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Nucleon Exchange in Deuteron Stripping Reactions

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In the normal theory of stripping reactions it is assumed that the outgoing nucleon comes from the incident deuteron. It is well known that this is not strictly justified (although it may be a good approximation in many cases) because of the need in principle to antisymmetrize the total wave function. This paper discusses the process of stripping with nucleon exchange that appears when the antisymmetry is taken into account. With the help of simplifying assumptions, expressions are obtained for the exchange amplitude in a direct transition between initial and final states of the system. The relation of this treatment to a theory involving compound nucleus formation is discussed.

I. INTRODUCTION

It is well known that the theory of stripping reactions developed by Butler,[1] though remarkably successful in describing the main features of many such reactions, does not by itself account for all that happens. In some cases the characteristic peaking of the angular distribution of outgoing particles in the region of small angles, demanded by Butler's theory, does not occur at all in practice. More usually, however, there is observed an angular distribution which has a general similarity to the Butler curve for the relevant energy and angular momentum transfer, but with the property that an appreciable intensity of outgoing particles takes place at the more backward angles for which the theoretical stripping intensity would be vanishingly small. The natural inference is that the stripping mechanism is also accompanied by a certain amount of compound nucleus formation, which will lead in general to a more or less isotropic angular distribution. A difficulty with this explanation is that the compound nucleus process ought to be incoherent with the stripping process if many overlapping levels of the compound nucleus are involved; and at the high excitations typical of stripping reaction energies this would be a normal condition. Experiments have, however, shown[2,3] the occurrence of broad resonances in (d,p) excitation curves which are very much more pronounced when measurements are made at the forward peaks of the angular distributions than at the minima. This suggests interference between the stripping amplitude and a second amplitude which exhibits resonances at various deuteron energies. The theory of pure stripping of course gives a monotonic variation of cross section with energy.

The purpose of the present paper is to discuss an enlargement of stripping theory to include the exchange of a nucleon in the deuteron with a nucleon inside the target nucleus. This process can be considered as a result of the complete antisymmetrization of the wave functions involved in the collision process. Although in principle this antisymmetrization and its consequences must always be considered, its physical importance will depend on the likelihood that the exchanging nucleons shall interact with each other. Since this depends on the possibility of an overlap of their wave functions, an exchange contribution to the reaction process will be important only if both nucleons in the incident deuteron (and not merely the nucleon being captured) come close to the target nucleus. Regarded in these terms, therefore, an exchange contribution to the stripping mechanism is a rather specific means of introducing something equivalent to what is normally thought of as compound nucleus formation. The Butler formulation of stripping theory specifically discards this exchange contribution, by taking it for granted that the outgoing nucleon in the final stage of a stripping process was originally contained in the incident deuteron.

One feature of the present treatment, which marks it off from a compound nucleus picture, is that it invokes no specific properties, such as angular momentum, parity, and characteristic energy, of the intermediate state. It speaks instead in terms of a direct transition from an initial state of two colliding particles to a final state of two separating particles. Under the conditions that normally hold in the study of stripping reactions, this approach seems to be rather reasonable, since the lifetimes of any intermediate states are almost certainly very short, the overlapping of levels is considerable, and an expression of complete ignorance concerning the intermediate stage is not far from the truth in most cases.

II. TOTAL SCATTERING AMPLITUDE IN A STRIPPING REACTION

Our presentation of the problem will have much in common with the formulation due to Gerjuoy,[4] who analyzed and related the various treatments of what we shall call normal or "direct" stripping. We first consider a plane wave of deuterons incident upon a target nucleus consisting of a "core" together with a single nucleon which we

[1] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).
[2] Stratton, Blair, Famularo, and Stuart, Phys. Rev. **98**, 629 (1955).
[3] J. B. Marion and G. Weber, Phys. Rev. **103**, 167, 1408 (1956).
[4] E. Gerjuoy, Phys. Rev. **91**, 645 (1953).

shall assume is the one involved in a possible exchange process. Let us for definiteness assume that we are concerned with a (d,p) reaction, so that the single exchangeable nucleon in the target nucleus is a proton. We then admit the possibility that the outgoing proton may have come either from the deuteron or from the target nucleus itself. We shall label the protons in such a way that proton 1 is always the one which is unbound in the final state. The wave function of the residual nucleus can thus be written as $\psi_r(2,n,c)$, where 2 refers to the proton that is bound in the final state, n refers to the captured neutron, and c refers to the core of the target nucleus. (We assume that the core suffers no internal rearrangements as a result of the reaction.) The scattering amplitude for the (d,p) reaction is then given by

$$A=\lim_{r_1\to\infty} r_1 e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}\chi_{\frac{1}{2}}^{*\sigma_1'}(\mathbf{s}_1)\int\psi_r^*(\mathbf{2nc})\Psi(\mathbf{12nc})d\mathbf{2}d\mathbf{n}d\mathbf{c}, \tag{1}$$

where Ψ is the total wave function of the system, and $\chi_{\frac{1}{2}}^{\sigma_1'}$ is a normalized proton spin function belonging to total spin $\frac{1}{2}$ and z-component σ_1'. (Where necessary, primes are used to distinguish symbols referring to the configuration after the collision.) The vectors **1**, **2**, **n**, and **c** represent the total space and spin coordinates of these particles.

The complete Hamiltonian of the system is

$$H=T_1+T_2+T_n+V_{nc}+V_{1c}+V_{2c}+V_{1n}+V_{2n}+V_{12}, \tag{2}$$

where T refers to the various kinetic energy operators and V to the interactions. (We assume the core to be infinitely heavy.) The Hamiltonian for the residual nucleus r is

$$H_r=T_2+T_n+V_{nc}+V_{2c}+V_{2n}. \tag{3}$$

Thus

$$H=H_r+T_1+V_{1c}+V_{1n}+V_{2n}. \tag{4}$$

We also have the eigenvalue equations:

$$H\Psi=E\Psi, \tag{5}$$

$$H_r\psi_r=E_r\psi_r. \tag{6}$$

These, together with Eq. (4), give

$$[T_1-(E-E_r)]\int\psi_r^*(\mathbf{2nc})\Psi(\mathbf{12nc})d\mathbf{2}d\mathbf{n}d\mathbf{c}=-\int\psi_r^*(\mathbf{2nc})[V_{1c}+V_{1n}+V_{12}]\Psi(\mathbf{12nc})d\mathbf{2}d\mathbf{n}d\mathbf{c}. \tag{7}$$

By a procedure similar to that of Gerjuoy, it can be shown that this leads to the following asymptotic form for A:

$$A(\mathbf{n}_1)=-\frac{1}{4\pi}\left(\frac{2M}{\hbar^2}\right)\int e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}\chi_{\frac{1}{2}}^{\sigma_1'}(\mathbf{s}_1)\psi_r^*(\mathbf{2nc})[V_{1c}+V_{1n}+V_{12}]\Psi(\mathbf{12nc})d\mathbf{1}d\mathbf{2}d\mathbf{n}d\mathbf{c}, \tag{8}$$

where M is the mass of a nucleon, and $\mathbf{n}_1$ denotes the direction of $\mathbf{k}_1$.

Now Ψ can be written in the form:

$$\Psi(\mathbf{12nc})=\Psi_d(\mathbf{1n})e^{\frac{1}{2}i\mathbf{k}_d\cdot(\mathbf{r}_1+\mathbf{r}_n)}\psi_t(\mathbf{2c})+\Phi(\mathbf{12nc}), \tag{9}$$

where Φ is everywhere outgoing. $\psi_d(\mathbf{1n})$ is the internal wave function of the deuteron and $\psi_t(\mathbf{2c})$ is the wave function of the target nucleus. Now this form of Ψ is not antisymmetrical with respect to interchange of protons 1 and 2. We therefore write

$$\Psi(\mathbf{12nc})=\psi_d(\mathbf{1n})e^{\frac{1}{2}i\mathbf{k}_d\cdot(\mathbf{r}_1+\mathbf{r}_n)}\psi_t(\mathbf{2c})-\psi_d(\mathbf{2n})e^{\frac{1}{2}i\mathbf{k}_d\cdot(\mathbf{r}_2+\mathbf{r}_n)}\psi_t(\mathbf{1c})+\Phi(\mathbf{12nc}). \tag{10}$$

This, together with (8), gives the required antisymmetrical form for A, since it can be shown that it is unnecessary to antisymmetrize the final wave function $\psi_r(\mathbf{2n})\exp(i\mathbf{k}_1\cdot\mathbf{r}_1)$ as well as Ψ.

In the Born approximation, we neglect the contribution which Φ makes to A, so that we have for the total scattering amplitude:

$$A(\mathbf{n}_1)=-\frac{1}{4\pi}\left(\frac{2M}{\hbar^2}\right)[F-G], \tag{11}$$

where

$$F=\int e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}\chi_{\frac{1}{2}}^{*\sigma_1'}(\mathbf{s}_1)\psi_r^*(\mathbf{2nc})[V_{1c}+V_{1n}+V_{12}]\psi_d(\mathbf{1n})e^{\frac{1}{2}i\mathbf{k}_d\cdot(\mathbf{r}_1+\mathbf{r}_n)}\psi_t(\mathbf{2c})d\mathbf{1}d\mathbf{2}d\mathbf{n}d\mathbf{c}, \tag{12}$$

$$G=\int e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}\chi_{\frac{1}{2}}^{*\sigma_1'}(\mathbf{s}_1)\psi_r^*(\mathbf{2nc})[V_{1c}+V_{1n}+V_{12}]\psi_d(\mathbf{2n})e^{\frac{1}{2}i\mathbf{k}_d\cdot(\mathbf{r}_2+\mathbf{r}_n)}\psi_t(\mathbf{1c})d\mathbf{1}d\mathbf{2}d\mathbf{n}d\mathbf{c}. \tag{13}$$

The outgoing proton intensity is then simply $|A|^2$. It is easy to show that we may write

$$V_{1c} \equiv V_{2c} + V_{nc}, \quad V_{1n} \equiv V_{nc} + V_{2n}, \tag{14}$$

without introducing any further approximations, although we shall not in fact make use of these relations in the present treatment.

We shall ignore the contribution to F and G produced by the V_{12} interaction, thus assuming that protons 1 and 2 interact with each other only indirectly. F and G can then each be split into two terms arising from V_{1c} and V_{1n} separately. Thus:

$$F = F_{1n} + F_{1c}, \quad G = G_{1n} + G_{1c}. \tag{15}$$

We shall consider these four terms separately, assuming an infinitely heavy target nucleus in each case so that center of mass corrections can be neglected.

III. EVALUATION OF G_{1n}

In this section the following notation will be used: $C_{j_1j_2}(j_3m_3;\, m_1m_2)$ is a Clebsch-Gordan coefficient as defined by Blatt and Weisskopf[5]; $f(r)$ and $g(r)$ are the normalized radial wave functions of bound proton and captured neutron, respectively, in the nucleus; χ_s^σ represents a normalized spin wave function characterized by a total spin s and its z component σ. The total angular momentum j of any system or subsystem is composed of orbital angular momentum l and spin s, the z components of these three quantities having quantum numbers m, λ, and σ, respectively. Suffixes will be used to identify parts of the system as follows: 1—a proton initially bound in the target nucleus and subsequently ejected from it; 2—the proton initially bound in the deuteron and subsequently captured by the target nucleus; n—the neutron contained in the deuteron; d—the deuteron, t—the target nucleus, r—the residual nucleus; c—the "core" of the target nucleus, depending for its description upon all the nucleon coordinates apart from those of **1**, **2**, and **n**.

G_{1n} can be expressed as the sum of a number of terms depending upon the z-component angular momentum quantum numbers defining the initial and final states of the whole system. Thus:

$$G_{1n} = \sum_{m_t m_d m_r \sigma_1'} G_{1n}{}^{m_t m_d;\, m_r \sigma_1'}. \tag{16}$$

We now expand the wave functions for the target and residual nuclei in terms of the complete set of simultaneous eigenfunctions of the total angular momenta and their z components, for the component parts of these two nuclei:

$$\psi_t{}^{m_t}(\mathbf{1c}) = \sum_{m_1 m_c \lambda_1 \sigma_1} C_{l_1\frac{1}{2}}(j_1m_1;\, \lambda_1\sigma_1) C_{j_1j_c}(j_tm_t;\, m_1m_c) \chi_{\frac{1}{2}}{}^{\sigma_1}(\mathbf{s}_1) \psi_{j_c}{}^{m_c}(\mathbf{c}) f^{l_1}(r_1) Y_{l_1}{}^{\lambda_1}(\Omega_1), \tag{17}$$

$$\psi_r{}^{m_r}(\mathbf{2nc}) = \sum_{m_{nc} m_2 m_n m_c'} \sum_{\lambda_n \lambda_2 \sigma_n' \sigma_2'} C_{l_n\frac{1}{2}}(j_nm_n;\, \lambda_n\sigma_n') C_{j_nj_c'}(j_{nc}m_{nc};\, m_nm_c') C_{l_2\frac{1}{2}}(j_2m_2;\, \lambda_2\sigma_2') C_{j_{nc}j_2}(j_rm_r;\, m_{nc}m_2)$$
$$\times \chi_{\frac{1}{2}}{}^{\sigma_2'}(\mathbf{s}_2) \chi_{\frac{1}{2}}{}^{\sigma_n'}(\mathbf{s}_n) \psi_{j_c'}{}^{m_c'}(\mathbf{c}) f^{l_2}(r_2) g^{l_n}(r_n) Y_{l_2}{}^{\lambda_2}(\Omega_2) Y_{l_n}{}^{\lambda_n}(\Omega_n). \tag{18}$$

We have here assumed jj coupling, and have supposed that n and c are first coupled to give the subsystem nc, which then couples with 2 to give r.

We also have

$$\Psi_d{}^{m_d}(\mathbf{2n}) = \sum_{\sigma_2\sigma_n} C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\, \sigma_2\sigma_n) \chi_{\frac{1}{2}}{}^{\sigma_2}(\mathbf{s}_2) \chi_{\frac{1}{2}}{}^{\sigma_n}(\mathbf{s}_n) \phi_d(r_{2n}), \tag{19}$$

where $\phi_d(r_{2n})$ is the spatial part of the internal wave function of the deuteron. Using (17), (18), and (19) in Eq. (13), and the usual orthonormality relations between the various spin wave functions, we obtain

$$G_{1n}{}^{m_t m_d;\, m_r \sigma_1'} = \sum_{m_1 m_2 m_n m_c m_{nc}} \sum_{\lambda_1 \lambda_2 \lambda_n \sigma_2 \sigma_n} C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\, \sigma_2\sigma_n) C_{l_1\frac{1}{2}}(j_1m_1;\, \lambda_1\sigma_1') C_{j_1j_c}(j_tm_t;\, m_1m_c) C_{l_n\frac{1}{2}}(j_nm_n;\, \lambda_n\sigma_n)$$
$$\times C_{j_nj_c}(j_{nc}m_{nc};\, m_nm_c) C_{l_2\frac{1}{2}}(j_2m_2;\, \lambda_2\sigma_2) C_{j_{nc}j_2}(j_rm_r;\, m_{nc}m_2)$$
$$\times \int e^{-i\mathbf{k}_1\cdot\mathbf{r}_1} f^{*l_2}(r_2) g^{*l_n}(r_n) Y_{l_2}{}^{*\lambda_2}(\Omega_2) Y_{l_n}{}^{*\lambda_n}(\Omega_n) V_{1n} e^{\frac{1}{2}i\mathbf{k}_d\cdot(\mathbf{r}_2+\mathbf{r}_n)} \phi_d(r_{2n}) f^{l_1}(r_1) Y_{l_1}{}^{\lambda_1}(\Omega_1) d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_n. \tag{20}$$

In order to separate the integrations over $d\mathbf{r}_1$, $d\mathbf{r}_2$, and $d\mathbf{r}_n$, we make the assumption that V_{1n} is of the form

$$V_{1n} = V_0 \delta(\mathbf{r}_1, \mathbf{r}_n) \tag{21}$$

[5] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 789.

where

$$\delta(\mathbf{r}_1,\mathbf{r}_n)=0, \quad (\mathbf{r}_1\neq\mathbf{r}_n); \quad \int_{-\infty}^{\infty}\delta(\mathbf{r}_1,\mathbf{r}_n)d\mathbf{r}_{1n}=1, \quad (\mathbf{r}_{1n}=\mathbf{r}_1-\mathbf{r}_n).$$

We also introduce Hulthèn's form[6] for $\phi_d(r_{2n})$, *viz.*:

$$\phi_d(r_{2n})=B\cdot\frac{1}{r_{2n}}e^{-\alpha r_{2n}}(1-e^{-\beta r_{2n}}), \tag{22}$$

where

$$B=\frac{1}{\beta}\left[\frac{\alpha(\alpha+\beta)(2\alpha+\beta)}{2\pi}\right]^{\frac{1}{2}}.$$

This can be expanded in terms of spherical harmonics of the angles Ω_2 and Ω_n, as follows:

$$\phi_d(r_{2n})=\frac{4\pi B}{(r_2r_n)^{\frac{1}{2}}}\sum_{\substack{l=0\\ r_2<r_n}}^{\infty}\sum_{\lambda=-l}^{+l}\{I_{l+\frac{1}{2}}(\alpha r_2)K_{l+\frac{1}{2}}(\alpha r_n)-I_{l+\frac{1}{2}}[(\alpha+\beta)r_2]K_{l+\frac{1}{2}}[(\alpha+\beta)r_n]\}Y_l^{\lambda}(\Omega_2)Y_l^{*\lambda}(\Omega_n). \tag{23}$$

$I(\alpha r)$ and $K(\alpha r)$ are Bessel functions of imaginary argument as defined by Watson.[7] Using Eqs. (23) and (21) in (20), and denoting the product of the seven Clebsch-Gordan coefficients in (20) by the symbol $\prod C_j^{1n}$, we have

$$G_{1n}^{m_tm_d;\,m_r\sigma_1'}=G_{1n}^{m_tm_d;\,m_r\sigma_1';\,\alpha}-G_{1n}^{m_tm_d;\,m_r\sigma_1';\,\alpha+\beta}, \tag{24}$$

$$G_{1n}^{m_tm_d;\,m_r\sigma_1';\,\alpha}=4\pi B\sum_{m_1m_2m_nm_c}\;\sum_{m_{nc}\lambda_1\lambda_2\lambda_n}\;\sum_{\sigma_2\sigma_nl\lambda}\prod C_j^{1n}\int f^{*l_2}(r_2)\frac{I_{l+\frac{1}{2}}(\alpha r_2)}{r_2^{\frac{1}{2}}}e^{\frac{1}{2}i\mathbf{k}_d\cdot\mathbf{r}_2}Y_{l_2}^{*\lambda_2}(\Omega_2)Y_l^{\lambda}(\Omega_2)d\mathbf{r}_2$$

$$\times\int g^{*l_n}(r_n)f^{l_1}(r_n)\frac{K_{l+\frac{1}{2}}(\alpha r_n)}{r_n^{\frac{1}{2}}}e^{i\mathbf{K}\cdot\mathbf{r}_n}Y_{l_n}^{*\lambda_n}(\Omega_n)Y_{l_1}^{\lambda_1}(\Omega_n)Y_l^{*\lambda}(\Omega_n)d\mathbf{r}_n, \tag{25}$$

where

$$\mathbf{K}=\tfrac{1}{2}\mathbf{k}_d-\mathbf{k}_1.$$

The products of spherical harmonics of the same argument may be expanded as sums of single spherical harmonics as follows:

$$Y_{l_n}^{*\lambda_n}(\Omega_n)Y_{l_1}^{\lambda_1}(\Omega_n)=(-)^{\lambda_n}\sum_{L''}\left[\frac{(2l_n+1)(2l_1+1)}{4\pi(2L''+1)}\right]^{\frac{1}{2}}C_{l_nl_1}(L''0;00)C_{l_nl_1}(L''\lambda_1-\lambda_n;\lambda_1,-\lambda_n)Y_{L''}^{\lambda_1-\lambda_n}(\Omega_n), \tag{26}$$

$$Y_{L''}^{\lambda_1-\lambda_n}(\Omega_n)Y_l^{*\lambda}(\Omega_n)=(-)^{\lambda}\sum_{L'}\left[\frac{(2L''+1)(2l+1)}{4\pi(2L'+1)}\right]^{\frac{1}{2}}C_{L''l}(L'0;00)C_{L''l}(L'\lambda_1-\lambda_n-\lambda;\lambda_1-\lambda_n,-\lambda)Y_{L'}^{\lambda_1-\lambda_n-\lambda}(\Omega_n), \tag{27}$$

$$Y_{l_2}^{*\lambda_2}(\Omega_2)Y_l^{\lambda}(\Omega_2)=(-)^{\lambda}\sum_{L}\left[\frac{(2l_2+1)(2l+1)}{4\pi(2L+1)}\right]^{\frac{1}{2}}C_{l_2l}(L0;00)C_{l_2l}(L\lambda_2-\lambda;\lambda_2,-\lambda)Y_L^{*\lambda_2-\lambda}(\Omega_2). \tag{28}$$

Also we can expand the two plane waves along the direction of $\mathbf{K}$:

$$e^{\frac{1}{2}i\mathbf{k}_d\cdot\mathbf{r}_2}=\textstyle\sum_{Pp}4\pi i^Pj_P(\frac{1}{2}k_dr_2)Y_P^{p}(\Omega_2)Y_P^{*p}(\mathbf{k}_d,\mathbf{K}), \tag{29}$$

[where $(\mathbf{k}_d,\mathbf{K})$ is the angle between $\mathbf{k}_d$ and $\mathbf{K}$],

$$e^{i\mathbf{K}\cdot\mathbf{r}_n}=\textstyle\sum_{P'}[4\pi(2P'+1)]^{\frac{1}{2}}i^{P'}j_{P'}(Kr_n)Y_{P'}^{0}(\Omega_n). \tag{30}$$

Using Eqs. (26) to (30) in (25) and making use of the orthonormality properties of the spherical harmonics, we

[6] L. Hulthèn, Arkiv. Mat., Astron. Fysik **28A**, No. 5 (1952).
[7] G. N. Watson, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, New York, 1944), Chap. 3.

have

$$G_{1n}{}^{m_tm_d;\,m_r\sigma_1';\,\alpha}=4\pi BV_0\sum_{m_1m_2m_nm_cm_{nc}}\sum_{\lambda_1\lambda_2\lambda_n\lambda}\sum_{\sigma_1\sigma_2\sigma_n}\sum_{lLL'L''}(-)^{\lambda_n}i^{L+L'}\left[\frac{(2l+1)^2(2l_1+1)(2l_2+1)(2l_n+1)}{(2L+1)}\right]^{\frac{1}{2}}\prod C_j{}^{1n}$$
$$\times C_{l_nl_1}(L''0;00)C_{L''l}(L'0;00)C_{l_2l}(L_0;00)C_{l_nl_1}(L''\lambda_1-\lambda_n;\lambda_1,-\lambda_n)$$
$$\times C_{L''l}(L'\lambda_1-\lambda_n-\lambda;\lambda_1-\lambda_n,-\lambda)C_{l_2l}(L\lambda_2-\lambda;\lambda_2,-\lambda)\int f^{*l_2}(r_2)\frac{I_{l+\frac{1}{2}}(\alpha r_2)}{r_2^{\frac{1}{2}}}j_L(\tfrac{1}{2}k_dr_2)r_2^2dr_2$$
$$\times\int g^{*l_n}(r_n)f^{l_1}(r_n)\frac{K_{l+\frac{1}{2}}(\alpha r_n)}{r_n^{\frac{1}{2}}}j_{L'}(Kr_n)r_n^2dr_nY_L{}^{*\lambda_2-\lambda}(\mathbf{k}_d,\mathbf{K}).\quad(31)$$

This equation, together with (16) and (24), gives the required expression for G_{1n}.

IV. EVALUATION OF G_{1c}

We now consider the scattering of the emitted proton by the core of the nucleus. This implies that the state of the core will be changed in the collision process. It will be assumed that this change of state is associated with only one of the nucleons composing the core, and we label this nucleon by the suffix ν, the rest of the core being denoted by ξ. We can write $\psi_{j_c}{}^{m_c}(\mathbf{c})$ in terms of the simultaneous eigenfunctions for ξ and ν, thus:

$$\psi_{j_c}{}^{m_c}(\mathbf{c})=\sum_{\lambda_\nu\sigma_\nu}C_{l_\nu\frac{1}{2}}(j_cm_c;\lambda_\nu\sigma_\nu)\chi_{\frac{1}{2}}{}^{\sigma_\nu}(\mathbf{s}_\nu)h^{l_\nu}(r_\nu)Y_{l_\nu}{}^{\lambda_\nu}(\Omega_\nu)\psi_0{}^0(\xi);\quad(32)$$

we have here assumed that the nucleons included in ξ are combined so as to give a state having a total j of zero; this is therefore a special assumption (made in the interests of simplicity) which will not be generally applicable. For the residual nucleus we can in this case write:

$$\psi_{j_c'}{}^{m_c'}(\mathbf{c})=\sum_{\lambda_\nu'\sigma_\nu'}C_{l_\nu'\frac{1}{2}}(j_c'm_c';\lambda_\nu'\sigma_\nu')h^{l_\nu'}(r_\nu)Y_{l_\nu}{}^{\lambda_\nu}(\Omega_\nu)\psi_0{}^0(\xi).\quad(33)$$

The total wave functions for the target and residual nuclei can therefore be written:

$$\psi_t{}^{m_t}(\mathbf{1c})=\sum_{m_1m_c\lambda_1\lambda_\nu}\sum_{\sigma_1\sigma_\nu}C_{l_1\frac{1}{2}}(j_1m_1;\lambda_1\sigma_1)C_{j_1j_c}(j_tm_t;m_1m_c)C_{l_\nu\frac{1}{2}}(j_cm_c;\lambda_\nu\sigma_\nu)\chi_{\frac{1}{2}}{}^{\sigma_1}(\mathbf{s}_1)\chi_{\frac{1}{2}}{}^{\sigma_\nu}(\mathbf{s}_\nu)\psi_0{}^0(\xi)$$
$$\times f^{l_1}(r_1)h^{l_\nu}(r_\nu)Y_{l_1}{}^{\lambda_1}(\Omega_1)Y_{l_\nu}{}^{\lambda_\nu}(\Omega_\nu),\quad(34)$$

$$\psi_r{}^{m_r}(\mathbf{2nc})=\sum_{m_2m_nm_{nc}m_c'}\sum_{\lambda_2\lambda_n\lambda_\nu'}\sum_{\sigma_2'\sigma_n'\sigma_\nu'}C_{l_n\frac{1}{2}}(j_nm_n;\lambda_n\sigma_n')C_{j_nj_c'}(j_{nc}m_{nc};m_nm_c')C_{l_2\frac{1}{2}}(j_2m_2;\lambda_2\sigma_2')C_{j_{nc}j_2}(j_rm_r;m_{nc}m_2)$$
$$\times C_{l_\nu'\frac{1}{2}}(j_c'm_c';\lambda_\nu'\sigma_\nu')\chi_{\frac{1}{2}}{}^{\sigma_2}(\mathbf{s}_2)\chi_{\frac{1}{2}}{}^{\sigma_\nu'}(\mathbf{s}_\nu)\chi_{\frac{1}{2}}{}^{\sigma_n'}(\mathbf{s}_n)\psi_0{}^0(\xi)f^{l_2}(r_2)g^{l_n}(r_n)h^{l_\nu'}(r_\nu)Y_{l_2}{}^{\lambda_2}(\Omega_2)Y_{l_n}{}^{\lambda_n}(\Omega_n)Y_{l_\nu'}{}^{\lambda_\nu'}(\Omega_\nu).\quad(35)$$

Using Eqs. (19), (34), and (35) in (13), together with the various orthonormality relations between the spin wave functions, we obtain

$$G_{1c}{}^{m_tm_d;\,m_r\sigma_1'}=\sum_{m_1m_2m_n}\sum_{m_cm_c'm_{nc}}\sum_{\lambda_1\lambda_2\lambda_n\lambda_\nu\lambda_\nu'}\sum_{\sigma_2\sigma_n\sigma_\nu}C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\sigma_2\sigma_n)C_{l_1\frac{1}{2}}(j_1m_1;\lambda_1\sigma_1')C_{j_1j_c}(j_tm_t;m_1m_c)C_{l_\nu\frac{1}{2}}(j_cm_c;m_\nu\sigma_\nu)$$
$$\times C_{l_n\frac{1}{2}}(j_nm_n;\lambda_n\sigma_n)C_{j_nj_c'}(j_{nc}m_{nc};m_nm_c')C_{l_2\frac{1}{2}}(j_2m_2;\lambda_2\sigma_2)C_{j_{nc}j_2}(j_rm_r;m_{nc}m_2)$$
$$\times C_{l_\nu'\frac{1}{2}}(j_c'm_c';m_\nu'\sigma_\nu)\int e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}f^{*l_2}(r_2)g^{*l_n}(r_n)h^{*l_\nu'}(r_\nu)Y_{l_2}{}^{*\lambda_2}(\Omega_2)Y_{l_n}{}^{*\lambda_n}(\Omega_n)Y_{l_\nu'}{}^{*\lambda_\nu'}(\Omega_\nu)$$
$$\times V_{1c}e^{\frac{1}{2}i\mathbf{k}_d\cdot(\mathbf{r}_2+\mathbf{r}_n)}\phi_d(r_{2n})f^{l_1}(r_1)h^{l_\nu}(r_\nu)Y_{l_1}{}^{\lambda_1}(\Omega_1)Y_{l_\nu}{}^{\lambda_\nu}(\Omega_\nu)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_nd\mathbf{r}_\nu,\quad(36)$$

and

$$G_{1c}=\sum_{m_tm_dm_r\sigma_1'}G_{1c}{}^{m_tm_d;\,m_r\sigma_1'}.\quad(37)$$

We now write:

$$V_{1c}=U_0\delta(\mathbf{r}_1,\mathbf{r}_\nu),\quad(38)$$

where

$$\delta(\mathbf{r}_1,\mathbf{r}_\nu)=0,\quad(\mathbf{r}_1\neq\mathbf{r}_\nu);\quad\int_{-\infty}^{\infty}\delta(\mathbf{r}_1,\mathbf{r}_\nu)d\mathbf{r}_{1\nu}=1\cdot(\mathbf{r}_{1\nu}=\mathbf{r}_1-\mathbf{r}_\nu).$$

Again using the expansion for $\phi_d(r_{2n})$ given in Eq. (23), and denoting the product of the nine Clebsch-Gordan coefficients in (36) by $\prod C_j^{1c}$, we have

$$G_{1c}{}^{m_t m_d;\, m_r \sigma_1'} = G_{1c}{}^{m_t m_d;\, m_r \sigma_1';\,\alpha} - G_{1c}{}^{m_t m_d;\, m_r \sigma_1';\,\alpha+\beta}, \tag{39}$$

$$G_{1c}{}^{m_t m_d;\, m_r\sigma_1';\,\alpha} = 4\pi B U_0 \sum_{m_1m_2m_nm_cm_c'm_{nc}} \sum_{\lambda_1\lambda_2\lambda_n\lambda_\nu\lambda_\nu'} \sum_{\sigma_2\sigma_n\sigma_\nu l\lambda} \prod C_j^{1c} \int e^{-i\mathbf{k}_1\cdot\mathbf{r}_1} h^{*l_\nu'}(r_1) h^{l_\nu}(r_1) f^{l_1}(r_1) Y_{l_\nu'}^{*\lambda_\nu'}(\Omega_1)$$
$$\times Y_{l_\nu}^{\lambda_\nu}(\Omega_1) Y_{l_1}^{\lambda_1}(\Omega_1) d\mathbf{r}_1 \int e^{\frac{1}{2}i\mathbf{k}_d\cdot\mathbf{r}_2} \frac{I_{l+\frac{1}{2}}(\alpha r_2)}{r_2^{\frac{1}{2}}} f^{*l_2}(r_2) Y_{l_2}^{*\lambda_2}(\Omega_2) Y_l^{\lambda}(\Omega_2) d\mathbf{r}_2$$
$$\times \int e^{\frac{1}{2}i\mathbf{k}_d\cdot\mathbf{r}_n} \frac{K_{l+\frac{1}{2}}(\alpha r_n)}{r_n^{\frac{1}{2}}} g^{*l_n}(r_n) Y_{l_n}^{*\lambda_n}(\Omega_n) Y_l^{*\lambda}(\Omega_n) d\mathbf{r}_n. \tag{40}$$

We again express the products of spherical harmonics of the same argument as sums of single harmonics, and expand the three plane waves, this time along the direction of $\mathbf{k}_d$:

$$Y_{l_\nu'}^{*\lambda_\nu'}(\Omega_1) Y_{l_\nu}^{\lambda_\nu}(\Omega_1) = \sum_L (-)^{\lambda_\nu'} \left[\frac{(2l_\nu+1)(2l_\nu'+1)}{4\pi(2L+1)}\right]^{\frac{1}{2}} C_{l_\nu l_\nu'}(L0;00) C_{l_\nu l_\nu'}(L\lambda_\nu-\lambda_\nu';\lambda_\nu,-\lambda_\nu') Y_L^{\lambda_\nu-\lambda_\nu'}(\Omega_1), \tag{41}$$

$$Y_L^{\lambda_\nu-\lambda_\nu'}(\Omega_1) Y_{l_1}^{\lambda_1}(\Omega_1) = \sum_{L'} \left[\frac{(2L+1)(2l_1+1)}{4\pi(2L'+1)}\right]^{\frac{1}{2}} C_{Ll_1}(L'0;00) C_{Ll_1}(L'\lambda_\nu-\lambda_\nu'+\lambda_1;\lambda_\nu-\lambda_\nu',\lambda_1) Y_{L'}^{\lambda_\nu-\lambda_\nu'+\lambda_1}(\Omega_1), \tag{42}$$

$$Y_{l_2}^{*\lambda_2}(\Omega_2) Y_l^{\lambda}(\Omega_2) = \sum_{L''} (-)^{\lambda} \left[\frac{(2l+1)(2l_2+1)}{4\pi(2L''+1)}\right]^{\frac{1}{2}} C_{l_2 l}(L''0;00) C_{l_2 l}(L''\lambda_2-\lambda;\lambda_2,-\lambda) Y_{L''}^{*\lambda_2-\lambda}(\Omega_2), \tag{43}$$

$$Y_{l_n}^{*\lambda_n}(\Omega_n) Y_l^{*\lambda}(\Omega_n) = \sum_{L'''} \left[\frac{(2l_n+1)(2l+1)}{4\pi(2L'''+1)}\right]^{\frac{1}{2}} C_{l_n l}(L'''0;00) C_{l_n l}(L'''\lambda_n+\lambda;\lambda_n\lambda) Y_{L'''}^{*\lambda_n+\lambda}(\Omega_n), \tag{44}$$

$$e^{-i\mathbf{k}_1\cdot\mathbf{r}_1} = \sum_{Pp} 4\pi(-i)^P j_P(k_1 r_1) Y_P^{*p}(\Omega_1) Y_P^p(\mathbf{k}_1,\mathbf{k}_d), \tag{45}$$

$$e^{\frac{1}{2}i\mathbf{k}_d\cdot\mathbf{r}_2} = \sum_{P'} [4\pi(2P'+1)]^{\frac{1}{2}} i^{P'} j_{P'}(\tfrac{1}{2}k_d r_2) Y_{P'}^0(\Omega_2), \tag{46}$$

$$e^{\frac{1}{2}i\mathbf{k}_d\cdot\mathbf{r}_n} = \sum_{P''} [4\pi(2P''+1)]^{\frac{1}{2}} i^{P''} j_{P''}(\tfrac{1}{2}k_d r_n) Y_{P''}^0(\Omega_n). \tag{47}$$

Using Eqs. (41) to (47) in (40), together with the orthonormality relations for the spherical harmonics, we obtain

$$G_{1c}{}^{m_t m_d;\, m_r\sigma_1';\,\alpha} = 4\pi B U_0 \sum_{m_1m_2m_nm_cm_c'm_{nc}} \sum_{\lambda_1\lambda_\nu\lambda_\nu'\sigma_2\sigma_n\sigma_\nu} \sum_{l\lambda LL'L''L'''} (-)^{\lambda+\lambda_\nu'+L'} i^{L'+L''+L'''}$$
$$\times \left[\frac{(2l+1)^2(2l_1+1)(2l_2+1)(2l_n+1)(2l_\nu+1)(2l_\nu'+1)}{(2L'+1)}\right]^{\frac{1}{2}} \left(\prod C_j^{1c}\right) C_{l_\nu l_\nu'}(L0;00) C_{Ll_1}(L'0;00)$$
$$\times C_{l_2 l}(L''0;00) C_{l_n l}(L'''0;00) C_{l_\nu l_\nu'}(L\lambda_\nu-\lambda_\nu';\lambda_\nu,-\lambda_\nu') C_{Ll_1}(L'\lambda_\nu-\lambda_\nu'+\lambda_1;\lambda_\nu-\lambda_\nu',\lambda_1)$$
$$\times C_{l_2 l}(L''0;\lambda,-\lambda) C_{l_n l}(L'''0;-\lambda,\lambda) \int h^{*l_\nu'}(r_1) h^{l_\nu}(r_1) f^{l_1}(r_1) j_{L'}(k_1 r_1) r_1^2 dr_1$$
$$\times \int f^{*l_2}(r_2) \frac{I_{l+\frac{1}{2}}(\alpha r_2)}{r_2^{\frac{1}{2}}} j_{L''}(\tfrac{1}{2}k_d r_2) r_2^2 dr_2 \int g^{*l_n}(r_n) \frac{K_{l+\frac{1}{2}}(\alpha r_n)}{r_n^{\frac{1}{2}}} j_{L'''}(\tfrac{1}{2}k_d r_n) r_n^2 dr_n Y_{L'}^{\lambda_\nu-\lambda_\nu'+\lambda_1}(\mathbf{k}_1,\mathbf{k}_d). \tag{48}$$

This, together with Eqs. (37) and (39), is the required result.

It can be seen that $|G_{1c}|^2$ will not have such a strong angular dependence as $|G_{1n}|^2$, since in Eq. (48) there is no analog of the factor $j_L'(Kr_n)$ in Eq. (31), which varies rapidly with the angle $(\mathbf{k}_1,\mathbf{k}_d)$ because of the change produced in K. Rough calculations using Eq. (48) suggested that $|G_{1c}|^2$ tends to be fairly isotropic in practice; at worst it contains only spherical harmonics of low order which give rise to an angular distribution not unlike those produced through compound nucleus processes.

To obtain the total exchange intensity, we have from Eqs. (11), (15), (16), and (37):

$$|A(\mathbf{n}_1)|^2=\left(\frac{1}{4\pi}\cdot\frac{2M}{\hbar^2}\right)^2\Big|\sum_{m_tm_dm_r\sigma_1'}(G_{1n}{}^{m_tm_d;\,m_r\sigma_1'}+G_{1c}{}^{m_tm_d;\,m_r\sigma_1'})\Big|^2. \tag{49}$$

Since the final state of the system can be expressed in terms of the complete set of eigenfunctions describing the initial system, the cross terms in Eq. (49) belonging to different sets of the quantum numbers m_t, m_d, m_r, σ_1' cancel out, so that we can write this equation in the more tractable form:

$$|A(\mathbf{n}_1)|^2=\left(\frac{1}{4\pi}\cdot\frac{2M}{\hbar^2}\right)^2\sum_{m_tm_dm_r\sigma_1'}|G_{1n}{}^{m_tm_d;\,m_r\sigma_1'}+G_{1c}{}^{m_tm_d;\,m_r\sigma_1'}|^2. \tag{50}$$

V. EVALUATION OF $|F_{1n}|^2$

Both F_{1n} and F_{1c} have been considered by a number of workers, the theories of Butler[1] and of Bhatia *et al.*[8] being concerned with the former, and the modifications introduced by Horowitz and Messiah,[9] Francis and Watson,[10] Tobocman,[11] Grant[12] and others taking into account the additional contribution from F_{1c}. It is however of interest to re-evaluate the ordinary direct stripping intensity using as nearly as possible the same approximations as those used in obtaining the expression for G_{1n} in Sec. III of the present paper. This enables a rough estimate to be made of the ratio of the cross sections for the direct and exchange processes.

Since the states of the nuclear "core" and proton 2 are unchanged by the direct collision process, we shall not need in this case to expand the total wave function describing the target nucleus in terms of eigenfunctions of the constituent subsystems. We merely replace $\psi_t(\mathbf{2c})$ by $\psi_{jt}(\mathbf{2c})$ in Eq. (12). Now for the direct process:

$$\psi_r{}^{m_rm_t}(\mathbf{2nc})=\sum_{m_n\lambda_n\sigma_n'}C_{l_n\frac{1}{2}}(j_nm_n;\lambda_n\sigma_n')C_{j_tj_n}(j_rm_r;m_tm_n)\chi_{\frac{1}{2}}{}^{\sigma_n'}(\mathbf{s}_n)\psi_{jt}{}^{m_t}(\mathbf{2c})g^{l_n}(r_n)Y_{l_n}{}^{\lambda_n}(\Omega_n). \tag{51}$$

Also

$$\psi_d{}^{m_d}(\mathbf{1n})=\sum_{\sigma_1\sigma_n}C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\sigma_1\sigma_n)\chi_{\frac{1}{2}}{}^{\sigma_1}(\mathbf{s}_1)\chi_{\frac{1}{2}}{}^{\sigma_n}(\mathbf{s}_n)\phi_d(r_{1n}). \tag{52}$$

Using these two equations in Eq. (12), we obtain

$$F_{1n}{}^{m_tm_d;\,m_r\sigma_1'}=\sum_{m_n\lambda_n\sigma_n}C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\sigma_1'\sigma_n)C_{l_n\frac{1}{2}}(j_nm_n;\lambda_n\sigma_n)C_{j_tj_n}(j_rm_r;m_tm_n)$$
$$\times\int e^{-i\mathbf{k}_1\cdot\mathbf{r}_1}g^{*l_n}(r_n)Y_{l_n}{}^{*\lambda_n}(\Omega_n)V_{1n}e^{\frac{1}{2}i\mathbf{k}_d\cdot(\mathbf{r}_1+\mathbf{r}_n)}\phi_d(r_{1n})d\mathbf{r}_1d\mathbf{r}_n. \tag{53}$$

With Eqs. (21), this gives

$$F_{1n}{}^{m_tm_d;\,m_r\sigma_1'}=V_0\phi_d(0)\sum_{m_n\lambda_n\sigma_n}C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\sigma_1'\sigma_n)C_{l_n\frac{1}{2}}(j_nm_n;\lambda_n\sigma_n)C_{j_tj_n}(j_rm_r;m_tm_n)W(\lambda_n), \tag{54}$$

where

$$W(\lambda_n)=\int e^{-i\mathbf{k}\cdot\mathbf{r}_1}g^{*l_n}(r_1)Y_{l_n}{}^{*\lambda_n}(\Omega_1)d\mathbf{r}_1, \tag{55}$$

and

$$\mathbf{k}=\mathbf{k}_d-\mathbf{k}_1.$$

Then

$$|F_{1n}|^2=V_0{}^2[\phi_d(0)]^2\sum_{\sigma_n\bar{\sigma}_n\lambda_n\bar{\lambda}_n}\sum_{m_n\bar{m}_n}\sum_{m_tm_dm_r\sigma_1'}C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\sigma_1'\sigma_n)C_{l_n\frac{1}{2}}(j_nm_n;\lambda_n\sigma_n)C_{j_tj_n}(j_rm_r;m_tm_n)$$
$$\times C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\sigma_1'\bar{\sigma}_n)C_{l_n\frac{1}{2}}(j_n\bar{m}_n;\bar{\lambda}_n\bar{\sigma}_n)C_{j_tj_n}(j_rm_r;m_t\bar{m}_n)W(\lambda_n)W^*(\bar{\lambda}_n), \tag{56}$$

the bars denoting quantum numbers associated with $F_{1n}{}^*$. The sum rules for Clebsch-Gordan coefficients give the

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relations:

$$\sum_{m_d\sigma_1'} C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\sigma_1'\sigma_n)C_{\frac{1}{2}\frac{1}{2}}(j_dm_d;\sigma_1'\bar{\sigma}_n)=\frac{2j_d+1}{2}\delta(\sigma_n,\bar{\sigma}_n),$$

$$\sum_{m_rm_t} C_{j_tj_n}(j_rm_r;m_tm_n)C_{j_tj_n}(j_rm_r;m_t\bar{m}_n)=\frac{2j_r+1}{2j_n+1}\delta(m_n,\bar{m}_n), \tag{57}$$

$$\sum_{m_n\sigma_n} C_{l_n\frac{1}{2}}(j_nm_n;\lambda_n\sigma_n)C_{l_n\frac{1}{2}}(j_nm_n;\bar{\lambda}_n\sigma_n)=\frac{2j_n+1}{2l_n+1}\delta(\lambda_n,\bar{\lambda}_n).$$

These, together with Eq. (56), give

$$|F_{1n}|^2=V_0^2[\phi_d(0)]^2\sum_{\lambda_n}\frac{2j_d+1}{2}\cdot\frac{2j_r+1}{2l_n+1}W(\lambda_n)W^*(\lambda_n). \tag{58}$$

Expanding the plane wave in $W(\lambda_n)$ along the direction of $\mathbf{k}$, we have

$$e^{i\mathbf{k}\cdot\mathbf{r}_1}=\sum_l[4\pi(2l+1)]^{\frac{1}{2}}i^lj_l(kr_1)Y_l^0(\Omega_1). \tag{59}$$

Thus

$$|F_{1n}|^2=2\pi V_0^2[\phi_d(0)]^2(2j_d+1)(2j_r+1)\left|\int g^{*l_n}(r_1)j^{l_n}(kr_1)r_1^2dr_1\right|^2. \tag{60}$$

It should be noted that the expressions in Eqs. (58) and (60) above, and the corresponding expressions for $|G|^2$, are all to be divided by the factor $(2j_d+1)(2j_t+1)$ if the cross section evaluated as an average over the initial spin states is required. The familiar statistical weight factor in the direct stripping cross section may then be recognized.

The internal motion of the deuteron has been partially ignored in Eq. (60), whereas in the evaluation of G_{1n} it has not. Since we shall be concerned to compare the probabilities of direct and exchange processes, the insertion of a correction factor in $|F_{1n}|^2$ is properly called for. At the peak of the normal stripping curve we shall postulate a correction factor γ^2 given by[13]

$$\gamma^2=\left(\frac{K_0^2+\alpha^2}{K_1^2+\alpha^2}\right)^2, \tag{61}$$

where K_0, K_1 are the values of $|\frac{1}{2}\mathbf{k}_d-\mathbf{k}_1|^2$ at 0° and at the peak of the $|F_{1n}|^2$ distributions, respectively. γ^2 then corresponds to the usual factor resulting from internal motion of the deuteron in the theory of direct stripping. It also seems more realistic to assume that the interaction concerned in producing the stripping reaction becomes effective at a distance about equal to the range of nuclear forces. To give approximate expression to this idea, we replace $\phi_d(0)$ by $\phi_d(r_0)$ in Eq. (60), where $r_0=1.5\times10^{-13}$ cm. Thus finally we put

$$|F_{1n}|^2=2\pi\left(\frac{K_0^2+\alpha^2}{K_1^2+\alpha^2}\right)^2V_0^2[\phi_d(r_0)]^2(2j_d+1)(2j_r+1)\left|\int g^{*l_n}(r_1)j^{l_n}(kr_1)r_1^2dr_1\right|^2. \tag{62}$$

VI. DISCUSSION

Equations (31) and (62) provide a basis for comparing the properties of direct and exchange contributions to stripping processes under the assumptions that we have made. One important feature has, however, been omitted in our discussion of the exchange process. It has been pointed out by Lane[14] that, if there are Z protons in the target nucleus, then the value of $|G_{1n}|^2$ for a (d,p) reaction should be enhanced by a factor Z^2 over the value obtained assuming that only one of these protons can exchange with the proton in the deuteron. For the purposes of the present treatment, in which the exchanging proton is assigned to a definite orbital state in the target nucleus, it seems more reasonable to multiply the exchange amplitude by the number of equivalent protons for the transition considered. (One can, of course, envisage a more elaborate treatment of the problem in which one uses the complete wave function of the target nucleus throughout, instead of arbitrarily dividing the nucleus, as we have done, into one odd particle and a "core").

One of the main questions of interest is the relative magnitude of G and F, and it is by no means obvious that exchange will be of negligible importance, as is often assumed. It is difficult to generalize on the basis

[13] We should strictly include some terms involving β (see reference 1), but their effect on γ^2 would be very slight.

[14] A. M. Lane (unpublished, 1955).

of an equation as complex as (31), but it may be noted that an exchange process will (granted the assumptions underlying our calculation) be encouraged if $l_n = l_1$, i.e., if the captured neutron enters the same shell as the exchangeable proton. The extensive recoupling of angular momenta in the exchange process will evidently tend to discourage it, as will the limited amount of overlap in the radial integrals appearing in G_{1n} and G_{1c}. There is little doubt that the direct reaction amplitude will exceed the exchange amplitude in any case where both are possible. On the other hand, it may happen that restrictions imposed by conservation of angular momentum will render a direct stripping process impossible in circumstances that do not rule out exchange. Indeed, it was the recognition of this as a practical possibility in a particular reaction—$B^{10}(d,p)$ leading to the first excited state of B^{11}—that led to the present investigation. The detailed calculations (reported separately[15]) for this case indicate that one may have an exchange stripping cross section that is of the order of 10% of a direct stripping cross section under comparable conditions.

Reference must be made to the work of Grant[12] and of Madansky and Owen[16] who have approached this same problem in different ways. Grant makes the formal separation of the reaction amplitude into direct and exchange terms, as we do, but proceeds to identify the exchange part explicitly as a compound nucleus effect. This is then evaluated in the normal way as a transition through an intermediate state, but the properties of the intermediate are suppressed through summation over all possibilities. The final result is an angular distribution whose form is essentially similar to G_{1c} of the present paper [Eq. (48)], being dominated by spherical harmonics of fairly low order. Madansky and Owen, on the other hand, concentrated on a simple and ingenious treatment of a true stripping process in which the roles of the deuteron and the target nucleus are interchanged. This then corresponds to our exchange process, in that the outgoing nucleon is assumed to come from the target nucleus, but from the way in which the problem is formulated by Madansky and Owen it necessarily follows that their angular distributions are in general peaked towards the backward direction with respect to the incident deuteron. The treatment in the present paper, on the other hand, will give peaking in the forward direction. To this extent the two methods (both of them approximations) may perhaps be regarded as complementary. Both types of peaking seem to occur in practice.

It is hardly necessary to stress the drastic nature of the simplifications in the present treatment. The main aim has been to draw attention to the general features of an exchange contribution to stripping reactions, namely the possible combination of a forward peak, rather similar to that of a conventional stripping process, together with a more or less isotropic angular distribution of the compound nucleus type. As has been pointed out by Thomas,[17] there is no clear division of nuclear configuration space into separate regions belonging to compound nucleus and to surface transfer processes. Peaslee,[18] in his review of nuclear reactions at intermediate energies, similarly emphasizes that features of the optical and the statistical models, representing two extremes, will both normally appear in some compromise form in actual reactions. The present treatment of a (d,p) or (d,n) reaction as a combination of direct and exchange processes is one way of realizing such a synthesis.

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