$J \geqslant 3$, $J \geqslant 4$ and $J \geqslant 5$, respectively, but their contribution would certainly be small and is neglected here. For the expressions $V(f_{7/2}{}^3, J_2) - V_0 = V(f_{7/2}{}^3, J_2) - V(f_{7/2}{}^3, J_2 = 7/2)$ with $J_2 = 7/2$, 5/2, and 3/2 we thus take the experimental values of Ca^{43} quoted above.

The matrix elements of the $d_{3/2} f_{7/2}{}^3$ interaction were calculated in reference 3 in terms of the expectation values of the proton-neutron interaction in the $d_{3/2} f_{7/2}$ configuration. They are given by the expression (7) of that paper by putting $j' = 3/2$, $j = 7/2$, and $m = 3$ and writing J_2 instead of J_1; the whole expression (7) should then be multiplied by $m(=3)$. The additional quantum numbers α_1, α_1', and β_1 are not necessary in this case and should be ignored. The various fractional parentage coefficients are given in the literature.[7] The values of $V_J = \langle d_{3/2} f_{7/2} J | V | d_{3/2} f_{7/2} J \rangle$ were determined to be[3]

$$V_2 = -1.79, \ V_3 = -1.04, \ V_4 = -0.47, \ V_5 = -1.09 \text{ Mev.}$$

Considering only the states with $J_2 = 7/2$, 5/2, and 3/2 we obtain three-by-three matrices for $J = 2$ and $J = 3$, two-by-two matrices for $J = 1$ and $J = 4$, while the states $J = 5$ and $J = 0$ can be built only with $J_2 = 7/2$ and $J_2 = 3/2$, respectively. The diagonalization is easily done numerically.

The ground state turns out to be one of the $J = 2$ states in agreement with the experimental findings. The next state above it at 0.43 Mev has $J = 1$, a state at 0.58 Mev has $J = 3$, and a state at 0.68 has $J = 4$. All other states lie higher than 1 Mev above the ground state. It is of some interest to note the amount of the wave functions of the various J_2 states in the wave function of the $J = 2$ ground state. The squares of the amplitudes are 63% for the state with $J_2 = 7/2$, 26% for the state with $J_2 = 5/2$, and 11% for the $J = 3/2$ state.

[7] See, for example, A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) **A214**, 515 (1952).

PHYSICAL REVIEW VOLUME 107, NUMBER 6 SEPTEMBER 15, 1957

Elastic Scattering of 7.5-Mev Protons*

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(Received April 23, 1957)

We have measured the angular distributions of 7.5-Mev protons elastically scattered from a series of elements with atomic numbers $Z = 13$ to $Z = 49$. The measurements were made, and the results are presented in terms of the ratio of the differential cross section from a particular nucleus to that of the scattering from a point charge nucleus of the same total charge. The data are interpreted in terms of the scattering from a complex-potential well with the slight variations in nearby elements interpreted as arising from a compound-elastic contribution to the over-all elastic scattering.

I. INTRODUCTION

OVER the last few years a considerable body of experimental data has been accumulated concerning the elastic scattering of protons with energies from about 10–30 Mev.[1,2] The significant feature of these data is a regularity in the differential cross section as a function of angle (for a given energy), that is, by and large, independent of the particular nucleus which was investigated except as a given nucleus represents a particular size scattering center. More specifically, one finds that apparently the most crucial variable describing the differential cross section empirically is the quantity $x = kA^{\frac{1}{3}} \sin(\theta/2)$, where k is the incident particle wave number, A the nuclear mass number, and θ the angle of observation. At any given energy this variable, x, has a characteristic value for a given order maximum or minimum in the differential cross section which is independent of particular nucleus. The semiempirical rule holds to a high degree of accuracy for all angles, *not only small ones*, and over a large range of elements. The $A^{\frac{1}{3}}$ dependence of the variable x is usually interpreted as a manifestation of saturation properties of nuclear forces, or in other words, nuclear matter has a constant density independent of the number of nucleons present. One should realize, however, that because of the interference effects between the Coulomb and nuclear scattering, this type experiment does not necessarily offer either as direct a measure of the nucleon-nucleus interaction radius as does elastic neutron scattering[3] or a measure of the charge distribution such as high-energy electron scattering.[4] A further drawback to the elementary interpretation of elastic proton scattering experiments

* This Work was supported in part by a joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
† The experimental section of this paper constituted a part of the thesis submitted in partial fulfillment of the S. M. requirements of Massachusetts Institute of Technology.
[1] B. B. Kinsey and T. Stone, Phys. Rev. **103**, 975 (1956). This reference gives a rather complete listing of elastic proton scattering experiments in the region 10–30 Mev.
[2] N. Hintz (private communication) Phys. Rev. **100**, 1794 (1955).
[3] One has a large number experiments such as those analyzed by Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).
[4] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).

which may also arise from the above-mentioned interference is that the radius parameter r_0 in the relation $r=r_0A^{\frac{1}{3}}$, even for a given energy, does depend upon the particular order number of the maximum or minimum at which the variable x is being evaluated. This particular point is further discussed in Sec. III.

In addition to these higher energy measurements there are, however, several experiments in an energy range around 5.25 Mev[5] which do not seem to follow the general rules outlined above. We were therefore motivated to perform a further series of measurements at an energy below 10 Mev to attempt to delineate the region of validity of these rules, and to understand further the 5.25-Mev experiments. In addition, since a large amount of the data, even the 5.25-Mev data, can be and has been analyzed using the phenomenological approach of a "cloudy crystal ball" scattering process, as represented by the scattering from complex-potential well,[6,7] we hoped to extend the range of energies for which analysis in this manner is possible. Such analyses in turn allow one to determine the energy dependence of the potential well parameters representing the scattering nucleus.

II. EXPERIMENTAL MATTERS

A. Equipment

For these experiments we used the M.I.T. cyclotron and emergent beam apparatus[8] to accelerate and focus a beam of 15.1-Mev H_2^+ ions. These ions were then stripped in the target foil enabling us to study the interactions of 7.55-Mev protons with the various target nuclei. A NaI(Tl) scintillation spectrometer was used in conjunction with conventional electronic equipment and twenty-channel pulse-height analyzer similar to the Atomic Instrument Model-520 pulse-height analyzer.

In any experiment where one is measuring a differential cross section, one must know either the integrated incident flux of particles striking the target or at least a quantity proportional to the flux. This latter quantity will suffice when one is measuring the cross sections by a comparison technique. In our experiments, in which a modification of the usual comparison technique was used, we determined a quantity proportional to the integrated incident flux by means of another scintillation counter whose angle was kept fixed at about 23° relative to the incident beam direction, and which "saw" the same area of the target, namely, the whole target, as did the spectrometer counter. This counter differed from the spectrometer counter in that its phosphor was an organic one, Pilot "B",[9] and after amplification in a conventional linear amplifier the photomultiplier signal was fed to an integral discriminator which was so biased as to count only those particles elastically scattered. This setting was easily attained only because, for the elements which were studied in these experiments, the elastic scattering differential cross sections were by far the predominant processes at an angle of 23°. We will show later that the assumption of monitoring only on elastically scattered particles was indeed valid to within about 3%.

B. Procedures

In the actual accumulation of data we relied upon the comparison method which is described in detail in the thesis of one of the authors (W.F.W.). The manner in which the data were taken was to observe the spectrum of particles elastically scattered into the spectrometer counter from a thin gold target both before and after we examined the spectrum from the element under study. From these spectra, the number of protons elastically scattered per proton incident on the monitor counter was determined by summing the appropriate channels in the 20-channel pulse-height analyzer. An appropriate spectrum of the particles elastically scattered from gold is shown in Fig. 1. This spectrum also shows the particles elastically scattered from a very thin layer of carbon and oxygen on the target foil. The full width at half-maximum of the elastic proton peak from Au is less than 0.3 Mev.

Then having the number of protons scattered through a given angle per monitor proton for both the target under study and for gold, we divided these two quantities giving a number R, which was the ratio of the

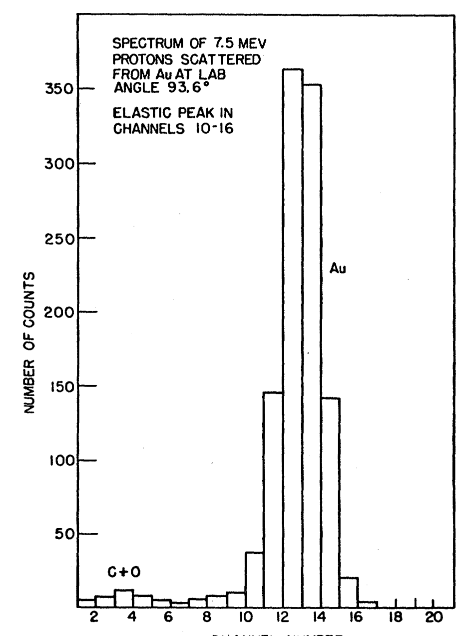

FIG. 1. Energy spectrum of 7.55-Mev protons elastically scattered through angle of 93.6° from a thin Au target.

[5] D. A. Bromley and N. S. Wall, Phys. Rev. **102**, 1560 (1956).
[6] D. A. Saxon and M. A. Melkanoff, Phys. Rev. **101**, 507 (1956).
[7] A. E. Glassgold *et al.*, Phys. Rev. **106**, 1207 (1957).
[8] H. J. Watters, Phys. Rev. **103**, 1763 (1956).
[9] Available from Pilot Chemical Corporation, Waltham, Massachusetts.

differential cross sections, aside from corrections arising from center-of-mass effects in both the monitor and spectrometer counter. It should be noted that because of the rapid dependence upon angle of the differential cross sections for elastic scattering, the repeat run on the gold target makes this method sensitive to most types of drifts in beam direction, electronics, etc., and we could therefore discard any data showing such instabilities.

It can be shown[10] that using such a comparison method to determine the elastic scattering cross section relative to Rutherford scattering enables one to find the desired ratio directly from the experimental ratio, R, in a manner which is insensitive to slight systematic errors in angle determination. In fact, because of this insensitivity to small systematic angular errors the conversion factors to transform the raw data from the laboratory system to the center-of-mass system for an element as light as Al are negligibly different from unity, so that the experimental number, R, directly gives the desired cross-section ratio. The proof of these statements is to be found in the aforementioned thesis. The statement that R is actually $(d\sigma/d\Omega)/(d\sigma/d\Omega)_{\mathrm{Coul}}$ assumes the angle at which the monitor is placed is one for which the unknown element scattering differential cross section is also Rutherford. If at 23°, the monitor angle in these experiments, the scattering were different from Rutherford, then the above ratio would have approached some constant value different from unity at small angles. However, for all the elements studied in these experiments the 23° scattering was found to be Rutherford.

Since at sufficiently small angles, that is, for classical apsidal distances large compared to the nuclear radius, the scattering should be pure Rutherford and deviations arising from target thickness effects or the fact that we did not use a true line "beam" and point detector[11] would be readily detectable using this method. For some targets at small angles such effects were observed and by appropriate choice of target thickness we were able to eliminate them.

C. Energy Spread and Resolution

In only one case of the nuclei studied in these experiments is it known with any degree of certainty that the levels of the compound nucleus are so closely spaced that our experiments definitely averaged over a large number of levels. This one nucleus, indium, was studied by C. P. Browne and R. Sharp of the M.I.T.-O.N.R. Van de Graaff Group. They found that in our energy range, ~7.0 Mev, the levels in the compound nucleus Sn^{116} (In being 95% In^{115}) show no discrete structure with a characteristic width greater than about 10 kev. In fact their yield curve showed no structure at all except for a gradual decrease in the cross section below that expected for pure Rutherford scattering. In order to assure ourselves that the nuclear property we investigated in these experiments did not depend critically upon the energy of the incident particles, we used targets of a thickness such that the energy spread because of loss in the target, and spread in the incident beam, amounted to as much as 200 kev.

For many of the elements studied here the first and higher excited states of the nuclei were well resolved as illustrated in Fig. 2. The nuclei in this category were Al, Ti, Fe, Co, and Ni. In the heavier elements studied the intensity of scattered protons corresponding to inelastic scattering, leaving the nucleus with about 0.5-Mev excitation, was a fraction usually much smaller than 20% of the elastically scattered protons. On this basis with the spectrometer having an energy resolution of better than 4% at 7.5 Mev, we are able to rule out any significant contribution ($\lesssim 1\%$) to the elastic scattering cross section arising from our inability to eliminate completely inelastic events in either detector.

D. Errors

On the basis of the above arguments and taking into account the statistical errors arising from the total number of counts observed at each point, it is believed that the relative cross sections we have measured are accurate to within a standard deviation of ±3%. The only other significant error, which might be involved, is our assumption that the elastic scattering from gold is pure Rutherford scattering. We have measured it and found the product of the scattered intensity at a given angle with the fourth power of the cosecant of half the angle to be constant to within about 3% over the angular range 30°–160°. In addition, measurements[2] at 9.8 Mev indicate that gold does obey Rutherford scattering to within 5%. It seems that this assumption is then quite valid, and therefore, we have

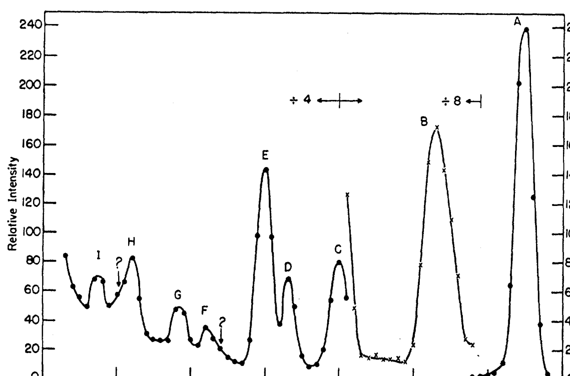

FIG. 2. Scattered proton energy spectrum of 7.55-Mev incident protons on a 1.88 mg/cm² Al target. The scattering angle was about 90°. The various letters indicate well-known levels in Al^{27}, A being the ground state. Level B is unresolved combination of the 0.853- and 1.01-Mev levels. Comparison of the width of this peak to that of the elastic indicates its unresolved nature.

[10] The thesis of one of the authors (W.F.W.) gives the experimental setup and techniques in detail.

[11] B. Dayton and G. Schrank, Phys. Rev. **101**, 1358 (1956).

in fact a measurement not only of the relative cross section to $\pm 3\%$, but also the absolute cross section for these various nuclei.

III. RESULTS

The results of these experiments are shown in Fig. 3. This is a composite drawing of all our results and clearly shows the regularities present at higher energies. The angular distribution for several elements is shown in detail in Fig. 4 which has the experimental points plotted.

The data from which these figures were drawn have been fitted by a least-squares procedure to a series of the form[12]:

$$\frac{d\sigma}{d\Omega}\Big/\left(\frac{d\sigma}{d\Omega}\right)_C = \sum_{n=0}^{10} a_n P_n(\cos\theta).$$

This analysis can be summarized by pointing out that the major contributions come from $n=0$ through $n\simeq 5$ for all but the heaviest nuclei. A coefficient of value zero or one with an uncertainty larger than its magnitude, signifies for such partial waves a contribution not significantly different from pure Rutherford scattering. In particular, it should be noted that any expression derived by the usual sort of partial wave expansion would contain not just the Legendre polynomials to the first power but would contain the squares of the Legendre polynomials as well as cross terms. One cannot, therefore, assign any specific significance to the highest value of n present in such an expansion. On the other hand, one can say qualitatively that large n values do *imply* large numbers of partial waves entering into the nuclear scattering process. On this basis, at 7.5 Mev, one can conclude from this analysis that of the order of at least 4 partial waves should be considered in a partial wave analysis. This is quite consistent with a classical interpretation which says that the highest partial wave, l', entering the nucleus is $l'\simeq kR=4$, for an element like copper.

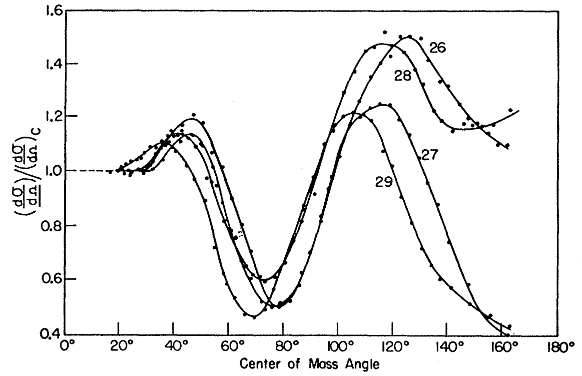

FIG. 4. The differential cross section of 7.55-Mev elastically scattered protons from Fe, Co, Ni, and Cu relative to Rutherford scattering. The experimental points are indicated for these elements to indicate the accuracy of the data.

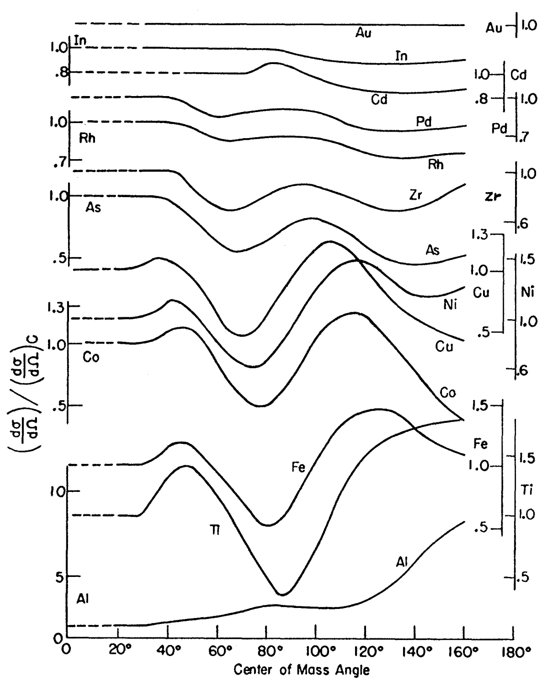

FIG. 3. A composite drawing for all the elements studied in these experiments of the differential cross section compared to Rutherford scattering. The curves all have a common abscissa but the ordinates are separate and are indicated for each element.

As Kinsey and Stone[1] have recently shown, at proton energies of 14.5, 20, and 31.5 Mev the observed maxima and minima of the angular distributions follow the scheme that the quantity $2kR'\sin(\theta/2)$ is constant. In this notation k is the wave number of the free protons, R' the nuclear radius plus the reduced wavelength of the incident protons, and θ is the angle of scattering at which a particular order maximum or minimum occurs. For the data shown in Fig. 3, it was empirically found that the quantity $A^{\frac{1}{3}}\sin(\theta/2)$ had the value 2.40+0.05 at the first minimum for the elements Al through Pd. Similarly for the elements Fe through Cd, this parameter, for the prominent maximum, was found to have the value 3.27 ± 0.06, and finally for the minimum present at back angles in the elements As through In, $A^{\frac{1}{3}}\sin(\theta/2)$ has the value 4.24 ± 0.13. The errors indicated represent the standard deviation of the pertinent values for each element and angle averaged with equal statistical weight.

IV. DISCUSSION

A

In general the results of these experiments agree with the concept demonstrated in the higher energy experiments that the proton scattering process depends upon some general property of nuclei and the result of the scattering is relatively independent of the detailed structure of a particular nucleus except for its size.

[12] The numerical values of these coefficients are available upon request.

This of course is one of the basic ideas of any attempt to describe the scattering on the basis of representing the nucleus as a complex-potential well.

We now, however, concentrate on the dissimilarities of the angular distributions of protons scattered from nuclei with approximately the same value of Z which might appear to contradict this hypothesis. Figure 4 shows in detail the angular distributions of Fe ($Z=26$), Co ($Z=27$), Ni ($Z=28$), and Cu ($Z=29$). As one can see there are the obvious similarities such as the position of the maxima and minima, and the general shape of the curves. However, two pronounced characteristics should be noticed showing, we believe, some sort of systematic behavior. In particular, the even-Z targets show: (1) higher back-angle maxima, as well as less shallow minima in that region, and (2) over most of the angular range their relative cross sections are somewhat higher. Furthermore, we would like to consider not only the present experiments but the more drastic behavior noted at 5.25 Mev,[5] and the large back-angle cross sections observed for Ni and Fe, as compared to nearby odd-Z isotopes in the 17-Mev experiments of Dayton and Schrank.[11]‡ The scattering data at 5.4 Mev[13] and 9.8 Mev[2] do not, however, show quite the same marked differences between Ni and Cu as the other data, although it should be noted that Glassgold *et al.*[7] experienced no difficulty in fitting the Cu data at 9.8 Mev whereas they were not able to obtain a satisfactory fit for Ni on the basis of the complex-potential well scattering.

Considering all of these observations one might invoke any one of several possible explanations for the behavior of the even-Z elements as compared to the others. The spin-dependence of the nuclear interaction or the dependence upon the level density in the compound nucleus are two explanations which at first glance seem reasonable. The first of these explanations, if true, would seriously affect any polarization observed in the elastic scattering from a pair of nuclei such as Cu and Ni.[14] A further possible way of investigating any spin-dependence might be to study the elastic alpha-particle scattering from these two nuclei. If the scattering of the alpha particles showed no difference between Ni and Cu then one might attribute the proton behavior to a spin-interaction. If a pronounced spin-independence is suspected, however, then elastic deuteron scattering might also be of interest.

The possibility that the above behavior has to do with properties of the compound nucleus is almost certainly invalid for the reason that a similar anomalous behavior exists over a very large region of bombarding energy, i.e., from 5.25 Mev to 17 Mev.[15,16] Furthermore since the even-Z targets form compound systems with an odd number of nucleons one would expect such nuclei to exhibit more complex structure in the compound nucleus. We therefore would expect that if our incident beam averages over the levels of the compound nucleus, the fluctuations would be more pronounced in the case of the odd-Z nuclei where the compound system has fewer levels. Since the complex-potential well analysis of elastic proton scattering has disregarded these fluctuations,[6,7] and has satisfactorily obtained agreement between such analysis and experiment for odd-Z nuclei in the range of Cu and Ni, it might be said that it is the even-Z nuclei which exhibit anomalous behavior. An additional recent piece of experimental evidence indicating that it is the even-Z nuclei which are anomalous is the investigation of the nearby series of elements V ($Z=23$), Cr ($Z=24$), Fe ($Z=26$), and Co ($Z=27$) which were studied at low energies.[17] In these experiments Preskitt found that the differential cross section varied much more rapidly with energy for the even-Z elements than for the odd-Z elements. In fact this behavior may account for the difference in the results at 5.25 Mev[5] and 5.4 Mev.[13]

Aside from the more trivial explanation that all of these experiments do not reflect a general property of nuclei, and we are just observing peculiarities of the individual nuclei, there is a possible explanation of all of the observations to which we have just referred. This explanation assumes the applicability of the complex-potential well representation to the problem at hand. In fact one essentially assumes that the probability for forming the compound nucleus, as well as the description of most of the elastic scattering, is that given by the complex-potential representation. The difference between the even-Z and odd-Z target nuclei is then postulated to arise from just a consideration of the compound-elastic scattering. A similar need for a consideration of the compound-elastic scattering has recently been put forward by several authors in their analyses of neutron interactions.[18]

The presence of compound-elastic scattering would account for the difficulty in fitting the 10-Mev scattering data from Ni with just shape-elastic scattering. It might also explain the need for a somewhat lower value for the imaginary part of the potential for Ni as observed by Glassgold *et al.*[7] In addition this explanation would call for more elastic scattering in the even-Z targets since the relative probability for the compound

‡ One should also mention here the fact that in these measurements another even-Z element Zn ($Z=30$) does not show the same behavior as Fe and Ni.

[13] Kliucharev, Bolotin, and Lutsik, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 573 (1956) [translation: Soviet Phys. JETP **3**, 463 (1956)].

[14] This particular point is being investigated by one of the authors (N.S.W.) and C. W. Darden, III, but as yet there are no definite results.

[15] There are only two experiments at energies above 2 Mev or 3 Mev, out of the range of individual resonances, that do show a marked energy dependence of the differential cross section, namely, the scattering of 9-Mev protons from Mg (see reference 16), and the results referred to in reference 17.

[16] Greenlees *et al.* (private communication).

[17] C. Preskitt (private communication).

[18] Beyster, Walt, and Selmi, Phys. Rev. **104**, 1319 (1956); W. S. Emmerich and R. M. Sinclair, Phys. Rev. **104**, 1399 (1956).

nucleus to break up into the entrance channel is higher simply because there are fewer possible exit channels for the compound nucleus. Consequently there would be an increased cross section at back angles for Ni and Fe as compared to Co and Cu. The fact that there are fewer exit channels for the compound nucleus to break up into, and therefore that the contribution of compound-elastic scattering to the elastic scattering in even-Z targets is more significant is borne out by the recent experiments of Buechner and Sperduto,[19] concerning the energy spectra of the isotopes of iron. Of course to relate their observations to our analysis requires the reasonable assumption that *any* odd-mass-number nucleus has a more complex level structure that an even-mass-number nucleus, particularly near the ground state.

If we assume that the difference between Ni and Cu does arise from compound elastic scattering, then the data we have can be used to evaluate the contribution of compound-elastic cross section relative to that of the shape-elastic differential cross section. To estimate this effect quantitatively we find the difference between the absolute cross sections for Ni and Cu at back angles. In the range from about 110° to 160° we find this value to be roughly constant at 0.013×10^{-24} cm^2 steradian^{-1}. Assuming isotropy for this differential cross section gives a total compound-elastic cross section of about 0.16×10^{-24} cm^2. This should be compared to a nuclear area, $\pi(R_N+\lambda\!\!\!^-)^2$, of about 1.5×10^{-24} cm^2. One need not assume isotropy for this differential cross section but instead evaluate it by the same procedure over the whole angular range. The large differential cross section of Rutherford scattering at small angles means that a small error in the differential cross section for either Ni or Cu will result in an apparent compound-elastic cross section which may be orders of magnitude larger than the actual cross section. It is for this reason that the effect of what we have called compound-elastic scattering is most obvious at back angles, and that we have chosen to evaluate it there.

More properly one should compare the difference between the Ni and Cu differential cross section at back angles to the observed differential cross sections. Such a comparison gives 0.013×10^{-24} cm^2 steradian^{-1} for the compound-elastic cross section as compared with 0.036×10^{-24} cm^2 steradian^{-1} the differential cross section for Ni at about 130°. In other words the compound-elastic contribution to the elastic scattering is less than the shape-elastic contribution, but not significantly less if our interpretation of the difference in the Ni and Cu cross sections is correct.

If we regard the ground state as being essentially no different from one of the excited states with respect to the breakup of the compound nucleus, then another estimate of the compound-elastic contribution might come from comparing inelastic and elastic cross sections at back angles. For Ni it was found that the intensity of the unresolved first excited states of Ni^{58} and Ni^{60} is 54% of the elastic scattering from these isotopes. On this basis the cross section for the excitation of these first excited states is of the order of 0.01×10^{-24} cm^2 steradian^{-1}.[20] Equating this inelastic cross section to the compound-elastic cross section neglects several effects which cast some doubt on this procedure however. Among these neglected effects are the different penetrabilities for the protons leaving the compound nucleus, and the possibility that the inelastic scattering proceeds through direct reaction as distinguished from compound nucleus formation.[21] Since we are only attempting here to estimate the compound-elastic contribution, it seems that the really significant point is not that our estimate of the compound-elastic contribution as derived from the inelastic scattering consideration agrees with the difference between the Ni and Cu elastic cross sections, but more importantly the inelastic contribution in the scattering of protons from Cu as determined in an identical measurement is down by a factor of from two to three relative to Ni. Since, in the copper measurement, we were looking at the combination of more than one unresolved level, experimentally, the factor of two or three in the Ni-Cu comparison indicates the inelastic scattering in Cu to a given level is probably even less than the observed factor. This observation is then in agreement with the spirit of our interpretation that calls for scattering to any one level in the odd-Z targets to be less than that for the even-Z ones.

The most reasonable objection to this whole line of reasoning lies in the fact that other channels, open for the decay of the compound nucleus, involve the emission of neutrons. On the other hand the total cross section for (p,n) reactions on Cu^{63} at this energy is only of the order of $\frac{1}{5}$ of the nuclear area.[22] This indicates that all the emergent neutron reactions are not significantly more important relative to the decay of the compound nucleus than what we have called compound-elastic scattering or for that matter inelastic scattering to a single level. One can say, therefore, that inelastic scattering, neutron emission, and compound-elastic scattering are roughly each of the same importance relative to the decay of the compound nucleus, and therefore the scattering from even-Z targets includes compound-elastic scattering in a significant quantity. It is not as obvious, however, why at higher energies the neutron emission reactions and inelastic scattering should not dominate the decay of the compound nucleus and thereby make the neglect of any compound-elastic contribution negligible, although it is true that between 7.5 Mev and 17 Mev the cross section for reactions on Cu in which one or more

[19] W. W. Buechner and A. Sperduto, Bull. Am. Phys. Am. Phys. Soc. Ser. II, 1, 39 and 223 (1956).

[20] The difference in Rutherford scattering for these two elements, a factor of $(29/28)^2=1.075$, has been neglected. That is to say, the value cited here is $(d\sigma/d\Omega)_{Ni}-(d\sigma/d\Omega)_{Cu}$.

[21] S. T. Butler, Phys. Rev. **107**, 272 (1957).

[22] S. N. Ghoshal, Phys. Rev. **80**, 939 (1950).

neutrons are emitted only increases by about a factor of three.[22]

B

It is of some interest to compare our results to the prediction of a simple optical model calculation. A relatively straightforward calculation of the scattering of high-energy neutrons gives as the differential cross section:

$$\frac{d\sigma}{d\Omega}=k^2R'^4\left[\frac{J_1(2kR'\sin(\theta/2))}{2kR'\sin(\theta/2)}\right]^2, \tag{1}$$

where R' is the interaction radius, k the wave number of the incident neutron, $J_1(2kR'\sin(\theta/2))$ the ordinary Bessel function of the first order of the argument indicated, and θ is the angle of scattering of the incident particle. As has been indicated, the above data, at the pertinent points of the angular distributions, show a constancy of this argument if R' can be expressed as a constant times $A^{\frac{1}{3}}$. If what we were observing was truly a diffraction pattern then one might be able to assign a radius to the scattering center from the values of this variable.

Assuming the validity of this formula in the proton scattering experiments and equating $2kR'\sin(\theta/2)$ to the value characteristic of the zeros or maxima of the square of the Bessel function, one finds the values for the parameter r_0 in the equation $r_0=RA^{-\frac{1}{3}}$, 1.33 ± 0.03, 1.35 ± 0.03, and 1.37 ± 0.05, all in units of 10^{-13} cm, for the values of $A^{\frac{1}{3}}\sin(\theta/2)$ given in Sec. III.

In general, if one is dealing with a nuclear force radius these values are quite reasonable. However, it is believed that this agreement is nothing more than fortuitous. One reason for this belief is the fact that with a reduced wavelength of $\lambda\!\!\!^{-}=1.66\times10^{-13}$ cm, a sizable fraction of the nuclear radius, $(\lambda\!\!\!^{-}/R_N\sim0.3)$ the $R=r_0A^{\frac{1}{3}}$ law should show a systematic deviation because of the fact that the interaction radius should be $(R_N+\lambda\!\!\!^{-})$ where R_N is the nuclear radius. Of course in the precise region of validity of Eq. (1), $\lambda\!\!\!^{-}/R_N\ll1$ and therefore the actual question of whether R_N or $(R_N+\lambda\!\!\!^{-})$ should be used in this equation is immaterial. An additional reason for being suspect of this agreement is that application of this same analysis to the higher energy data yields very different values for r_0. Furthermore Eq. (1) as applied to elastic proton scattering neglects not only Coulomb scattering by the nucleus, but the interference term between Coulomb and nuclear scattering as well.

Before leaving this analysis, however, two further points should be made. One is that though the derivation of Eq. (1) above does contain a small-angle approximation, the result seems to be valid over the whole range. If one calculates neutron scattering in analogy with the scattering of light as has been done by Fernbach, Serber, and Taylor,[23] then the argument of the Bessel function is not $2kR\sin(\theta/2)$ but $kR\sin\theta$. Again this is a small angle theory and in the region where both of these formulas are truly valid there is no difference. However, the argument $A^{\frac{1}{3}}\sin\theta$ does not show the constancy at even the angle of the first minimum that the $A^{\frac{1}{3}}\sin(\theta/2)$ variable does.

The other point to be made is that as Glassgold *et al.*[7] have shown, Vr_0^2 (V is the depth of the real part of a complex-potential well and r_0 has the same significance as above) represents a certain constant of the system. That is, they can obtain a reasonably good fit to some of the 10-Mev proton scattering data with various values of either V or r_0 providing Vr_0^2 keeps a constant value. Furthermore they have found the same value of Vr_0^2 for all nuclei. This fact combined with the observed constancy of r_0, independent of nucleus implies, but does not prove, the constancy of the real part of the nuclear potential as a function of mass number A. It is necessary to note that the location of the maxima and minima has been found to be insensitive to the imaginary part of the potential.[7]

C

Since a large number of partial waves seem to be entering into the elastic scattering process we have not attempted any calculations on the basis of the complex-potential well. Melkanoff *et al.*[24] however, have obtained good fits to some of these data, which aside from enabling one to determine the energy dependence of the well parameters, shows some interesting and not entirely unexpected results. In particular they also find Ni difficult to fit. Furthermore they find that lighter nuclei demand a 1-Mev to 2-Mev depth for the imaginary part of the potential well, whereas the heavier elements call for a depth of ~5 Mev. Pending a more complete analysis by them, nothing more can be said regarding the values of the parameters. Furthermore the question of the neglect of compound-elastic scattering as well as any spin-orbit interaction also creates doubt concerning the validity of this type of analysis for our experiments, and therefore the actual significance of any parameters derived on this basis.

V. ACKNOWLEDGMENTS

We wish to thank Professor V. F. Weisskopf and Professor H. Feshbach for several interesting and stimulating conversations. We would also like to acknowledge the assistance of Mr. E. F. White, Mr. F. J. Fay, and the late Mr. A. R. Nummela of the M.I.T. Cyclotron Group.

We also wish to thank Dr. D. A. Bromley of Chalk River National Laboratories for his services in the least-squares analysis of the experimental data.

[23] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[24] Melkanoff *et al.* (private communication).