambda\Sigma}=V_{\Sigma\Lambda}$. These four equations must be solved simultaneously for u_Σ, u_Λ, w_Σ, and w_Λ in order to obtain the phase shifts which yield the hyperon-nucleon scattering cross sections.

A considerable simplification results if we neglect tensor forces. Then we have only two simultaneous equations to solve in the triplet $T=\frac{1}{2}$ state and only one equation in the triplet $T=\frac{3}{2}$ state. It can be seen from Figs. 2 and 3 that the Λ-nucleon and Σ-nucleon tensor potentials are much smaller than the triplet central potentials. This is also true of the $T=\frac{3}{2}$ potentials (proton-proton odd-state potentials). The tensor potential shown in Fig. 4 is small compared to the diagonal central potentials, but it is not small compared to the nondiagonal triplet central potential. By neglecting tensor forces in this case, we underestimate the triplet state hyperon-exchange cross section.

In the singlet spin state we need make no further approximations. Then in both singlet and triplet states, Eqs. (12) (and the equations formed by permuting Λ and Σ) reduce to the following form:

$$\begin{aligned} \frac{1}{2\mu_\Sigma}\frac{d^2u_\Sigma}{dr^2}+(E_\Sigma-V_{\Sigma c})u_\Sigma &= V_{\Lambda\Sigma c}u_\Lambda, \\ \frac{1}{2\mu_\Lambda}\frac{d^2u_\Lambda}{dr^2}+(E_\Lambda-V_{\Lambda c})u_\Lambda &= V_{\Lambda\Sigma c}u_\Sigma, \end{aligned} \quad (13)$$

where $V_{\Sigma c}$, $V_{\Lambda c}$, and $V_{\Lambda\Sigma c}$ are of course different in the triplet and singlet states.[11]

We have solved Eqs. (13) numerically on an electronic computor at Indiana University by integrating out from the repulsive core, point by point. In discussing the solutions of the equations, it is convenient to regard the incident particle as a Λ and to distinguish between energy regions below and above threshold for hyperon-exchange scattering.

In the first region, the Λ-nucleon kinetic energy is insufficient to produce a real Σ. Then Eqs. (13) have, at any given energy, only one solution that does not blow up at infinity. From this solution we obtain a phase shift which describes Λ-nucleon elastic scattering. In the second energy region, we obtain two independent solutions, each one of which corresponds to a mixture of Λ and Σ hyperons approaching the scatterer and a different mixture leaving the scatterer. From these solutions we construct two linear combinations, one of which corresponds to a pure incoming Λ and an outgoing mixture of Λ and Σ, and the other to a pure incoming Σ and outgoing mixture. The phase shifts in this case are complex, and from them we obtain the elastic scattering and hyperon-exchange scattering cross sections.

[11] R. G. Newton has discussed some of the general properties of equations of this form (to be published).

IV. RESULTS AND DISCUSSION

A. Applications to Hyperfragments

An analysis of the binding energies of the Λ in hyperfragments can be carried out to yield some information about the Λ-nucleon potential. Dalitz[12] has shown, under the assumption that the Λ-nucleon potential is short range and is weaker than the nucleon-nucleon potential, that the hyperfragment binding gives an estimate for the volume integral $U\equiv -\int d^3rV(r)$ of the Λ-nucleon potential. We can use this information to estimate the magnitude of the coupling constant h. However, this estimate cannot be readily carried out directly. For example, our potentials have repulsive cores and therefore their volume integrals are infinite. It would be a difficult task to calculate hyperfragment binding energies with these singular and nondiagonal potentials.

We determine h in the following manner. Consider the triplet and singlet S-wave scattering of a Λ by a nucleon below threshold for production of a Σ. The calculated scattering phase shifts, as a function of energy for various values of h, determine "equivalent" potentials, such as Gaussian or square-well potentials, which are the same as the actual potentials as far as their low-energy scattering properties are concerned. These "equivalent" potentials are well behaved; their volume integrals exist and they are relatively easy to calculate with. We obtain the "equivalent" potential by making an effective-range approximation from the computed phase shifts and then determining, for example, the square well which has the same scattering length a and effective range r_0. The scattering length and effective range obtained from the phase shifts in triplet and singlet states are shown in Table I for $fh/4\pi=0.095$. The effective-range approximation yields cross sections which are at most 10% higher than the computed cross sections at an energy of 15 Mev in the center of mass. Square wells with the same scattering length and effective range as the actual potentials give the cross sections with an error of about 5% at 15 Mev. The volume integrals U and ranges b of the triplet and singlet equivalent square wells are given in Table II. As has been pointed out by various authors,[13,14] the range of the Λ-nucleon potential, as well as its volume integral, is important for predicting hyperfragment binding energies. As a result of the range determined by our theory, we obtain volume integrals somewhat higher than those deduced assuming Λ-N force of very short range.‡

TABLE I. The scattering-length and effective-range parameters describing the low-energy Λ-nucleon scattering in triplet and singlet states. The units are 1.4×10^{-13} cm.

	Scattering length a	Effective range r_0
Triplet	-0.55	2.7
Singlet	-1.7	1.7

TABLE II. Volume integrals and ranges of square-well potentials with the same low-energy scattering properties as the calculated Λ-nucleon triplet and singlet potentials.

	Volume integral U in Mev cm$^3\times10^{-39}$	Range b in 1.4×10^{-13} cm
Triplet	240	1.36
Singlet	440	1.32

An empirical analysis of observed hyperfragment binding does not reveal whether the Λ-nucleon potential is more attractive in the triplet or singlet state. If singlet forces are favored, we find empirically that $U_s\approx380$ Mev cm$^3\times10^{-39}$ and $U_t\approx220$ for potentials of range about a pion Compton wavelength. If triplet forces are favored, we find $U_t\approx300$ while $U_s\approx0$. (U_s is not very well determined by experiment in this case.) These numbers are based on H and He hyperfragments. The analysis of heavier fragments seems less conclusive, at present. It is seen that the chosen coupling constant together with repulsive cores from the two-nucleon case puts our theoretical results of Table II in rough agreement with the observed volume integrals which favor singlet forces. For a coupling strength $fh/4\pi=0.09$ (nearly the ^1S constant used by Brueckner and Watson[8]), the volume integrals of the equivalent square wells are a little over half those shown in Table II.

With these forces the hyperdeuteron ($\Lambda+p$) is not bound. (Note, however, that if the measured Q value for Λ decay should increase, all hyperfragment binding energies would increase and this result might be changed.)

B. Hyperon-Nucleon Scattering

At low energies, only S-wave Λ-nucleon scattering is important. At higher energies, such that a Σ can be produced, it is still sufficient to consider only S-wave scattering provided the kinetic energy of the Σ-nucleon system is small. Using the coupling constant $fh/4\pi=0.095$, we have calculated the S-wave hyperon-nucleon scattering amplitudes in definite isotopic spin states. It remains to relate these amplitudes to cross sections which can be observed. Fourteen different processes of the form

$$Y+N\rightarrow Y+N \tag{14}$$

may occur, not counting the inverse reactions. Here Y means either a Λ or Σ hyperon. Making use of charge symmetry, which states that the cross section for a particular reaction is unaltered if we replace Σ^+ by Σ^- and proton by neutron, the number of distinct cross

[12] R. H. Dalitz, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956).

[13] Marc Ross and D. B. Lichtenberg, Midwest Conference of Theoretical Physics, Iowa City, March 14, 1957 (unpublished).

[14] D. W. Downs, Bull. Am. Phys. Soc. Ser. II, **2**, 175 (1957).

‡ *Note added in proof.*—See, for example, L. Brown and M. Peshkin, Phys. Rev. **107**, 272 (1957).

TABLE III. Calculated total S-wave hyperon-nucleon scattering cross sections in millibarns as a function of energy in the center-of-mass system. The cross sections are averaged over triplet and singlet spin states. They are not corrected for Coulomb effects or the mass differences of the Σ^-, Σ^+, and Σ^0.

Energy (Mev)	$\Lambda p\to\Lambda p$ $\Lambda n\to\Lambda n$	$\Sigma^+p\to\Sigma^+p$ $\Sigma^-n\to\Sigma^-n$	$\Sigma^-p\to\Lambda n$ $\Sigma^+n\to\Lambda p$	$\Sigma^0p\to\Lambda p$ $\Sigma^0n\to\Lambda n$	$\Sigma^-p\to\Sigma^-p$ $\Sigma^+n\to\Sigma^+n$	$\Sigma^0p\to\Sigma^0p$ $\Sigma^0n\to\Sigma^0n$	$\Sigma^-p\to\Sigma^0n$ $\Sigma^+n\to\Sigma^0p$
1.4	170	750	17	8.7	210	420	120
7	83	200	6.6	3.3	87	130	29
14	46	98	4.6	2.3	50	67	15
28	19	39	2.8	1.4	24	27	7.1

sections reduces to seven. With our charge-independent model, where the Hamiltonian (and therefore the cross sections) depends only upon the isotopic spin configuration of the hyperon-nucleon system and not on its orientation in isotopic spin space, the seven cross sections σ can be written in terms of four isotopic-spin scattering amplitudes with one relative phase between them. We have

$$\begin{aligned}
\sigma(\Sigma^+p\to\Sigma^+p)&=|\alpha_3|^2,\\
\sigma(\Sigma^+n\to\Sigma^+n)&=\tfrac{1}{9}|\alpha_3+2\alpha_1|^2,\\
\sigma(\Sigma^0p\to\Sigma^0p)&=\tfrac{1}{9}|2\alpha_3+\alpha_1|^2,\\
\sigma(\Sigma^+n\to\Sigma^0p)&=2/9|\alpha_3-\alpha_1|^2,\\
\sigma(\Sigma^+n\to\Lambda p)&=\tfrac{2}{3}|\alpha_{10}|^2,\\
\sigma(\Sigma^0p\to\Lambda p)&=\tfrac{1}{3}|\alpha_{10}|^2,\\
\sigma(\Lambda p\to\Lambda p)&=|\alpha_0|^2,
\end{aligned} \tag{15}$$

where α_3 is the $T=\frac{3}{2}$ scattering amplitude and α_1, α_{10}, and α_0 are the three $T=\frac{1}{2}$ amplitudes. Here α_1 is the amplitude for a Σ-nucleon configuration in both initial and final states, α_{10} comes from a Σ-nucleon initial configuration and a Λ-nucleon final configuration, and α_0 is the Λ-nucleon amplitude in initial and final states. The expressions for the Σ-nucleon scattering cross sections in terms of isotopic spin amplitudes are formally the same as the corresponding expressions for pion-nucleon scattering. If we combine the individual amplitudes according to Eqs. (15) and appropriately average the triplet and singlet cross sections, we obtain the total S-wave cross sections shown in Table III. These cross sections are not corrected for Coulomb effects or for the mass differences of the three Σ hyperons.

There are almost no direct data on hyperon-nucleon scattering with which to compare the cross sections of Table III.§ For example, Alvarez *et al.*[15] have one apparent Σ^-p interaction in flight. The Σ^- track ends suddenly, corresponding to a charge-exchange scattering. There is more information on the Σ^-p interaction at rest. Alvarez *et al.* have ten events at rest, three of which correspond to the process

$$\Sigma^-+p\to\Lambda+n, \tag{16}$$

and four to the process

$$\Sigma^-+p\to\Sigma^0+n. \tag{17}$$

The remaining three events cannot be identified, presumably because the Λ which is produced (either directly or indirectly via the decay of the Σ^0) decays neutrally. Events classified as occurring at rest are either very low-energy events in flight (say, $\lesssim 1$ Mev) in which the Σ^- appears to have stopped, or events in which the Σ^- is captured from a Bohr orbit. Theoretically, the potentials leading to the processes (16) and (17) are the same, provided we neglect the Σ-Λ mass difference Δ. This can be verified directly by operating with $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ on the Σ^0n_s and Λn wave functions and noting that diagonal and nondiagonal terms are equal in the two cases. Then if we also neglected Δ in solving the Schrödinger equation, we would find that the matrix elements for processes (16) and (17) would be equal. More simply,[16] we note that the wave function of the initial Σ^-p system can be written K^-np. Similarly we write for the final Σ^0n and Λn wave functions $2^{-\frac{1}{2}}(K^-p+K^{0*}n)n$ and $2^{-\frac{1}{2}}(K^-p-K^{0*}n)n$, respectively, where K^{0*} is used to represent the charge conjugate wave function to K^0 (i.e., the wave function for a $\bar{K}^0$ particle). But since we are postulating that the potential is due only to the exchange of pions, with the K playing only a geometrical role, there is no mechanism for converting a K^- into a $\bar{K}^0$. Then the wave functions of the Λn and Σn systems must have equal amplitudes so that the K^{0*} part of the final wave functions will cancel.

If we assume that, neglecting Λ and Σ mass differences, the matrix elements for processes (16) and (17) are equal, the ratio (Λ/Σ) of the number of Λ's to Σ^0's produced is simply the ratio of the phase-space factors. Since the energy release for processes (16) and (17) is 80 Mev and 7 Mev, respectively (assuming that $M(\Sigma^-)-M(\Sigma^0)=8$ Mev), with corresponding momenta $k_\Lambda=300$ Mev and $k(\Sigma^0)=90$ Mev, we have for scattering in an S state (either in flight or from a Bohr orbit)

$$(\Lambda/\Sigma)=M_\Lambda k_\Lambda/(M(\Sigma^0)k(\Sigma^0))\approx 3.$$

This value is about four times the experimental ratio $(\Lambda/\Sigma)\approx\frac{3}{4}$. If the Σ^-p interaction goes from a P-state Bohr orbit, then (Λ/Σ) is increased by the additional centrifugal-barrier factor $[k_\Lambda/k(\Sigma^0)]^2\approx 11$. A rough estimate indicates that the nuclear interaction of Σ^- in the P state will completely predominate over the radiative transition to the S state.

However, although it is a good approximation to neglect Δ in calculating the potentials, it is a poor approximation to neglect it in solving the Schrödinger

§ *Note added in proof.*—One can, however, look at hyperon-nucleon final state interactions. See E. Henley, Phys. Rev. **106**, 1083 (1957).

[15] Alvarez, Bradner, Falk-Vairant, Gow, Rosenfeld, Solmitz, and Tripp, Nuovo cimento **5**, 1026 (1957). See also P. Falk-Vairant *et al.*, Bull. Am. Phys. Soc. Ser. II, **2**, 222 (1957), and Fry, Schneps, Snow, and Swami, Phys. Rev. **100**, 939 (1955).

[16] We should like to thank M. Gell-Mann for pointing out to us that simpler arguments exist for obtaining this and certain other results.

equation. We find that the $\Sigma^- p$ system is just bound (by $\approx\frac{1}{2}$ Mev) in the ^1S, $T=\frac{3}{2}$ state. This has the effect of greatly enhancing the singlet cross section for process (17), near zero energy, so that the singlet ratio becomes $(\Lambda/\Sigma)_s\approx 0$. In the triplet state, the ratio of the cross sections is still given by approximately the ratio of the phase-space factors as before: $(\Lambda/\Sigma)_t\approx 3$. Then, if a significant fraction of the $\Sigma^- p$ interactions are actually low-energy events in flight, agreement could be obtained with the result of Alvarez *et al.* If the resonance in the $T=\frac{3}{2}$ state occurs very nearly at zero energy, a crude calculation (including Coulomb distortion of the $\Sigma^- p$ wave function) indicates that the in-flight lifetime for process (17) may be of the same order of magnitude as the time required for a kilovolt-energy Σ^- to be slowed down and captured into a Bohr orbit ($\approx 10^{-12}$ sec).[17] Thus, while we have not made any detailed calculations, we feel that it is possible that the explanation for the low ratio of Λ to Σ^0 production is the $T=\frac{3}{2}$ zero-energy resonance in Σ-N scattering.

Hyperon-nucleus interactions can also give information on hyperon-nucleon scattering. In particular, one can examine production of hyperons in nuclear matter by K particles or pions. We are in the process of investigating these questions. At present we can just remark that the spectrum of hyperons produced by K particles unfortunately seems to lead to very weak conditions on the hyperon-nucleus potential, particularly since most of the production occurs at the nuclear surface.

C. Further Discussion

An interesting experimental question is the possible binding of $\Sigma^- n$ and $\Sigma^+ p$ systems. If the pion-hyperon coupling constant is equal to (or greater than) the pion-nucleon constant, the $\Sigma^- n$ system is bound. This can be seen as follows: the $\Sigma^- n$ singlet potential is the same as the nucleon-nucleon singlet even-state potential. (The neglect of the mass difference Δ does not affect the $\Sigma^- n$ potential, since there cannot be an intermediate Λ.) The nucleon-nucleon potential just fails to bind the neutron-proton system in the singlet state. But the same potential will bind the $\Sigma^- n$ system because of its heavier mass. Indeed, for the purpose of determining if a bound state exists, we can just consider that the two-nucleon singlet even potential is increased as the reduced mass, i.e., by the factor $2M_\Sigma(M_\Sigma+M_N)=1.12$, which is sufficient for binding.[18] The question of whether the $\Sigma^+ p$ system will be bound is very delicate because of the additional Coulomb repulsion. If we use our derived potential with a coupling constant $fh/4\pi=0.095$, instead of taking exactly the two nucleon potential, the $\Sigma^+ p$ system is definitely bound.

[17] A. S. Wightman, Phys. Rev. **77**, 521 (1950).
[18] J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 201.

The fact that the pion-hyperon coupling constant comes out so close to the pion-nucleon coupling constant makes it tempting to assume that the couplings are equal. If this is the case, the hyperon-nucleon potentials can be written in terms of linear combinations of the observed nucleon-nucleon potentials according to Eqs. (9). That is, we do not have to rely on a field-theoretical calculation but can relate the hyperon-nucleon potentials to the experimentally determined nucleon-nucleon potentials. In the two-nucleon case, the potentials are well known only in even orbital angular momentum states. But the hyperon-nucleon potentials, except the $T=\frac{3}{2}$ singlet potential, involve the two-nucleon potentials in odd as well as even states. Because of the lack of experimental information about the two-nucleon odd-state potentials, we have preferred to rely on a field-theoretical calculation, even in the case that pions have the same coupling with all baryons.[19]

If the pion-hyperon and pion-nucleon coupling strengths are in fact the same, we have to take the Λ-nucleon cores a few percent smaller than the nucleon-nucleon cores. Since the Λ-nucleon cores involve the two-nucleon cores in odd as well as even states, there is no inconsistency in this result.

We now consider what factors in the hyperon-nucleon problem, other than the unknown two-nucleon odd-state cores, might lead to our choice of smaller Λ-nucleon cores (or a larger coupling constant). We made approximations which are different from those that have been made in the case of the two-nucleon potential. The neglect of tensor forces may partially explain the need for a smaller core in the triplet state, but we also need a smaller core in the singlet state. If we do not neglect the mass difference between the Σ and Λ, the Λ-nucleon potential becomes smaller, so that this effect is in the wrong direction. Finally, there is the effect of K-meson exchange. If the results of second-order perturbation theory are valid, a pseudoscalar K meson will cause an additional attraction between the Λ and nucleon in both triplet and singlet states.[20] Whether this attraction is sufficient depends on the K-meson coupling strength. However, before we conclude that some K-particle forces are necessary to account for the Λ-nucleon force, it is necessary to improve deductions from the binding energies in hyperfragments.

ACKNOWLEDGMENTS

The authors would like to thank Professor Roger G. Newton for helpful discussions. Their thanks are also due to Professor Marshall Wrubel for making available the facilities of the Indiana Research Computing Center.

[19] There is some indication that the odd-state two-nucleon potentials will be empirically determined in the near future. J. Gammel and R. Thaler, *Proceedings of The Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957).
[20] G. Wentzel, Phys. Rev. **101**, 835 (1956).