Beta-Gamma Correlations from Oriented Nuclei

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The measurement of the correlation function for β-γ transitions in oriented nuclei is proposed as a means of determining the validity of time-reversal invariance in the β interaction. Such experiments also provide a sensitive test of the type of Fermi interaction. The correlation function for allowed transitions ($\Delta J=0$, no) is presented in general form, with a discussion of the conclusions which can be drawn from such experiments.

RECENT developments[1] in β decay have raised two important questions about the β interactions: do they possess time-reversal invariance, and is the Fermi interaction S, or V, or $S+V$? Several experiments have been discussed[2] which could shed light on these questions. We intend to discuss here another experiment, the measurement of the β-γ correlation function of oriented nuclei, which can in principle give clear-cut answers to both questions. Such experiments are a relatively simple generalization of the original experiments of Wu *et al.*,[1] in which the β and γ distributions were measured individually but not in coincidence.

Before presenting the general result, let us discuss a special case for illustration. Consider the transition $1^+(\beta)1^+(\gamma)0^+$, assuming both Fermi and Gamow-Teller contributions present. The correlation function can be shown to be

$$W(\mathbf{p},\mathbf{k},\mathbf{J})\sim(A+B)R_0-\frac{1}{\sqrt{2}}(\tfrac{1}{2}A-B)R_2[\tfrac{3}{2}(\mathbf{J}\cdot\mathbf{k})^2-\tfrac{1}{2}]$$
$$+\frac{1}{\sqrt{3}}\left(\frac{v}{c}\right)\left(\frac{1}{\sqrt{2}}C+D\right)R_1\mathbf{J}\cdot\mathbf{p}$$
$$-\frac{1}{4\sqrt{3}}\left(\frac{v}{c}\right)(\sqrt{2}C-D)R_1(3\mathbf{J}\cdot\mathbf{k}\mathbf{p}\cdot\mathbf{k}-\mathbf{J}\cdot\mathbf{p})$$
$$-\tfrac{3}{4}\left(\frac{v}{c}\right)ER_2(\mathbf{J}\cdot\mathbf{p}\times\mathbf{k})(\mathbf{J}\cdot\mathbf{k}).$$

Here $\mathbf{J},\mathbf{p},\mathbf{k}$ are unit vectors in the directions of the orientation axis, the electron momentum, and the photon momentum, respectively. The parameters R_0, R_1, R_2 are the statistical tensors[3] of the initial state, and are related to the populations of the nuclear levels $a(m)$ by

$$R_k=\sum\nolimits_m (-)^{J-m}C(JJk;\,m,\,-m)a(m).$$

We have assumed that the orientation of the initial state is such that the system has axial symmetry about $\mathbf{J}$. Finally, the constants $A\cdots E$ are defined by[4]

$$A=[(\alpha_{TT}+\alpha_{AA})\pm\gamma W^{-1}(\alpha_{TA}+\text{c.c.})]|M_{GT}|^2,$$
$$B=[(\alpha_{SS}+\alpha_{VV})\pm\gamma W^{-1}(\alpha_{SV}+\text{c.c.})]|M_F|^2,$$
$$C=[\pm(\beta_{TT}-\beta_{AA})-i(\alpha Z/p)(\beta_{TA}-\text{c.c.})]|M_{GT}|^2,$$
$$D=[(\beta_{ST}-\beta_{VA})M_FM_{GT}^*+\text{c.c.}]\mp i(\alpha Z/p)[(\beta_{SA}-\beta_{VT})M_FM_{GT}^*-\text{c.c.}],$$
$$E=-i[(\beta_{ST}-\beta_{VA})M_FM_{GT}^*-\text{c.c.}]\mp(\alpha Z/p)[(\beta_{SA}-\beta_{VT})M_FM_{GT}^*+\text{c.c.}],$$

where the upper (lower) sign is for electrons (positrons).

The correlation function contains five unknown parameters $A\cdots E$, which can in principle be measured independently by a sequence of experiments. The β intensity determines $A+B$; the gamma anisotropy determines $(\tfrac{1}{2}A-B)$; the electron asymmetry measures $[(1/\sqrt{2})C+D]$; the β-γ coincidence asymmetries determine $(\sqrt{2}C-D)$ and E individually. We are treating the statistical tensors $R_0\cdots R_2$ as known here, which requires a knowledge of the magnetic moments, hyperfine coupling, etc. Presumably by studying the temperature dependence of these effects, these statistical tensors can be determined.

The most interesting term is the last, proportional to E. It gives rise to a distribution of photon momenta asymmetric under reflection in the plane of $\mathbf{J},\mathbf{p}$. Its presence can be detected by setting the orientation axis along the z direction, the electron counter in the x direction, and the photon counter in the y-z plane. The contribution from the last term is maximum for an angle of 45 degrees between $\mathbf{J}$ and $\mathbf{k}$, and changes sign under reflection of the photon counter in the x-z plane. Notice that if a polarizing field is present, proper account of the curvature of electron orbits must be made in determining the $\mathbf{J},\mathbf{p}$ plane. There is, however, no need to use a polarizing field; since the "E" term is proportional to statistical tensors of even rank, it will be present for an aligned source as well as a polarized source.

Considerable information can be inferred from the magnitude and energy dependence of these five con-

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[1] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

[2] Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957); M. E. Ebel and G. Feldman, Nuclear Phys. (to be published); R. B. Curtis and R. R. Lewis, Phys. Rev. **107**, 543 (1957); M. Morita and R. S. Morita, Phys. Rev. **107**, 139 (1957).

[3] U. Fano, Phys. Rev. **90**, 577 (1953); L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

[4] We use the notation $\alpha_{xy}=C_xC_y^*+C_x'C_y'^*$, $\beta_{xy}=C_xC_y'^*+C_x'C_y^*$. Otherwise, the notation is identical with that of T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

stants. Let us discuss the situation assuming $C_A=0$, and $|M_F||M_{GT}|\neq 0$, for simplicity. We can conclude:

(1) if $E\sim 1$, then time-reversal invariance is violated, and the Fermi interaction is S;

(2) if $E\sim\alpha Z/p$, then there is some V present, and either $C_S=0$ or time-reversal invariance is valid;

(3) if E has both 1 and $\alpha Z/p$ terms, then time-reversal invariance is violated and the Fermi interaction is $S+V$;

(4) if E is zero, then the Fermi interaction is S, and time reversal is valid.

Note that D and E are the real and imaginary parts of the same number and therefore cannot both vanish for all electron energies except for the unlikely possibility of exact cancellation of $\beta_{ST}-\beta_{VA}$ and $\beta_{SA}-\beta_{VT}$. Further conclusions about the Fermi interaction can be drawn from a study of the velocity dependence of D. Perhaps it should be pointed out that the violation of time-reversal invariance in this case could be due either to the β interaction or to the nuclear forces, or both. The experiment measures only a combination of coupling constants and nuclear matrix elements, and cannot distinguish between these possibilities.

In a more general case, the correlation function has a more complex angular dependence, and contains further statistical tensors, but still depends on the β interaction through the same five parameters $A\cdots E$. By similar measurements the same information can be obtained. The general result for the correlation function for the allowed transition $J(\beta)J(\gamma)J'$, assuming a pure γ transition of multiple order L, is

$$W(\mathbf{p},\mathbf{k},\mathbf{J})=\sum_\nu\Bigg[R_{2\nu}\left\{A\left[1-\frac{\nu(2\nu+1)}{J(J+1)}\right]+B\right\}P_{2\nu}(\mathbf{J}\cdot\mathbf{k})$$
$$-\left(\frac{v}{c}\right)R_{2\nu-1}\left[\frac{(2J+2\nu+1)(2J-2\nu+1)}{4J(J+1)(4\nu-1)(4\nu+1)}\right]^{\frac{1}{2}}\{2\nu[J(J+1)]^{-\frac{1}{2}}C-D\}\{\mathbf{p}\cdot\mathbf{k}P_{2\nu}'(\mathbf{J}\cdot\mathbf{k})-\mathbf{p}\cdot\mathbf{J}P_{2\nu-1}'(\mathbf{J}\cdot\mathbf{k})\}$$
$$-\left(\frac{v}{c}\right)R_{2\nu+1}\left[\frac{(J+\nu+1)(J-\nu)}{J(J+1)(4\nu+1)(4\nu+3)}\right]^{\frac{1}{2}}\{(2\nu+1)[J(J+1)]^{-\frac{1}{2}}C+D\}\{\mathbf{p}\cdot\mathbf{k}P_{2\nu}'(\mathbf{J}\cdot\mathbf{k})-\mathbf{p}\cdot\mathbf{J}P_{2\nu+1}'(\mathbf{J}\cdot\mathbf{k})\}$$
$$-\left(\frac{v}{c}\right)R_{2\nu}[4J(J+1)]^{-\frac{1}{2}}E\{(\mathbf{J}\cdot\mathbf{p}\times\mathbf{k}P_{2\nu}'(\mathbf{J}\cdot\mathbf{k})\}\Bigg]F_{2\nu}(LJ'J),$$

where the sum on ν is from zero to the lesser of J, L. Here R_ν vanishes for $\nu<0$ and $\nu>2J$. $P_\nu(\mathbf{J}\cdot\mathbf{k})$ is the Legendre polynomial of order ν, and $P_\nu'(\mathbf{J}\cdot\mathbf{k})$ is its first derivative with respect to its argument. $F_\nu(L,J',J)$ is the function tabulated by Biedenharn and Rose[3] and by Ferentz and Rosenzweig.[5]

For allowed transitions with $\Delta J=\pm 1$, the coefficients B, D, E vanish. Since the constants A, C can be measured by observing the electron asymmetry alone, no further information about the β interaction can be obtained from the β-γ correlation function.

[5] M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL 5324, 1954 (unpublished).

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[6] M. Morita (private communication).