Method for Testing Time-Reversal Invariance in Beta Decay*

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One of the possible tests of time-reversal invariance in weak interactions is the electron-neutrino angular correlation, for which no polarized nuclei and no measurements of the polarization of emitted electrons are required. If the electron-neutrino angular correlation function, e.g., $1+(p/3W)f(g)\cos\theta$ (for $\Delta J=\pm 1$ transitions) has p and/or Z dependences in the measurements of $f(g)$ [see Eq. (7)], invariance under time reversal T is not conserved. Conversely, if invariance under T holds, there are no p and Z dependences of $f(g)$. In experiments, it is advisable to compare the $f(g)$'s for electron and positron decays. Furthermore, the experiment will be another possible test of charge-conjugation invariance if time-reversal invariance does not hold.

1. INTRODUCTION

TO test the conservation of parity in weak interactions, several crucial experiments were suggested by Lee and Yang.[1–3] Recent results on the beta decay of polarized Co^{60} nuclei[4] and the electron angular distribution of $\pi-\mu-e$ decay[5] showed that the weak interactions such as beta decay and $\pi-\mu-e$ decay are indeed not invariant under space inversion (P) and charge-conjugation (C). However, the question of invariance under time-reversal (T) is still unanswered.

From the theoretical studies of several authors[3,6,7] together with the results of the recent experiments,[4,5] the remaining possibilities of the invariance properties for weak interactions (H) are, either (i) H is invariant under T, PC, and CP, but not invariant under P and C; or (ii) H is invariant under PCT and its permutations, but not invariant under each of the single operators, P, C, and T.

As is well known, the invariance with respect to T imposes the restriction that the 10 coupling constants C_i and C_i' must be real (apart from a trivial common phase factor which can be normalized to unity). Therefore, we can test invariance with respect to T by measuring the values of $(iC_i^*C_j+\text{c.c.})$ $(i\neq j)$ or $(iC_i^*C_j'+\text{c.c.})$ $(i=j$ and $i\neq j)$ in experiments. In terms of measurable quantities, it is necessary to measure terms that change sign under T, which can be constructed out of the following five quantities: polarization of decaying nuclei $\langle\mathbf{J}\rangle$, polarization of emitted electron $\boldsymbol{\sigma}$, momenta of electrons and neutrinos $\mathbf{p}_e$, $\mathbf{p}_\nu$, and the factor $\alpha Z/p$ which gives the Coulomb distortion of the electron wave functions. A general rule for constructing terms that are not invariant under T has been given by Lee and Yang.[8]

In this connection, one method which could be used to test invariance with respect to T was proposed by Lee and Yang.[2,3] This is to measure the p or Z dependence of the asymmetry parameters β in the beta angular distributions from polarized nuclei.[9]

Jackson, Treiman, and Wyld[10] investigated also possible tests of invariance with respect to T in connection with the quantities $\langle\mathbf{J}\rangle\cdot(\mathbf{p}_e\times\mathbf{p}_\nu)$, $\boldsymbol{\sigma}\cdot(\mathbf{p}_e\times\mathbf{p}_\nu)$, and $\boldsymbol{\sigma}\cdot(\langle\mathbf{J}\rangle\times\mathbf{p}_e)$.

Here we will show another method which could be used to test the invariance under time reversal. This method was once proposed by one of the authors[11] for just such a test but was based on the old theory of beta decay where parity is conserved. The principal quantity to be measured in this method is the term $\mathbf{p}_e\cdot\mathbf{p}_\nu(\alpha Z/p)$. Consequently, this method needs neither polarized nuclei nor any measurements of the polarization of emitted electrons. This may help greatly in simplifying the experimental procedures. Furthermore, the $\alpha Z/p$ term is not so small in certain circumstances as one suspected at first.

2. ELECTRON-NEUTRINO ANGULAR CORRELATION

Let us consider the electron-neutrino angular correlation by taking Eq. (A1) of reference 1 as the interaction of beta decay. If we use the same notation as reference 1, the energy and angle distribution of the electron in an allowed transition is given as follows[12]:

$$N(W,\theta)dW\sin\theta d\theta=(\xi L_0/4\pi^3)F(Z,W)pW(W_0-W)^2 \times\{1+(a+a_T)(p/W)\cos\theta+(b/W)\}dW\sin\theta d\theta, \quad (1)$$

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[9] This is also possible in transitions with $\Delta J=0$, where the asymmetry parameter has $(iC_T^*C_A'+\text{c.c.})$, $(iC_S^*C_A'+\text{c.c.})$ and similar terms in its $(\alpha Z/p)$ term. The cross terms among interactions of Fermi and Gamow-Teller type like these can be expected in some other phenomena of beta decay from unpolarized nuclei (to be published).
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[12] The fourth term of (2) in reference 11 should be multiplied by a factor $-\frac{1}{3}$, as the first term of (4) in this paper. In (A4) of reference 1, (4) was not considered.

with

$$L_0=(2p^2F)^{-1}(g_{-1}^2+f_1^2),$$

$$\xi=(|C_S|^2+|C_V|^2+|C_S'|^2+|C_V'|^2)|M_{\mathrm{F}}|^2+(|C_T|^2+|C_A|^2+|C_T'|^2+|C_A'|^2)|M_{\mathrm{G.T.}}|^2, \quad (2)$$

$$a\xi=\tfrac{1}{3}(|C_T|^2-|C_A|^2+|C_T'|^2-|C_A'|^2)|M_{\mathrm{G.T.}}|^2-(|C_S|^2-|C_V|^2+|C_S'|^2-|C_V'|^2)|M_{\mathrm{F}}|^2, \quad (3)$$

$$a_T\xi=\pm[\tfrac{1}{3}(iC_T{}^*C_A+iC_T'{}^*C_A')|M_{\mathrm{G.T.}}|^2-(iC_S{}^*C_V+iC_S'{}^*C_V')|M_{\mathrm{F}}|^2+\text{c.c.}](\alpha Z/p), \quad (4)$$

$$b\xi=\pm\gamma[(C_S{}^*C_V+C_S'{}^*C_V')|M_{\mathrm{F}}|^2+(C_T{}^*C_A+C_T'{}^*C_A')|M_{\mathrm{G.T.}}|^2+\text{c.c.}]; \quad (5)$$

α is the fine-structure constant. These equations are correct without any assumption for the magnitude of Z, but with neglect of the finite de Broglie wavelength effect and finite nuclear-size correction. The upper (lower) signs in (4) and (5) refer to electron (positron) decay. As we expected, Eq. (4) shows dependence on T and also on C. As the recent experimental results[4,5] showed noninvariance with respect to C in weak interactions, this term remains. Equation (5) is equivalent to the Fierz interference term in the old theory with parity conservation, except that $C_S{}^*C_V$ is replaced by $C_S{}^*C_V+C_S'{}^*C_V'$ and so on. If invariance under C holds, then (5) automatically vanishes. From this phenomenon, however, we can get no information concerning invariance under P, because of lack of interferences between C_i and C_j'.[1]

To simplify the situation, let us assume that C_i and C_i' have equal absolute magnitudes and a very small phase difference, i.e., $C_i\approx\pm C_i'$. Furthermore, we define $C_A/C_T=g\exp(i\varphi)$, where g and φ are real. From (1), $\cdots$, (5), the electron-neutrino angular correlation function $W(\theta)$, for $\Delta J=\pm1$ for example, becomes as follows:

$$W(\theta)=1+(p/3W)f(g)\cos\theta, \quad (6)$$

with

$$f(g)=[1\mp2g(\alpha Z/p)-g^2]/(1+g^2), \quad (7)$$

where again the upper (lower) sign refers to electron (positron) decay, and we assumed $\varphi=\pi/2$,[11] which is in conformity with experimental results (no Fierz term). The value of $f(g)$ is $+1$ for a pure tensor and -1 for a pure axial vector. If the beta interaction is invariant under T, then C_A and C_T are in phase. Then, there are no momentum and charge dependences in $f(g)$ (and a $1/W$ term appears in the beta spectrum which restricts the value of $|C_A/C_T|$). If we observe a p or Z dependence of $f(g)$ in the experiment, then time-reversal invariance does not hold in beta decay.

From the data for the electron-neutrino angular correlation in the decay of He^6,[13] which has a very small αZ, we can obtain $g^2\lesssim\frac{1}{3}$. Therefore, if we choose some beta-active nuclide with a medium or high value of Z,[14] we can test the p or Z dependence of $f(g)$ and, consequently, the invariance under time-reversal. It is more advisable to measure and compare the $f(g)$'s of electron and positron decays in the same energy region:

$$f_1(g)-f_2(g)=(2g\alpha Z/p)(Z_1+Z_2)/(1+g^2), \quad (8)$$

where the subscripts 1 and 2 refer to positron and electron decays. Assuming invariance with respect to T or C, $f_1(g)-f_2(g)$ is equal to zero.

3. CONCLUSION

The p and Z dependences of $f(g)$ in the electron-neutrino angular correlation will reveal the invariance of beta interaction under T. If the measurement of the electron-neutrino angular correlation indicates that invariance under T does not hold, this phenomenon also indicates noninvariance under charge-conjugation. It is noticed that if the experimental $f(g)$ does not show any momentum or charge dependence, we cannot obtain any definite conclusion for invariance under time-reversal from this experiment alone. This is due to the possibility that although C_A and C_T are out of phase, the absolute magnitude of C_A/C_T is very small. The situation is similar for other methods to test time-reversal invariance given in references 2 and 10.

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[14] Beta-active nuclide of Z at least 15–40 should be used.