Magnetic Moment of the Deuteron and Nucleon-Nucleon Spin-Orbit Potentials*

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The change in the magnetic moment of the deuteron arising from the presence of a spin-orbit potential in the nucleon-nucleon interaction has been estimated for two recent proposals. The effects are found to be substantial.

THE proposal of Case and Pais[1] that nucleon-nucleon forces include a spin-orbit term has been recently revived by Signell and Marshak[2] and by Gammel and Thaler.[2] Both sets of authors found that substantial agreement with experiments could be obtained.

The spin-orbit potential employed by Signell and Marshak to obtain agreement with experiment for both n-p and p-p scattering up to 150 Mev is

$$V_{\mathrm{SM}}=\frac{V_0}{x}\frac{d}{dx}\left(\frac{e^{-x}}{x}\right)\mathbf{L}\cdot\mathbf{S}, \qquad r\geqslant r_c$$
$$=\frac{V_0}{x}\frac{d}{dx}\left(\frac{e^{-x}}{x}\right)\bigg|_{r=r_c}\mathbf{L}\cdot\mathbf{S}, \quad r\leqslant r_c \tag{1}$$

where $x=r/r_0$, r is the internucleon distance, and $\mathbf{L}$ and $\mathbf{S}$ are the orbital and spin angular momentum operators divided by $\hbar$. The parameters were chosen as follows:

$$V_0=30\text{ Mev};\quad r_c=0.21\times10^{-13}\text{ cm};\quad r_0=1.07\times10^{-13}\text{ cm}.$$

Gammel and Thaler have examined p-p and n-p scattering up to 310 Mev. Their spin-orbit force is

$$V_{\mathrm{GT}}=-V_0\frac{e^{-\mu r}}{\mu r}\mathbf{L}\cdot\mathbf{S}, \tag{2}$$

where, for the isotopic singlet state,

$$V_0=5000\text{ Mev},\quad \mu=3.7\times10^{13}\text{ cm}^{-1}.$$

The central part of the triplet potential contains a repulsive core of radius $r_c=0.4\times10^{-13}$ cm.

It has long been known[3] that the introduction of spin-orbit forces leads to a modification of the magnetic moment operator and, therefore, to a deviation of the calculated deuteron magnetic moment from the familiar expression

$$\mu_D-(\mu_n+\mu_p)=-\tfrac{3}{2}(\mu_n+\mu_p-\tfrac{1}{2})p_D, \tag{3}$$

where μ_n and μ_p are the magnetic moments of the neutron and proton, respectively; while p_D is the D-state probability. The experimental values of the quantity on the left side of Eq. (3) is

$$[\mu_D-(\mu_n+\mu_p)]_{\exp}=-0.0224\text{ nm} \tag{4}$$

(nm = nuclear magneton).

We shall now estimate the addition $(\Delta\mu)_{SL}$ to the right-hand side of Eq. (3) arising from the spin-orbit force

$$V_{SL}=V(r)\mathbf{S}\cdot\mathbf{L}=\frac{1}{2\hbar}V(r)(\mathbf{S}\times\mathbf{r})\cdot(\mathbf{p}_1-\mathbf{p}_2)$$
$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2. \tag{5}$$

Taking particle one to be the proton, the additional electromagnetic interaction ΔH_{em} arising from (5) is

$$\Delta H_{\mathrm{em}}=-\frac{e}{2\hbar c}V(r)(\mathbf{S}\times\mathbf{r})\cdot\mathbf{A}, \tag{6}$$

where $\mathbf{A}$ is the vector potential. For a uniform magnetic field $\mathbf{H}$, $\mathbf{A}=-\frac{1}{2}(\mathbf{r}_1\times\mathbf{H})$ so that the magnetic moment operator is

$$(\Delta\boldsymbol{\mu})_{SL}=-\frac{e}{8\hbar c}V(r)(\mathbf{S}\times\mathbf{r})\times\mathbf{r}. \tag{7}$$

The contribution of this term to the magnetic moment of the deuteron is

$$(\Delta\mu)_{SL}=-\frac{e}{16\hbar c}\langle V(r)[(\mathbf{S}\cdot\mathbf{r})^2-r^2(\mathbf{S}\cdot\mathbf{J})]\rangle. \tag{8}$$

Inserting the deuteron wave function,

$$\psi=[\psi_S(r)+\tfrac{1}{4}\sqrt{2}S_{12}\psi_D(r)]\chi_m^{(1)}, \tag{9}$$

where ψ_S is the S-radial wave function, ψ_D the D-radial wave function, S_{12} the tensor operator, and $\chi_m^{(1)}$ the triplet spin wave function, we obtain

$$\Delta\mu_{SL}=\frac{e}{12\hbar c}\left[\langle S|r^2V|S\rangle-\frac{1}{\sqrt{2}}\langle S|r^2V|D\rangle+\tfrac{1}{2}\langle D|r^2V|D\rangle\right], \tag{10}$$

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[1] K. M. Case and A. Pais, Phys. Rev. **80**, 203 (1950).

[2] P. S. Signell and R. E. Marshak, Phys. Rev. **106**, 832 (1957); J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 290 (1957), also private communication. I am indebted to both sets of authors for preprints describing their results.

[3] Blanchard, Avery, and Sachs, Phys. Rev. **78**, 292 (1950); C. H. Blanchard and R. Avery, Phys. Rev. **81**, 35 (1951); N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).

where

$$\langle a|O|b\rangle = \int \psi_a^* O \psi_b d\mathbf{r}.$$

Since the D-state wave function will generally be small in the region where r^2V is most important[4] we may, to a good approximation, keep only the first term in (10):

$$(\Delta\mu)_{SL} \simeq \frac{e}{12\hbar c}\langle S|r^2V|S\rangle. \quad (11)$$

We note immediately that since the two potentials (1) and (2) are negative definite, $(\Delta\mu)_{SL}$ is negative. Since this is to be added on to the right side of Eq. (3), it is clear that p_D will need to be reduced in order to match the experimental value (4). Clearly, if $\Delta\mu_{SL}$ is too large in absolute value, it will become impossible to obtain (4).

To evaluate (11) we need the S-state wave function. This we assume to be of the form

$$\psi_S = \frac{N}{r}[e^{-\alpha(r-r_c)} - e^{-\beta(r-r_c)}], \quad r \geqslant r_c$$
$$= 0, \quad r \leqslant r_c. \quad (12)$$

Actually, in the SM case the wave function is not exactly zero for $r < r_c$ (though it should be very small), so that for that case we shall underestimate $(\Delta\mu)_{SL}$ slightly. The constants in the wave function (12) are

$$\alpha^{-1} = 4.31\times 10^{-13} \text{ cm},$$

while

$$N = \left(\frac{2\alpha}{1-\alpha\rho'}\right)^{\frac{1}{2}}, \quad (13)$$

$$\beta = \frac{3}{\rho'}\left(\frac{1}{1+(4/9)\alpha\rho'}\right), \quad (14)$$

where

$$\rho' = \rho - 2r_c, \quad (15)$$

and ρ, the triplet effective range, is 1.70×10^{-13} cm. The integral in (11) may be easily carried out and evaluated. The results are

$$\Delta\mu_{SL} = -0.056 \text{ nm} \quad \text{(Signell and Marshak potential)}$$
$$= -0.036 \text{ nm} \quad \text{(Gammel and Thaler potential).} \quad (16)$$

These values are so large as to preclude the possibility of matching the experimental value of the deuteron magnetic moment even if p_D were reduced to zero.

[4] Note that the usual centrifugal D-state barrier is reinforced by the spin-orbit potential of Eq. (1) and Eq. (2) since these contribute central repulsive terms in the Schrödinger equation for ψ_D. Hence, they will reduce the value of p_D to a smaller value than that obtained by S. Gartenhaus, Phys. Rev. **100**, 900 (1955).

We have omitted so far any reference to mesonic and relativistic effects. In the most recent paper on this subject, Sugawara[5] estimates these effects to be between one to two percent of the deuteron magnetic moment. The most certain term arising from relativistic effects is negative and is about three times the mesonic contribution which is also negative. The latter, based as it is on the Tamm-Dancoff approximation, is not reliable.

Finally, we should emphasize the approximations employed in the present calculation. We dropped the SD interference term and the DD term in going from Eq. (10) to Eq. (11), and, secondly, we employed the hardly precise wave function (12). These calculations should, of course, be redone using the exact wave function. In addition, we have not taken the finite size of the proton into account. However, it does not seem likely that the qualitative nature of our results will be changed.

Assuming this to be the case, we feel that the Gammel and Thaler potential is superior to the Signell and Marshak potential. Both are, however, suspect since agreement with experiment could be reached only if the mesonic contribution to the deuteron magnetic moment is opposite in sign to that obtained by Sugawara and one order of magnitude larger. Essentially two ways out of the above dilemma seem available. In one we note that the spin-orbit force is only one of many possible velocity-dependent potentials. If we require that the velocity-dependent potential should not contribute via the S state to the deuteron magnetic moment, the potential must be bilinear in $\mathbf{L}$. For example, the potential

$$V(r)[\boldsymbol{\sigma}_1\cdot\mathbf{L}\boldsymbol{\sigma}_2\cdot\mathbf{L}+\boldsymbol{\sigma}_2\cdot\mathbf{L}\boldsymbol{\sigma}_1\cdot\mathbf{L}]$$

would be quite suitable and could, of course, when combined with a suitable central and tensor force, yield a repulsive $^3P_0$ force, which Gammel and Thaler require. A second possibility, suggested by Breit,[6] is that one can interpret the above results as meaning that the center of charge of the deuteron does not precisely follow the motion of the proton so that the substitution of $\mathbf{p}-e\mathbf{A}/c$ for $\mathbf{p}$ is not correct. This hypothesis raises a rather fundamental question so that it becomes important to determine whether other phenomena exist in which it is measurably important. In any event it is clear that the requirement that the magnetic moment of the deuteron be predicted correctly will be useful in determining the form and meaning of the spin-orbit potential.

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Note added in proof.—The point raised by this note was also advanced by R. G. Sachs at the Seventh Annual Rochester Conference on High-Energy Physics.

[5] M. Sugawara, Progr. Theoret. Phys. (Japan) **14**, 535 (1955).
[6] G. Breit (private communication).