Circular Polarization of Internal Bremsstrahlung

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(Received June 4, 1957)

A calculation is made of the circular polarization of the internal bramsstrahlung in allowed β-decay with the most general β-interaction. The polarization is found to rise from zero at the low-energy end of the spectrum to a maximum value at the high-energy end. This maximum value depends critically on the relative magnitudes of the coupling constants in the β-interaction. With the two-component neutrino theory and only the scalar and tensor interactions present, the polarization is complete at the high-energy end of the spectrum. Plots have been made, with this β-coupling, of the Born approximation result for the circular polarization as a function of photon energy for S^{35} and P^{32}. The effects of the nuclear Coulomb field on the γ-ray spectrum and polarization are discussed.

1. INTRODUCTION

THE bremsstrahlung accompanying β decay will be circularly polarized only if parity is not conserved in the β interaction. Assuming nonconservation of parity, we shall calculate the degree of this circular polarization for the internal bremsstrahlung in allowed decay. The calculation will be carried out in the Born approximation, but we shall also discuss the corrections to the Born approximation due to the effect of the nuclear Coulomb field. The correction to the first order in $Z\alpha$ is easily found and gives a simple result when all the coupling constants in the β interaction are real (or in phase). In our calculations we employ the Green's function of the second order Dirac equation. This technique, which was developed by Glauber and Martin[1] for radiative K-capture, is particularly suitable in finding the Coulomb field corrections.

In calculating the degree of polarization we have used the most general β interaction. With certain interaction combinations the result is independent of the coupling constants and the nuclear matrix elements. This is the case, for example, if we take the two-component neutrino theory to be correct and the vector and axial vector interactions to be absent. In this special case we have plotted the Born-approximation result for the degree of polarization as a function of energy for S^{35} and P^{32}.

An experimental confirmation that the internal bremsstrahlung is circularly polarized would present further evidence that parity is not conserved in the β interaction. Such experiments might also supply us with a check on the two-component neutrino theory. Furthermore, since the vector and axial vector interactions in the two-component theory polarize in the direction opposite to that of the scalar and tensor interactions, we might obtain an indication of the amount of a possible vector-axial vector admixture to the β interaction, which at the present is assumed to be predominantly tensor and scalar.

[1] R. J. Glauber and P. C. Martin, Phys. Rev. **104**, 158 (1956); and P. C. Martin and R. J. Glauber, Phys. Rev. (to be published).

2. CALCULATIONS IN THE BORN APPROXIMATION

The over-all process of internal bremsstrahlung may be pictured as taking place in two stages. In the first stage we have the nuclear transformation from an initial state i, whereby a neutron goes into a proton with the creation of an antineutrino and an electron in an intermediate state l. In the second stage we have a transition of the electron from the state l to the final state f with the emission of a photon.

The standard second-order perturbation theory gives the following expression for the probability, $S(k,\mathbf{e})dk$, that a photon will be emitted with energy between k and $k+dk$ and with polarization $\mathbf{e}$:

$$S(k,\mathbf{e})=\frac{1}{(2\pi)^8}k^2\int_m^{E_0-k}dE(E_0-E-k)^2Ep \times\int d\Omega_k\int d\Omega_e\int d\Omega_\nu\sum_{s_e,s_\nu,n}|M|^2, \quad (1)$$

where

$$M=\sum_l\frac{\langle f|H_\gamma|l\rangle\langle l|H_\beta|i\rangle}{E_l-E-k}.$$

Here E is the energy and p the momentum of the electron in the final state f; E_l is the energy of the electron in the intermediate state l, and E_0 is the total energy available in the decay. The summations are over the final spin states of the electron, s_e, and the antineutrino, s_ν, as well as the magnetic quantum numbers of the final nuclear states, labeled n. The expression is also to be considered as averaged over the initial nuclear states. The integrations are over the momentum directions of the electron, Ω_e, the neutrino, Ω_ν, and the photon, Ω_k. The electromagnetic interaction term is given by[2]

$$\langle f|H_\gamma|l\rangle=(2\pi e^2/k)^{\frac{1}{2}}\int\psi_f^\dagger(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}\boldsymbol{\alpha}\cdot\mathbf{e}\psi_l(\mathbf{r})d(\mathbf{r}). \quad (2)$$

[2] We use units in which $\hbar=1$ and $c=1$, and the Dirac matrices, $\boldsymbol{\alpha}=\rho_1\boldsymbol{\sigma}$, $\beta=\rho_3$, $\gamma_5=-\rho_1$. Hermitian conjugate is designated by a superscript dagger.

The general β interaction term in allowed decay is

$$\langle l|H_\beta|i\rangle=\sum_j\langle O_j\rangle_{fi}\psi_l^\dagger(0)O_j[C_j+C_j'\gamma_5]\chi(0). \quad (3)$$

Here $\langle O_j\rangle_{fi}$ is the matrix element of the β-coupling operator, O_j, taken between the final and initial nuclear states. $\psi(0)$ and $\chi(0)$ are the electron and neutrino wave functions evaluated at the origin. The expression (3) must be modified (by taking into account the finite size of the nucleus) for relativistic Coulomb wave functions, which are singular at the origin.

Following Glauber and Martin,[1] we introduce the Green's function, $\mathcal{G}(\mathbf{r})$, for the second order Dirac equation, describing propagation from the origin to the point $\mathbf{r}$. The matrix element M defined in (1) then becomes

$$M=(2\pi e^2/k)^{\frac{1}{2}}\int d\mathbf{r}\psi_E^\dagger(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}\times[-2i\mathbf{e}\cdot\nabla+i\boldsymbol{\sigma}\cdot(\mathbf{e}\times\mathbf{k})+k\boldsymbol{\alpha}\cdot\mathbf{e}]\times\mathcal{G}_{E+k}(\mathbf{r})\sum_j\langle O_j\rangle_{fi}O_j[C_j+C_j'\gamma_5]\chi(0). \quad (4)$$

Let us for the moment neglect the nuclear Coulomb field. The Green's function is then simply the unit matrix times a scalar, which satisfies

$$[\nabla^2+(E+k)^2-m^2]\mathcal{G}_{E+k}(\mathbf{r})=-\delta(\mathbf{r}), \quad (5)$$

whose outgoing wave solution is

$$\mathcal{G}_{E+k}(\mathbf{r})=e^{i\mu r}/4\pi r. \quad (6)$$

The momentum, μ, is defined by $\mu=[(E+k)^2-m^2]^{\frac{1}{2}}$.

To simplify the following expressions we shall temporarily restrict ourselves to the two component neutrino theory, $C'=-C$. M is then,

$$M=\left(\frac{2\pi e^2}{k}\right)^{\frac{1}{2}}u^\dagger\int d\mathbf{r}e^{-i(\mathbf{p}+\mathbf{k})\cdot\mathbf{r}}\times[-2i\mathbf{e}\cdot\nabla+i\boldsymbol{\sigma}\cdot(\mathbf{e}\times\mathbf{k})+k\boldsymbol{\alpha}\cdot\mathbf{e}]\times\frac{e^{i\mu r}}{4\pi r}\sum_j C_j\langle O_j\rangle_{fi}O_j(1-\gamma_5)v. \quad (7)$$

Here v and u are the plane-wave spinors of the neutrino and the electron in the final state. The spatial integration is easily carried out and yields

$$M=(2\pi e^2/k)^{\frac{1}{2}}(|\mathbf{p}+\mathbf{k}|^2-\mu^2)^{-1}u^\dagger\times[2\mathbf{e}\cdot\mathbf{p}+i\boldsymbol{\sigma}\cdot(\mathbf{e}\times\mathbf{k})+k\boldsymbol{\alpha}\cdot\mathbf{e}]\times\sum_jC_j\langle O_j\rangle_{fi}O_j(1-\gamma_5)v. \quad (8)$$

We introduce the circular polarization vectors, $\mathbf{e}^\pm$, defined by

$$\mathbf{e}^\pm=\tfrac{1}{2}\sqrt{2}(\mathbf{e}_1\pm i\mathbf{e}_2), \quad (9)$$

where $\mathbf{e}_1\times\mathbf{e}_2=\mathbf{k}/k$. Because of the identity,

$$i\boldsymbol{\sigma}\cdot(\mathbf{e}_1\pm i\mathbf{e}_2)\times\mathbf{k}=\mp k\boldsymbol{\sigma}\cdot(\mathbf{e}_1\pm i\mathbf{e}_2), \quad (10)$$

Eq. (8) can be written

$$M=(2\pi e^2/k)^{\frac{1}{2}}(|\mathbf{p}+\mathbf{k}|^2-\mu^2)^{-1}u^\dagger[2\mathbf{p}+k(\rho_1\mp1)\boldsymbol{\sigma}]\cdot\mathbf{e}^\pm\sum_jC_j\langle O_j\rangle_{fi}O_j(1-\gamma_5)v. \quad (11)$$

For the scalar and tensor interactions,

$$O_j(1-\gamma_5)=(1+\gamma_5)O_j=(1-\rho_1)O_j. \quad (12)$$

For the vector and axial vector interactions,

$$O_j(1-\gamma_5)=(1-\gamma_5)O_j=(1+\rho_1)O_j. \quad (13)$$

Hence

$$M=\left(\frac{2\pi e^2}{k}\right)^{\frac{1}{2}}\frac{1}{|\mathbf{p}+\mathbf{k}|^2-\mu^2}u^\dagger\times\{[2\mathbf{p}-(1\pm1)k\boldsymbol{\sigma}]\cdot\mathbf{e}^\pm\sum_{j=S,T}C_j\langle O_j\rangle_{fi}O_j+[2\mathbf{p}+(1\mp1)k\boldsymbol{\sigma}]\cdot\mathbf{e}^\pm\sum_{j=V,A}C_j\langle O_j\rangle_{fi}O_j\}\times(1+\rho_1)v. \quad (14)$$

We see that the scalar and tensor interactions polarize in the $\mathbf{e}^+$ direction, the vector and axial vector interactions polarize in the $\mathbf{e}^-$ direction. Since the scalar and tensor interactions are at the present assumed to dominate, we expect the internal bremsstrahlung to be polarized in the $\mathbf{e}^+$ direction.[3]

It is apparent already at this stage that the polarization is more pronounced the higher the photon energy becomes. In the low-energy limit there is clearly no circular polarization. To make the resulting expressions more manageable, we shall now consider only the tensor and scalar interactions. The result of an arbitrary interaction mixture can be found from an obvious extension of the following formulas, and will be given later.

We sum over the spin states in the usual way. The integrations over the neutrino and electron directions are trivial. With the scalar and tensor interactions, we obtain

$$S(k,\mathbf{e}^\pm)=\frac{e^2(|C_S|^2|\langle\beta\rangle|^2+|C_T|^2|\langle\beta\boldsymbol{\sigma}\rangle|^2)}{4\pi^4k}\times\int_m^{E_0-k}dE(E_0-E-k)^2p\int_{-1}^1d(\cos\theta)\times\left\{\frac{p^2\sin^2\theta[E+(1\pm1)k]}{(E-p\cos\theta)^2}+\frac{(1\pm1)k^2}{E-p\cos\theta}\right\}. \quad (15)$$

Here θ is the angle between $\mathbf{k}$ and $\mathbf{p}$. The nuclear terms $|\langle\beta\rangle|^2$ and $|\langle\beta\boldsymbol{\sigma}\rangle|^2$ are averaged over initial and summed over final states. (15) shows clearly the dependence of the circular polarization on θ. In particular the radiation

[3] There are conflicting definitions of "right" and "left" circular polarizations. We identify "right" polarization with the vector $\mathbf{e}^+$.

in the $\pm$ **p**-direction is completely polarized at all energies. When we integrate over θ, the result can be written as follows,

$$S(k,\mathbf{e}^{\pm})=(e^2/4\pi^4k)[|C_S|^2|\langle\beta\rangle|^2+|C_T|^2|\langle\beta\boldsymbol{\sigma}\rangle|^2]\times[\mathcal{J}_1+(1\pm1)\mathcal{J}_2], \quad (16)$$

where

$$\mathcal{J}_1=4\int_1^x dE(x-E)^2[E^2\ln(E+p)-Ep],$$
$$\mathcal{J}_2=2k\int_1^x dE(x-E)^2[(k+2E)\ln(E+p)-2p]. \quad (17)$$

Here we have set $m=1$, and $E_0-k=x$. We define the momentum s by $s=(x^2-1)^{\frac{1}{2}}$. The integrals $\mathcal{J}_1$ and $\mathcal{J}_2$ when carried out, yield

$$\mathcal{J}_1=\ln(x+s)\left[\frac{2}{15}x^5-\frac{1}{4}x\right]-s\left[\frac{107}{450}x^4-\frac{223}{900}x^2-\frac{8}{75}\right],$$

$$\mathcal{J}_2=k\left\{\ln(x+s)\left[E_0\left(\frac{2}{3}x^3+x\right)-\frac{1}{3}x^4+\frac{1}{8}\right]-s\left[E_0\left(\frac{11}{9}x^2+\frac{4}{9}\right)-\frac{19}{36}x^3+\frac{23}{72}x\right]\right\}. \quad (18)$$

If we define the degree of polarization, $\mathcal{P}(k)$, by

$$\mathcal{P}(k)=[S(k,\mathbf{e}^+)-S(k,\mathbf{e}^-)]/[S(k,\mathbf{e}^+)+S(k,\mathbf{e}^-)],$$

it follows that

$$\mathcal{P}(k)=\mathcal{J}_2/(\mathcal{J}_1+\mathcal{J}_2). \quad (19)$$

A plot has been made of $\mathcal{P}(k)$, $kS(k,\mathbf{e}^+)$ and $kS(k,\mathbf{e}^-)$ as functions of k for S^{35} with $k_{\max}=167$ kev (Fig. 1),

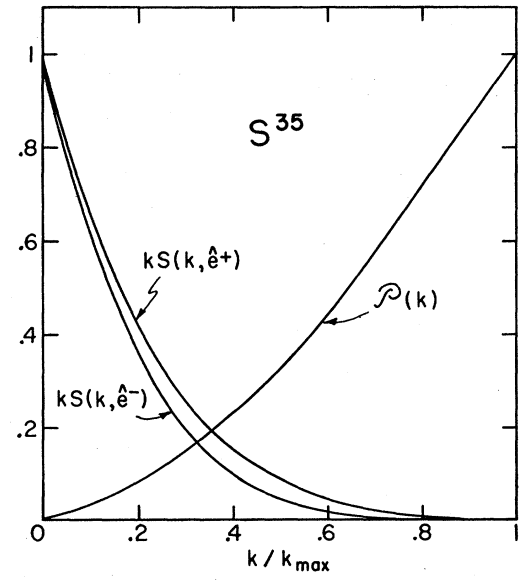

FIG. 1. The internal bremsstrahlung spectra, $kS(k,\mathbf{e}^+)$ and $kS(k,\mathbf{e}^-)$, and circular polarization, $\mathcal{P}(k)$, as functions of photon energy, k, for S^{35} ($k_{\max}=167$ kev), with $a=1$ and $b=0$.

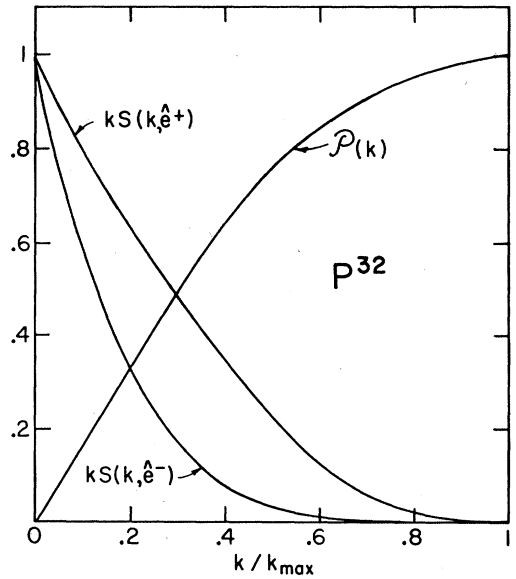

FIG. 2. The internal bremsstrahlung spectra, $kS(k,\mathbf{e}^+)$ and $kS(k,\mathbf{e}^-)$, and circular polarization, $\mathcal{P}(k)$, as functions of photon energy, k, for P^{32} ($k_{\max}=1.70$ Mev), with $a=1$ and $b=0$.

and P^{32} with $k_{\max}=1.70$ Mev (Fig. 2). Note that (19) and Figs. 1 and 2 are based on the two-component neutrino theory and the scalar-tensor interaction mixture. The total probability that a photon of energy k will be emitted is

$$S(k)=S(k,\mathbf{e}^+)+S(k,\mathbf{e}^-)=(e^2/2\pi^4k)\times[|C_S|^2|\langle\beta\rangle|^2+|C_T|^2|\langle\beta\boldsymbol{\sigma}\rangle|^2][\mathcal{J}_1+\mathcal{J}_2]. \quad (20)$$

This probability was first calculated by Knipp and Uhlenbeck and by Bloch.[4] Our result agrees with theirs when we note that $|C_j|^2|\langle O_j\rangle|^2$ in the old theory becomes $2|C_j|^2|\langle O_j\rangle|^2$ in the two-component theory.

Following Knipp and Uhlenbeck, we can rewrite the expressions for $S(k)$ in terms of the β-decay probability, $P(E+k)$, that an electron of energy $E+k$ will be emitted, and the conditional probability, $\Phi(E,k)$, that the electron will radiate a photon of energy k:

$$S(k)=\int_m^{E_0-k} dEP(E+k)\Phi(E,k), \quad (21)$$

where

$$P(E+k)=\pi^{-3}[|C_S|^2|\langle\beta\rangle|^2+|C_T|^2|\langle\beta\boldsymbol{\sigma}\rangle|^2]\times(E_0-E-k)^2(E+k)\mu, \quad (22)$$

and

$$\Phi(E,k,\mathbf{e}^{\pm})=e^2[2\pi\mu k(E+k)]^{-1}\{[2E^2+(1\pm1)k(2E+k)]\times\ln(E+p)-2p[E+(1\pm1)k]\}. \quad (23)$$

The differential radiative probability can be inferred

[4] J. K. Knipp and G. E. Uhlenbeck, Physica **3**, 425 (1936); F. Bloch, Phys. Rev. **50**, 272 (1936).

from (14):

$$d\Phi(E,k,\theta,\mathbf{e}^{\pm})=d\Omega_k\frac{e^2p}{8\pi^2\mu k(E+k)}\times\left\{\frac{p^2\sin^2\theta[E+(1\pm1)k]}{[E-p\cos\theta]^2}+\frac{(1\pm1)k^2}{E-p\cos\theta}\right\}. \quad (24)$$

One advantage of writing $S(k)$ as in (21) lies in the fact that the experimentally observed β spectrum can be inserted directly and unambiguously. When we go through the above steps with the most general β interaction, the formulas (7) through (24) will change in an obvious manner. Thus (16) and (19) become[5]

$$S(k,\mathbf{e}^{\pm})=(e^2/8\pi^4k)\xi[\mathcal{J}_1+(1\pm a)\mathcal{J}_2+b\mathcal{J}_3], \quad (25)$$

and

$$\mathcal{P}(k)=\frac{a\mathcal{J}_2}{\mathcal{J}_1+\mathcal{J}_2+b\mathcal{J}_3}, \quad (26)$$

where

$$\mathcal{J}_3=4\int_1^x dE(x-E)^2[E\ln(E+p)-p]$$

$$=\ln(x+s)\left(\frac{1}{3}x^4+x^2+\frac{1}{8}\right)-s\left(\frac{25}{36}x^3+\frac{55}{72}x\right). \quad (27)$$

$\mathcal{J}_1$ and $\mathcal{J}_2$ are given by (17) and (18), and

$$\xi=[|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2]|M_F|^2+[|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2]|M_{GT}|^2,$$

$$a\xi=[-C_S^*C_S'-C_SC_S'^*+C_V^*C_V'+C_VC_V'^*]|M_F|^2+[-C_T^*C_T'-C_TC_T^{*\prime}+C_A^*C_A'+C_AC_A'^*]|M_{GT}|^2,$$

$$b\xi=[C_S^*C_V+C_SC_V^*+C_S^{*\prime}C_V'+C_S'C_V^{*\prime}]|M_F|^2+[C_T^*C_A+C_TC_A^*+C_T^{*\prime}C_A'+C_T'C_A^{*\prime}]|M_{GT}|^2. \quad (28$$

We have set $|\langle\beta\rangle|^2=|\langle1\rangle|^2=|M_F|^2$ and $|\langle\beta\boldsymbol{\sigma}\rangle|^2=|\langle\boldsymbol{\sigma}\rangle|^2=|M_{GT}|^2$.

The Fierz interference term b is known to be quite small. If we set $b=0$, (26) still differs from (19), but only by the constant multiplicative factor a. From experiments measuring the circular polarization at any one energy, we should therefore be able to determine the constant a.

3. EFFECT OF THE NUCLEAR COULOMB FIELD

When we use the correct Coulomb field Green's function, $\mathcal{G}_{E+k}(\mathbf{r})$ and wave function, $\psi_E(\mathbf{r})$, in (4) and take all the coupling constants in the β interaction to be real, we find a very simple correction to the first order in $Z\alpha$. [Z is the nuclear charge and $\alpha(=e^2)$ is the fine structure constant.] One part of this Coulomb correction may be recognized and evaluated immediately. The absolute value of the final state electron wave function at the nucleus differs from unity when the Coulomb field is taken into account. This absolute value is found to occur as a factor in the wave function and consequently as a factor in the expression for the matrix element M. The value for $|M|^2$ obtained with the Born approximation should therefore be multiplied by the square of the absolute value of the final state electron wave function at the nucleus. This is simply the well-known Sommerfeld factor,

$$|\psi_E(0)|^2=\frac{2\pi Z\alpha E/p}{1-\exp(-2\pi Z\alpha E/p)},$$

or its relativistic analog, the Fermi function. Let us denote this factor by $F(Z,E)$. It should be noted that it is the energy and the momentum of the electron in its final state that occurs in this Sommerfeld factor, rather than the energy and momentum with which the electron is "born" at the nucleus. The remaining first-order Coulomb corrections in M are also found without difficulty. However, these remaining corrections both from the Green's function and the wave function are all found to be imaginary (or out of phase) with respect to the zero-order terms. They will therefore contribute to $|M|^2$ only as second and higher order corrections in $Z\alpha$. Our result agrees with the first order Coulomb correction to the total spectrum, $S(k)$, recently calculated by Lewis and Ford[6] with a rather elaborate third order perturbation procedure.

It should be stressed again that this result is only correct if the β-coupling constants are all in phase. If the theory is not invariant under time inversion (weak), we should expect any imaginary first order terms in M to contribute to the first order in $Z\alpha$ in $|M|^2$ and $S(k)$ as well. Assuming the β-coupling constants to be in phase, the formulas for the degree of polarization, (19) and (26), take the same form to the first order in $Z\alpha$, but with the factor $F(Z,E)=1+(\pi Z\alpha E/p)+\cdots$, inserted in the integrals $\mathcal{J}_1$, $\mathcal{J}_2$ and $\mathcal{J}_3$. Similarly we insert the factor $F(Z,E)$ in the integrand of (21),

$$S(k)=\int_m^{E_0-k}dEF(Z,E)P(E+k)\Phi(E,k). \quad (29)$$

If we use the observed β-decay spectrum, which should take the form,

$$P(Z,E+k)=F(Z,E+k)P(E+k),$$

and define $\Phi(Z,E,k)$ by

$$S(k)=\int_m^{E_0-k}dEP(Z,E+k)\Phi(Z,E,k),$$

[5] After completing the work reported here, we were informed about a letter by G. W. Ford [Phys. Rev. **107**, 320 (1957)], wherein he obtains the same result for the degree of polarization as in our formula (26).

[6] G. W. Ford and R. R. Lewis, Bull. Am. Phys. Soc. Ser. II, **1**, 195 (1956), and Atomic Energy Commission Report AEC-AT (11-1)-427 (unpublished). (See also S. B. Nilsson, Arkiv för Fysik, **10**, 467 (1956).

we see that to first order in $Z\alpha$:

$$\Phi(Z,E,k)=\frac{F(Z,E)}{F(Z,E+k)}\Phi(E,k). \quad (30)$$

The analogy with the Elwert correction in external bremsstrahlung is obvious. The theory of internal bremsstrahlung has been used primarily to predict photon intensity relative to the electron intensity.

We see from (30) that the ratio of photon intensity to electron intensity is less sensitive to the influence of the Coulomb field than either intensity by itself. There is a partial cancellation of the Coulomb effects on the two intensities in taking the ratio. Lewis and Ford[6] express the hope that a similar cancellation will take place in the higher order corrections. The second order terms which have been neglected, however, are not simply wave function normalization factors, and there is therefore no reason to anticipate further cancellations. The second order Coulomb terms are somewhat more difficult to calculate. A discussion of the $(Z\alpha)^2$ corrections will be given in a subsequent paper. It is our hope to investigate their bearing on the disagreement with the present theory, which has recently been reported by several experimenters.[7]

In closing we wish to express our gratitude to Professor R. J. Glauber, who suggested the problem, for many helpful discussions.

[7] K. Liden and N. Starfelt, Phys. Rev. **97**, 419 (1955); N. Starfelt and N. L. Svantesson, Phys. Rev. **97**, 708 (1955); H. Langevin-Joliot, thesis, Paris, 1956 (unpublished).

PHYSICAL REVIEW VOLUME 107, NUMBER 6 SEPTEMBER 15, 1957

Elastic Scattering of a Σ^+ Hyperon from a Free Proton*

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(Received June 3, 1957)

An event has been found in nuclear emulsion which is interpreted as an elastic scattering of a Σ^+ hyperon from a free proton. The center-of-mass scattering angle was 125 degrees and the energy of the hyperon at the point of scattering was 22.9 Mev.

DURING a systematic study of the interactions of K^- mesons in nuclear emulsion, an event was found which is interpreted as an elastic scattering of a Σ^+ hyperon from a free proton. A photomicrograph of this event is shown in Fig. 1. A K^- meson, which was identified by ionization *versus* range, came to rest and produced a star with only one prong, track A. After traversing 4.79 mm of emulsion, prong A interacted yielding prongs B and C. Prong B then decayed at rest into a lightly ionizing track D which interacted in flight after traversing 28.2 mm of emulsion. The decay was characteristic of a Σ^+ hyperon which decayed at rest into a π^+ meson. Prong C originated at the point of scattering and went 1.52 mm before stopping with a p-ending.

An important feature of the event is the coplanarity of the tracks A, B and C. Track A made an angle of only 0.2 ± 1.5 deg with the plane of tracks B and C. The coplanarity suggests a two-body collision. From the measured angles and ranges that appear in Table I and energy and momentum conservation laws, the mass of the scattered particle can be calculated in a number of ways.[1] The results of seven of these calculations are shown in Table II. In the second column appear the measured quantities that were used in each method for calculating the mass, in the third column the conservation laws that were used, and in the fourth column the resulting mass values. In the third column E, LM, and TM stand for the conservation laws of energy, longitudinal momentum (the total momentum parallel to the direction of the incident particle A), and transverse momentum (the total momentum perpendicular to the direction of A in the plane of ABC), respectively. In method 5 the mass of the scattered particle, B, depends on the direction of the incident particle, A, but not on its mass. In method 7 the calculated mass is independent of the direction of the incident particle but not of its mass. In all calculations except for method 5 the mass of the incident particle was assumed to be the same as that of the scattered particle B, and particle C was assumed to be a proton.

TABLE I. Range and angle measurements from event 659.

Prong	Range (mm)	θ^b (deg)
$A(\Sigma^+)$	4.79[a]	75.6 ± 1.5
$B(\Sigma^+)$	0.161 ± 0.003	152.4 ± 1.5
$C(p)$	$1.52\ \pm0.02$	131.8 ± 1.5

[a] This range was measured from the K^- capture point to the scattering point.
[b] Angle θ is the angle opposite the prong listed in the first column (i.e., the angle between the two other prongs).

* Work performed under auspices of U. S. Atomic Energy Commission.

[1] Gilbert, Violet, and White, Phys. Rev. **103**, 1825 (1956).