Possibility of Hyperfragment Formation in $K^- - d$ Reactions

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(Received May 27, 1957)

If there exist bound Σ^--neutron and/or Λ^0-proton systems, the interaction between slow K^--mesons and deuterons may be an effective means for their production. Estimates are given for the ratio Γ of production of bound states to the corresponding free states for the reactions $K^-+d\rightarrow\Sigma^-+n+\pi^+$ or $\Lambda^0+p+\pi^-$, where the K^- is supposed to be bound in a low-lying atomic orbit. The following connected alternatives must be considered: (1) K^- is scalar or pseudoscalar, (2) the capture takes place from an atomic s- or p-state, (3) the final bound state is ^3S or ^1S. The ^3S and ^1S cases are qualitatively different. Apart from this it turns out that Γ depends mainly on the binding energy of the hyperon-nucleon system and is insensitive to its effective range. If the (Σ^-,n) system is bound in a ^3S-state with an energy $\sim$1 Mev, Γ may be anticipated to be of order unity. It is also noted that the reactions in question may yield partially polarized hyperon beams.

I. INTRODUCTION

It is the purpose of this paper to show that a study of the interaction of slow K^- mesons with deuterons may be particularly suited to obtain information on hyperon-nucleon binding. Specifically, we have in mind the reactions

$$K^-+d\rightarrow\Sigma^-+n+\pi^+, \tag{1}$$

$$K^-+d\rightarrow\Lambda^0+p+\pi^-, \tag{2}$$

where the K^- meson is captured from a Bohr orbit.

Recently an event has been reported[1] which may be interpreted as the decay of a (Σ^+,p) hyperfragment. If these particles indeed can bind, the same should be true for the system (Σ^-,n). The latter case would in fact be favored, owing to the absence of Coulomb repulsion. At any rate, if the (Σ^-,n) shows binding which is at all appreciable, particularly in the ^3S state, we shall see that reaction (1) should be a good source for such two-body hyperfragments.

No bound (Λ^0,p) systems have so far been unambiguously identified.[2] Indications are that if bound states exist at all, the binding energy is not more than a few tenths of an Mev. But we shall see that even if the binding is no stronger than this, the chances are not negligible that such fragments could be produced in the reaction (2).

In order to estimate the relative frequencies of the effects in question, we shall employ an impulse approximation in which the reaction (1), for example, takes place "on the proton." That is to say, we consider the process

$$K^-+p\rightarrow\Sigma^-+\pi^+, \tag{3}$$

to be the primary mechanism for reaction (1). Such an assumption is perhaps not unreasonable. The reaction volume for K absorption is presumably small compared to the dimensions of the deuteron. The pion energies involved ($\leq$86 Mev) are well below the region of resonance in π-nucleon scattering. For this reason multiple π scattering effects should not be very large, a circumstance which favors the validity of the impulse approximation. In this spirit then, we shall treat the outgoing pion as free; and we neglect, for example, contributions to reaction (1) coming from K absorption "on the neutron" via $K^-+n\rightarrow\Sigma^-+\pi^0$, with subsequent exchange scattering $\pi^0+p\rightarrow n+\pi^+$.

It may be noted that direct experimental evidence for the validity of the present picture may come from a comparison of the rates of reactions like (1) with, for example, $K^-+d\rightarrow\Sigma^-+p$. The latter is favored by phase space but should be relatively suppressed if conditions are unfavorable for the virtual reabsorption of pions that have to be produced in intermediate states.

In the same way we shall assume that reaction (2) proceeds, in the sense of the impulse approximation, from the primary process

$$K^-+n\rightarrow\Lambda^0+\pi^-. \tag{4}$$

In both reactions, (1) and (2), the interaction between the final baryons is in principle taken into account by the use of the appropriate (bound or continuum) wave functions of the two-baryon system.

The matrix element for either of our reactions will have the form[3]

$$Q=\int v^*(\mathbf{r})e^{-i\mathbf{k}_0\cdot\mathbf{r}}Tu(\mathbf{r})d\mathbf{r}, \tag{5}$$

where u is the ground state function of the deuteron and v is the final hyperon-nucleon wave function. The arguments in both functions are the relative coordinates

[1] Baldo-Ceolin, Fry, Greening, Huzita, and Limentani, Nuovo cimento (to be published). We are grateful to the Padua group for keeping us informed of their results in advance of publication.

[2] For hyperfragment surveys see Fry, Schneps, and Swami, Phys. Rev. **99**, 1561 (1955); **101**, 1526 (1956); and **106**, 1062 (1957). Also Filipkowski, Gierula, and Zelinsky, Acta Phys. Polon. (to be published); V. L. Telegdi, *Proceedings of the Seventh Rochester Conference on High-Energy Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

[3] We put $\hbar=c=1$.

in question and, of course, v and u depend on the spin states involved. We shall assume Σ and Λ to have spin $\frac{1}{2}$ and K to have spin zero. Finally

$$\mathbf{k}_0=\left(\frac{m}{M+m}\right)\mathbf{p}_\pi, \tag{6}$$

where $\mathbf{p}_\pi$ is the pion momentum and m and M are respectively the nucleon and hyperon masses.

The operator T is the transition operator for the basic interaction (3) or (4), as the case may be. The structure of T depends on further dynamical details. In particular, it depends on whether the K meson capture takes place from an s- or p-state atomic orbit (these are the only two possibilities expected to have appreciable probability); and it also depends on the relative intrinsic parities of the particles concerned.

As to the latter, the intrinsic parities of strange particles in fact cannot be defined relative to that of the nucleon, if parity is not conserved in weak interactions.[4] But all that matters is the product parity of K meson and hyperon relative to the nucleon parity. For convenience of writing, we shall say that the hyperon parity is even, both for reactions (1) and (2)—without implying by this convention that the Λ^0 and Σ parities are in fact relatively even; and thus for either reaction we shall distinguish between K mesons of even and odd parity. There is some indication, however, that the Λ and Σ parities are indeed relatively even and that, relative to these, the K meson has odd parity.[5]

In the following section we will be concerned with estimating the frequency in reaction (1) for production of (Σ^-,n) in a bound state, if it exists, relative to production in the continuum. It will turn out that if there is a bound triplet S state, the frequency can be expected to be appreciable, largely independent of whether the reaction takes place from an s- or p-state orbit and of whether the K meson is scalar or pseudoscalar (in the sense defined above). The results of course depend on the assumed binding energy and on the details of the bound state wave function, but not too sensitively if the binding energy is more than a few tenths of a million volts. If the bound state is a spin singlet, appreciable production of (Σ^-,n) fragments can again be expected if the K meson is scalar—or, if pseudoscalar, if capture from p-state orbits is significant. Identical considerations apply for the production of (Λ^0,p) fragments in reaction (2), where, however, the probabilities are reduced because of the larger energy release involved here.

There is as yet little experimental evidence which bears directly on the question of the relative importance of capture from s- and p-state orbits. There is, however, some information on the magnitude and energy dependence of K^- absorption on hydrogen;[5] and an analysis similar to that carried out by Brueckner *et al.*[6] suggests that s- and p-state capture may be comparable, the latter perhaps being slightly favored.

Although our main interest here lies in the relative frequency for production of fragments, it should be noted that strong hyperon-nucleon forces, even if they are not sufficient to produce binding, would distort the continuum spectrum in a characteristic way in the reactions (1) and (2). It is perhaps premature at this stage to discuss this in detail, however; such effects require more quantitative information than is required for discussion of possible fragment production.

II. FRAGMENT PRODUCTION RATES

A

We begin by considering the case where the K meson is pseudoscalar and capture occurs from an atomic s-state. The T operator of Eq. (5) must transform like a scalar. In our approximation, according to which the primary reaction occurs on one specific nucleon, say nucleon "1" (see Eqs. (3) or (4)), T must have the general form

$$T=a+ib\left(\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\right)\cdot(\mathbf{p}_\pi\times\mathbf{p})+ib\left(\frac{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2}{2}\right)\cdot(\mathbf{p}_\pi\times\mathbf{p}), \tag{7}$$

where $\mathbf{p}$ is the gradient operator which acts on the relative coordinate $\mathbf{r}$. The coefficients a and b are scalar functions which may depend on $\mathbf{p}_\pi$ and $\mathbf{p}$. We shall, in fact, assume that the first term is essentially a constant and further that it is much larger, for the energies in question, than the last two terms. This is equivalent to the assumption that in reactions (3) or (4) the $l=0$ K-meson partial wave is predominant at low energies. There is some evidence[7] that the absorption cross section for negative K mesons on hydrogen increases rapidly with decreasing energy, which suggests an important s-wave contribution.

Insofar then as we retain only the first term in Eq. (7), we see that in reactions (1) and (2) the hyperon-nucleon system is produced only in a triplet spin state; so that for the case under discussion (pseudoscalar K meson, capture from an s-state) it is only if binding occurs in the triplet state that fragments can be produced. Let us suppose there is a ^3S bound state. The rate of production of fragments is then given by

$$R_b=2\pi|a|^2\int\frac{d\mathbf{p}_\pi}{(2\pi)^3dE}\left|\int v_B^*(\mathbf{r})e^{-i\mathbf{k}_0\cdot\mathbf{r}}u(\mathbf{r})d\mathbf{r}\right|^2, \tag{8}$$

[4] Concerning the definability of such intrinsic parities, see A. Pais, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

[5] See M. Gell-Mann, Phys. Rev. **106**, 1296 (1957). The argument concerns the K^+. On invariance grounds one will assign the same intrinsic properties to K^- as to K^+ (with respect to strong interactions).

[6] Brueckner, Serber, and Watson, Phys. Rev. **81**, 575 (1951).

[7] L. Alvarez, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

where v_B is the bound state hyperon-nucleon wave function and $d\mathbf{p}_\pi/dE$ is essentially the density of final states "on the energy shell." For reaction (1), $p_\pi=177$ Mev/c and $k_0=78$ Mev/c; for reaction (2), $p_\pi=265$ Mev/c, $k_0=121$ Mev/c.

The total rate for production in the continuum is given by

$$R_f=2\pi|a|^2\int\frac{d\mathbf{k}d\mathbf{p}_\pi}{(2\pi)^6dE}\left|\int v_{\mathbf{k}}^*(\mathbf{r})e^{-i\mathbf{k}_0\cdot\mathbf{r}}u(\mathbf{r})d\mathbf{r}\right|^2, \quad (9)$$

where $v_{\mathbf{k}}(\mathbf{r})$ is a continuum hyperon-nucleon wave function labeled by the relative momentum $\mathbf{k}$. If we were free to integrate over all $\mathbf{k}$, we could evidently use closure[8] on the function $v_{\mathbf{k}}$, namely,

$$\frac{1}{(2\pi)^3}\int v_{\mathbf{k}}^*(\mathbf{r})v_{\mathbf{k}}(\mathbf{r}')d\mathbf{k}=\delta(\mathbf{r}-\mathbf{r}')-v_B^*(\mathbf{r})v_B(\mathbf{r}'). \quad (10)$$

Actually the k-domain is bounded by over-all energy conservation. It follows that we overestimate R_f by applying closure to (9). If we do apply closure, the integration over p_π is to be replaced by an assignment of a suitable mean value to $|\mathbf{p}_\pi|$. If we take for this the maximum value p_π can attain, which is just the value corresponding to the pion energy for fragment production (aside from small corrections due to binding energy), we overestimate R_f once again.

Denote by Γ the ratio of the rates for producing bound hyperon-nucleon systems relative to free systems. Owing to the nature of our approximations, we find a lower limit for Γ. The result is

$${}^3\Gamma\gtrsim{}^3\Gamma_0=\frac{|{}^3X|^2}{1-|{}^3X|^2}, \quad (11)$$

where

$${}^3X=\int d\mathbf{r}^3v_B^*e^{-i\mathbf{k}_0\cdot\mathbf{r}}u, \quad (12)$$

with k_0 evaluated at maximum pion energy. The superscript *three* denotes that we are dealing with the case where the bound state is a spin triplet. It may be remarked that the lower limit on ${}^3\Gamma$ obtained here by closure is probably not too different from the value which would be obtained by a more careful treatment—since the very fact of attractive forces strong enough to produce binding would mean that the hyperon-nucleon system in the continuum would tend to have small relative energy. The average value of p_π is probably close to the maximum value, therefore, and the states which violate energy conservation would have little weight.

We can estimate ${}^3X$ as follows: Put

$$u(r)=(N/r)(e^{-\alpha r}-e^{-\beta r}), \quad (13)$$

$${}^3v_B(r)=(N'/r)(e^{-\alpha' r}-e^{-\beta' r}). \quad (14)$$

[8] For a similar discussion see G. F. Chew and H. W. Lewis, Phys. Rev. 84, 779 (1951).

Expression (13) is the usual approximation for the deuteron ground state wave function, where

$$\alpha=(M\epsilon)^{\frac{1}{2}}, \quad (15)$$

ϵ being the magnitude of the deuteron binding energy; β is related to the triplet effective range ρ by

$$\rho=\frac{4}{\alpha+\beta}-\frac{1}{\beta}; \quad (16)$$

with $\rho=1.7\times10^{-13}$ cm, $\beta=6.2\alpha$. The normalization factor N is given by

$$N=\left(\frac{\alpha}{2\pi(1-\alpha\rho)}\right)^{\frac{1}{2}}. \quad (17)$$

The high-momentum Fourier components of the deuteron wave function are not sensitively involved in the present considerations. Thus the rather phenomonological form (13) should be suitable for our purposes. Likewise it is perhaps reasonable to characterize the hyperon-nucleon bound state by a wave function (14) of similar structure. Specifically we have

$$\alpha'=\left(\frac{2mM}{m+M}\epsilon'\right)^{\frac{1}{2}} \quad (18)$$

where ϵ' is the magnitude of the hyperon-nucleon binding energy. The relations (16) and (17) again hold true for the primed quantities. We then find

$${}^3X=\frac{4\pi NN'}{k_0}\left[\tan^{-1}\left(\frac{k_0(\alpha+\alpha'+\beta+\beta')}{(\alpha+\alpha')(\beta+\beta')-k_0^2}\right)-\tan^{-1}\left(\frac{k_0(\alpha+\alpha'+\beta+\beta')}{(\alpha+\beta')(\beta+\alpha')-k_0^2}\right)\right]. \quad (19)$$

In Table I we list the value of ${}^3X$ for the (Σ^-,n) system for several values of the binding energy and for two choices of effective range: $\rho'=0$ and $\rho'=\rho$. The latter value is perhaps reasonable since the Σ, n forces presumably involve single pion exchange and should have a range comparable to that of nucleon-nucleon forces. In any case, the results are not too sensitive to the effective range. In Fig. 1 the fraction ${}^3\Gamma_0$ is plotted as a function of the binding energy.

For the (Λ^0,p) system the forces involve the exchange of at least two pions and here one expects a shorter effective range. In Table II we list values of X for this system, taking zero effective range and restricting ourselves to small binding energies.

TABLE I. Value of X for a (Σ^-,n) fragment.

ϵ' (Mev)	4.5	2.0	1.0	0.50	0.13
$X(\rho'=\rho)$	0.78	0.74	0.69	0.61	0.47
$X(\rho'=0)$	0.70	0.71	0.68	0.64	0.51

B

Here we turn to the situation where a pseudoscalar K meson is captured from a p-state orbit. The transition matrix must again transform like a scalar, but it must now be linear in $\nabla\psi_K(0)$, the gradient of the p-state K meson wave function evaluated at the deuteron. The general form, in our approximation, is then

$$T=c\mathbf{p}_\pi\cdot\nabla\psi_K(0)+id\left(\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\right)\cdot[\mathbf{p}_\pi\times\nabla\psi_K(0)]$$
$$+id\left(\frac{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2}{2}\right)\cdot[\mathbf{p}_\pi\times\nabla\psi_K(0)],\quad(20)$$

plus similar terms in which $\mathbf{p}_\pi$ is replaced by $\mathbf{p}$, the gradient operator which acts on the relative coordinate $\mathbf{r}$. We shall neglect these latter terms, which represent baryon recoil effects. This is not clearly justified; but we note that the average pion momentum in reactions (1) and (2) is in general much larger than the baryon momenta. At any rate, the inclusion of the additional terms does not alter the order of magnitude of the effects with which we are concerned. The coefficients c and d we shall again take to be constants.

The first two terms in Eq. (20) connect to final triplet states for the hyperon-nucleon system; the last term produces transitions to singlet states. Under the present circumstances then, if the magnitudes of c and d are comparable, fragments which are either triplet or singlet could be produced.

Suppose that the bound state is a spin triplet (we do not consider the possibility that there is both a triplet and a singlet bound state). We again apply closure to obtain an upper limit on the rate for production of the hyperon-nucleon system in the continuum, both for triplet and singlet states. The number of (triplet) fragment events, relative to continuum events (triplet plus singlet) satisfies the inequality

$$^3\Gamma\gtrsim\frac{(|c|^2+\frac{4}{3}|d|^2)|^3X|^2}{(|c|^2+\frac{4}{3}|d|^2)(1-|^3X|^2)+\frac{2}{3}|d|^2}.\qquad(21)$$

The numerical factors arise from averaging over the deuteron polarization and over the magnetic quantum number of the K-meson p-state wave function. Again the superscript *three* denotes that we are considering the case of spin-triplet fragments; and ^3X is again given by Eq. (19). The results now depend on the relative magnitudes of c and d, the no-spin-flip and spin-flip coefficients. In the case most unfavorable for production of spin-triplet fragments ($c=0$), we have

$$^3\Gamma\gtrsim\frac{|^3X|^2}{\frac{3}{2}-|^3X|^2},\qquad(21')$$

so the minimum value is not much smaller than in the case of capture from an s-state orbit [Eq. (11)].

TABLE II. Value of X for a (Λ^0,p) fragment.

ϵ' (Mev)	0.51	0.13	0.045
$X(\rho'=0)$	0.47	0.36	0.29

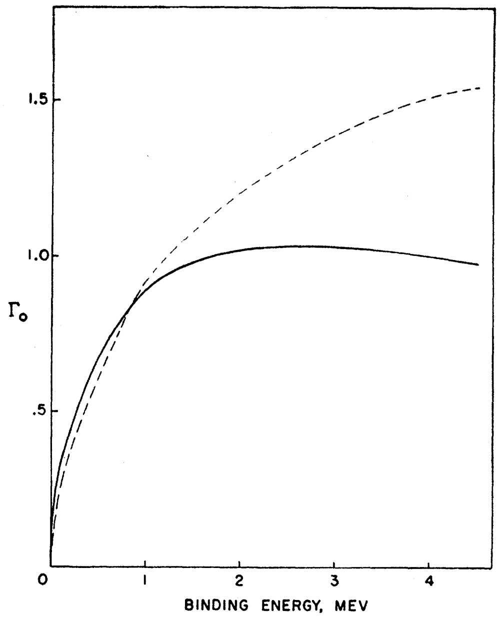

FIG. 1. Lower limit on number of (Σ^-,n) fragment events relative to continuum events, as a function of fragment binding energy, for the case of pseudoscalar K meson, s-state capture. Solid curve: effective range $\rho'=0$; dashed curve: $\rho'=1.7\times10^{-13}$ cm.

If the bound state is a spin singlet, the number of (singlet) fragment events, relative to continuum events (triplet plus singlet), satisfies the inequality

$$^1\Gamma\gtrsim\frac{\frac{2}{3}|d|^2|^1X|^2}{\frac{2}{3}|d|^2(1-|^1X|^2)+(|c|^2+\frac{4}{3}|d|^2)},\qquad(22)$$

where the superscript *one* denotes that we are considering spin-singlet fragments; and ^1X is given by an equation analogous to (11), with v_B the singlet bound state wave function. We assume that the latter can again be represented by the structure (14), so that ^1X, as a function of binding energy and effective range, is again given by Table I. We see that appreciable production of singlet fragments will occur only if the spin-flip coefficient d is comparable to or larger than the coefficient c.

C

We now turn to the case where the K meson is scalar. The T operator must then transform like a pseudoscalar. For capture from an s-state atomic orbit the general form, in our approximation, is

$$T=e\left(\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\right)\cdot\mathbf{p}_\pi+e\left(\frac{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2}{2}\right)\cdot\mathbf{p}_\pi, \tag{23}$$

plus similar terms with $\mathbf{p}_\pi$ replaced by $\mathbf{p}$. We shall again neglect the latter terms and assume that the coefficient e is a constant. For capture from a p-state orbit, T must be linear in $\nabla\psi_K(0)$ and we have

$$T=f\left(\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\right)\cdot\nabla\psi_K(0)+f\left(\frac{\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2}{2}\right)\cdot\nabla\psi_K(0), \tag{24}$$

where again we assume that the coefficient f is a constant.

For both cases, s- and p-state capture, we now have terms which connect to triplet and singlet states of the final hyperon-nucleon system. We proceed as before to obtain lower limits on Γ, the ratio of fragment events to continuum events. The results are identical in form for both s- and p-state capture.

If it is the triplet state which is bound, we find

$${}^3\Gamma\gtrsim\frac{|{}^3X|^2}{(1-|{}^3X|^2)+\frac{1}{2}}; \tag{25}$$

if it is the singlet state which is bound, we have

$${}^1\Gamma\gtrsim\frac{|{}^1X|^2}{(1-|{}^1X|^2)+2}. \tag{26}$$

III. SUMMARY AND DISCUSSION

The results of the preceding analysis can be summarized in the following way. If the hyperon-nucleon systems in reactions (1) or (2) can bind in a ${}^3S$ state, then appreciable production of such fragments can be expected, especially for the (Σ^-,n) system. Our estimates of the relative frequency of fragment production depend on whether the K meson is scalar or pseudoscalar, on whether capture occurs from an s- or p-state orbit, and on the details of the bound-state wave function. The results are not, however, too sensitive to any of these factors, provided the binding energy is at all appreciable. In Fig. 1 the lower limit on the relative frequency for (Σ^-,n) fragments is plotted as a function of binding energy for an especially favorable case: pseudoscalar K-meson, s-state capture. But for the other alternatives the situation is not too much less favorable, as can be seen by comparison of Eqs. (21′) and (25) with Eq. (11).

If binding occurs in a ${}^1S$ state and if the K meson is scalar, the orders of magnitude of Γ are the same as for ${}^3S$ fragments [see Eq. (26)]. For pseudoscalar K mesons, on the other hand, appreciable production of singlet fragments can be expected only if p-state capture is significant and if there is an important spin-flip term in the transition matrix element [see Eqs. (20) and (22)].

The essential approximations on which the present discussion have been based are the following: (1) the choice of a specific form for the bound state wave functions [we believe, however, that the representation of Eq. (14) is adequate for the present purposes and that no appreciable error is involved in this choice]; (2) the use of the impulse approximation; (3) the neglect of terms involving baryon recoil in the transition operators.

As to the last-mentioned, we may say that the inclusion of such terms in any case would not be likely to alter qualitatively the results obtained here on fragment production. However, it is important to note that they would give rise to another kind of effect which is not without interest. One sees, for example, that if in Eq. (7) both the a and b terms are retained, then their interference would give rise to a polarization of the free hyperon—a polarization perpendicular to the plane of the reaction. If parity conservation is violated in the subsequent weak decay of the hyperon, one could detect this polarization by means of an "up-down" asymmetry relative to the plane of the reaction. It would be worthwhile, and simple experimentally, to look for such effects.

In this paper we have concentrated on the question of fragment production rather than on the question of the spectrum for the case of unbound particles, because, as will by now be clear, the former problem can be handled by means of rather qualitative considerations. Of course, a study of the spectrum by itself will give further information which in part is connected with the various alternatives we had to consider under the headings A, B, C, of Sec. II. Thus, for example, if strong attractive forces favor hyperon-nucleon S-states, the distribution in angle θ between the pion and the relative hyperon-nucleon momentum will be composed of $L=0$ and 1. In special cases this distribution may even be simpler: if the K^- is pseudoscalar (scalar) and if capture from an s-(p-) state is strongly favored, the distribution in θ will tend to be isotropic.

These last comments on the spectrum apply equally well to such reactions as $K^-+d\rightarrow\Sigma^++n+\pi^-$ for which the fragment problem does not arise. Finally, it is clear that the problem of comparing the total rate of reactions like (1) with the various other processes that can happen upon K^--absorption by deuterons will also necessitate more detailed assumptions on the dynamics than were needed in the present discussion of fragment rates.