Time-Reversal Invariance and Beta-Gamma Angular Correlation*†

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The angular correlation functions between beta and gamma rays from oriented nuclei (with or without observing circular polarization) are given for use in testing the invariance of the beta interactions under time-reversal. If the beta interactions are noninvariant under time-reversal, the angular correlation functions have asymmetries, $W(\Theta,\theta,\varphi)\neq W(\Theta,\theta,-\varphi)$ and $W(\Theta,\theta,\varphi,P)\neq W(\Theta,\theta,-\varphi,P)$, where the axis of the nuclear orientation is chosen as the z-axis, and the beta and gamma rays are assumed to be emitted in the directions with polar angles Θ, $\Phi\equiv$zero, and θ, φ, respectively. P indicates the circular polarization of the gamma rays, and $P=+1$ (-1) for left (right) circular polarization. These asymmetries are of the order of $(p/W)\,\mathrm{Im}(C_T{}^*C_S'+C_T'{}^*C_S-C_A{}^*C_V'-C_A'{}^*C_V)$.

The angular correlation functions between beta and gamma rays from unoriented nuclei (with or without observing circular polarization) are also considered. However, they do not give us a clear-cut experiment to test the invariance of the beta interactions under time-reversal.

Another method to test time-reversal invariance is to measure the difference between the values of $\mathrm{Re}(C_i{}^*C_j)$ and $|C_i|\cdot|C_j|$, etc. This is discussed briefly.

I. INTRODUCTION

SINCE the first announcements of nonconservation of parity, P, and violation of invariance under charge conjugation, C, in weak interactions,[1–3] further experiments[4–7] based on different methods have confirmed these findings. However, the validity of invariance of weak interactions under time-reversal, T, has not yet been established.

As is well known, invariance with respect to time-reversal imposes the restriction that the ten coupling constants C_i and C_i' $(i=S,V,T,A,P)$ must be real (apart from a trivial common phase factor which can be normalized to unity). Therefore, we can test the invariance under time-reversal: on the one hand, by measuring the values of the imaginary parts of the products of the coupling constants, $C_i{}^{(\prime)*}C_j{}^{(\prime)}$ $(i\neq j)$ ($^{(\prime)}$ means with or without prime), or $C_i{}^*C_j'$ ($i=j$ and $i\neq j$); or on the other hand, by measuring the difference between values of $\mathrm{Re}(C_i{}^{(\prime)*}C_j{}^{(\prime)})$ and $|C_i{}^{(\prime)}|\cdot|C_j{}^{(\prime)}|$ $(i\neq j)$, or of $\mathrm{Re}(C_i{}^*C_j')$ and $|C_i|\cdot|C_j'|$ ($i=j$ and $i\neq j$). For the latter case, $\mathrm{Re}(C_i{}^{(\prime)*}C_j{}^{(\prime)})=|C_i{}^{(\prime)}|\cdot|C_j{}^{(\prime)}|$, etc., if the beta interactions are invariant under time-reversal and $\mathrm{Re}(C_i{}^{(\prime)*}C_j{}^{(\prime)})<|C_i{}^{(\prime)}|\cdot|C_j{}^{(\prime)}|$, etc., if the beta interactions are noninvariant under time-reversal. Two experiments for determining such quantities are the beta-ray angular distributions from oriented nuclei and electron (positron)-recoil experiments in $J\rightarrow J$ transitions considered together with beta-ray spectra.[8]

By exhaustive studies of angular distributions, angular correlations, and polarizations of emitted beta particles in beta decay[9–12] and in muon decay,[13,14]

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956); **105**, 1671 (1957). Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).

[2] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957); and Ambler, Hayward, Hoppes, Hudson, and Wu, Phys. Rev. **106**, 1361 (1957).

[3] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).

[4] Postma, Huiskamp, Miedema, Steenland, Tolhoek, and Gorter, Physica **23**, 259 (1957).

[5] Frauenfelder, Bobone, Goeler, Levine, Lewis, Peacock, Rossi, and De Pasquali, Phys. Rev. **106**, 386 (1957).

[6] L. A. Page and M. Heinberg, Phys. Rev. **106**, 1220 (1957).

[7] Goldhaber, Grodzins, and Sunyar, Phys. Rev. **106**, 826 (1957).

[8] According to the great successes of the explanation for the data on the beta decays of Co^{60},[2] Na^{22},[6] and Y^{90},[7] and of $\pi-\mu-e$ decay[3] by the two-component neutrino theory [T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957); L. Landau, Nuclear Phys. **3**, 127 (1957); A. Salam, Nuovo cimento **5**, 299 (1957)], it seems to be natural to put $C_i=-C_i'$. On the other hand, there is no clear-cut explanation of the data for the beta-ray angular distribution from oriented nuclei of Co^{58}.[2,4] Some of the possible explanations for the Co^{58} data are:

(1) Assuming invariance under time reversal: (a) $C_V{}^{(\prime)}=C_A{}^{(\prime)}=0$ and $|M_{\mathrm{GT}}|^2/|M_{\mathrm{F}}|^2\gtrsim 500$, which contradicts the result, $|M_{\mathrm{GT}}|^2/|M_{\mathrm{F}}|^2\lesssim 50$, obtained by gamma-ray anisotropy from aligned Co^{58} nuclei [D. Griffing and J. C. Wheatly, Phys. Rev. **104**, 389 (1956)]. (b) $C_S{}^{(\prime)}=C_A{}^{(\prime)}=0$ and the Fermi-type interaction is not scalar but vector. This argument cannot agree with the data of the electron (positron) recoil experiments on the neutron [J. M. Robson, Phys. Rev. **100**, 933 (1955)] and on Ne^{19} [J. S. Allen *et al.*, Phys. Rev. **97**, 109 (1955); M. Good *et al.*, Phys. Rev. **105**, 213 (1957); D. R. Hamilton *et al.*, Phys. Rev. **105**, 673 (1957)], and the beta spectrum of RaE [M. Yamada, Progr. Theoret. Phys. (Japan) **9**, 268 (1953)].

(2) Possible violation of time-reversal invariance: For this case, we can explain very many ways. For example, $C_V{}^{(\prime)}=C_A{}^{(\prime)}=0$ and the phase difference of $C_S{}^{(\prime)}$ and $C_T{}^{(\prime)}$ is nearly equal to $\pi/2$ or $3\pi/2$; or $C_A{}^{(\prime)}=0$ and the scalar, vector, and tensor interactions are out of phase. (The meaning of "out of phase" is that the phase difference is not equal to 0 or π.) The data of the electron (positron) recoil experiments and ft values on n^1 and Ne^{19} allow us to assume that $0\lesssim(|C_V|^2+|C_V'|^2)/(|C_S|^2+|C_S'|^2)\lesssim 1$, even in the case of $C_A{}^{(\prime)}=0$. A more detailed discussion has been given by the present authors [Soryusiron Kenkyu **14**, 489 (1957)]. The situation is almost the same for Co^{56}.

[9] M. Morita and R. S. Morita (unpublished). See Eq. (1) in Ambler, Hayward, Hoppes, Hudson, and Wu, reference 2. It has also been given by many other authors.

[10] Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957);

several tests of invariance under time-reversal have been already proposed, all of which involve the direct observation of the imaginary part of the product, $C_i^*C_j$, etc. Some of them are based on the p or Z dependence of the asymmetry effects in the relevant phenomena. However, as the p or Z dependence is usually of the order of $\alpha Z/p$ (where α is the fine structure constant) times smaller than the main asymmetry term, it is rather difficult to perform these experiments with sufficient accuracy.

Some other experiments are based on cross terms such as $(p/W)\,\mathrm{Im}(C_T^*C_A^{(\prime)})$ which also may be very small or zero, because $|C_V^{(\prime)}|$ and $|C_A^{(\prime)}|$ may be much smaller than $|C_S^{(\prime)}|$ and $|C_T^{(\prime)}|$.[15] Furthermore, when the angular distribution (or correlation) has $(p/W)\,\mathrm{Im}(C_T^*C_A^{(\prime)})$, etc., as a main term of asymmetry, it has also $(\alpha Z/W)\,\mathrm{Re}(C_T^*C_T^{(\prime)})$, etc., as a Coulomb correction.[16] Therefore, the lack of (or very small) asymmetry in these experiments does not give us any definite information on the invariance of beta interactions under time-reversal. Conversely, if some phenomenon has a term like $(p/W)\,\mathrm{Im}(C_T^*C_S^{(\prime)})$ as an asymmetry term, it has $(\alpha Z/W)\,\mathrm{Re}(C_A^*C_S^{(\prime)})$ as a Coulomb correction, which may be very small compared with the main term. The asymmetry in this phenomenon may be large. Consequently, this phenomenon may reveal the invariance or noninvariance of beta interactions under time-reversal.

One such experiment was proposed by Jackson *et al.*[10] This is the electron (positron) recoil experiment in polarized nuclei, which has $(p/W)\,\mathrm{Im}(C_T^*C_S+C_T'^*C_S'-C_A^*C_V-C_A'^*C_V')$ in its $\mathbf{J}\cdot(\mathbf{p}_e\times\mathbf{p}_\nu)$ term. The most suitable beta emitter for this investigation would be the neutron. However, the counting rate is expected to be rather limited for the polarized neutron flux currently available from reactors.

Here we shall discuss another possible method to test time-reversal invariance. This is to measure the angular correlation between the beta and gamma rays in the successive decays of oriented nuclei. The angular correlation function has an asymmetry which is of the form $\mathbf{J}\cdot(\mathbf{p}\times\mathbf{k})(\mathbf{J}\cdot\mathbf{k})^n$ with $n=1$ and 3. Here $\mathbf{J}$ is the orientation axis of the nucleus; $\mathbf{p}$ and $\mathbf{k}$ are the directions of the momenta of the emitted electron and the gamma rays. If one assumes the Coulomb correction to be very small, this asymmetry can be expected only if the beta interactions are not invariant under time-reversal, and it has the order of magnitude $(p/W)\,\mathrm{Im}(C_T^*C_S'+C_T'^*C_S-C_A^*C_V'-C_A'^*C_V)$ (see Sec. II). Both aligned and polarized nuclei may be used for this experiment.

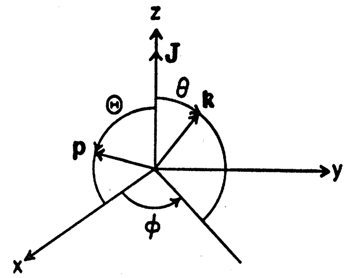

FIG. 1. Geometry for beta-gamma angular correlation from oriented nuclei. The orientation axis $\mathbf{J}$ of the nuclei is chosen as the z-axis. The beta and gamma rays are assumed to be emitted in the directions $\mathbf{p}$ and $\mathbf{k}$, with polar axes Θ, $\Phi\equiv$zero, and θ, φ, respectively.

When $\mathbf{J}$ and $\mathbf{k}$ are perpendicular, this asymmetry vanishes. In this case, however, if time-reversal invariance is violated in the beta decay, the beta-gamma angular correlation function can still exhibit a similar asymmetry, which is related to $P\mathbf{J}\cdot(\mathbf{p}\times\mathbf{k})$, by observing the circular polarization P of the gamma ray and using polarized nuclei. This asymmetry changes its sign for opposite signs of the circular polarization of the gamma ray, $P=\pm1$, and its order of magnitude is also $(p/W)\,\mathrm{Im}(C_T^*C_S'+C_T'^*C_S-C_A^*C_V'-C_A'^*C_V)$ (see Sec. III). In Sec. IV, various angular correlations in the successive decays of oriented or unoriented nuclei are related to each other. Some final remarks are given in Sec. V.

In Appendix I, the angular correlation functions between beta and gamma rays from oriented nuclei (with or without circular polarization) are given. For comparison, the angular correlation functions between beta and gamma rays from unoriented nuclei (with or without circular polarization) are also given in Appendix II.

II. BETA-GAMMA ANGULAR CORRELATION FROM ORIENTED NUCLEI[17]

We shall use the following definitions of orientation hereafter.

(1) a_m = relative population of the initial magnetic substate.

(2) Unoriented nuclei: $a_m=a$ (constant), for all m.

(3) Oriented nuclei:

Aligned nuclei: $a_m=a_{-m}$, $a_m\neq a_{m'}$ for $|m|\neq|m'|$.
Polarized nuclei: $a_m\neq a_{m'}$ for $m\neq m'$.

The orientation axis of the nuclei is chosen as the z-axis. The beta and gamma rays are assumed to be emitted in the directions with polar axes Θ, $\Phi\equiv0$, and θ, φ, respectively (see Fig. 1).

The angular correlation function between beta and gamma rays in the successive decay of the oriented

107, 326 (1957). B. T. Feld, Phys. Rev. (to be published); T. Kotani (unpublished).

[11] M. Morita, Progr. Theoret. Phys. (Japan) **10**, 364 (1953); M. Morita and R. S. Morita, Phys. Rev. **107**, 139 (1957).

[12] Alder, Stech, and Winther, Phys. Rev. **107**, 728 (1957).

[13] T. Kotani (unpublished).

[14] T. Kinoshita and A. Sirlin, Phys. Rev. **106**, 1110 (1957); **107**, 523 (1957).

[15] The beta-ray angular distribution from polarized Co^{60} nuclei,[2] and the measurements of electron polarization from unpolarized Co^{60},[5] Na^{22},[6] and Y^{90},[7] showed that $0\lesssim|C_A|^2/|C_T|^2\lesssim0.1$, even if the coupling constants are complex numbers. There has been no such indication for Fermi type prior to this.

[16] See various formulas given by several authors.

[17] After this work was completed, the authors have heard that the same problem has also been calculated by R. B. Curtis and R. R. Lewis [Phys. Rev. (to be published)].

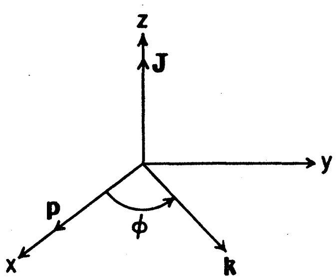

FIG. 2. A special geometry for beta-circularly polarized gamma angular correlation from polarized nuclei. The polarization axis **J** of the nuclei is chosen as the z axis. The beta and gamma rays are assumed to be emitted in the x-y plane. The angle between the directions, **p** and **k**, of the momenta of the beta and gamma rays is φ.

nuclei, $W(\Theta,\theta,\varphi)$, for a $2-\xrightarrow{\beta}2-\xrightarrow{\gamma}0$ transition is derived from (A1) of Appendix I by averaging over the circular polarization of the gamma ray.

$$W(\Theta,\theta,\varphi)=\tfrac{1}{2}\{W(\Theta,\theta,\varphi,P=+1)+W(\Theta,\theta,\varphi,P=-1)\}. \quad (1)$$

$W(\Theta,\theta,\varphi)$ is the P-independent part of (A1), so we do not rewrite it here.

In (A1), the first half, which contains square terms $|C_i|^2$ or $|C_i'|^2$ and cross terms $C_i^*C_j$ or $C_i'^*C_j'$ $(i\neq j)$, is the contribution from the correction factor of the beta spectrum, and it is equal to the gamma-ray angular distribution (beta particle unobserved) in the successive beta and gamma decays of oriented nuclei. The last half, which contains cross terms $C_i^*C_j'$ ($i=j$ and $i\neq j$), appears in the case of nonconservation of parity, which has already been demonstrated.[2–7] The terms which have different sign for the electron and positron decays appear in the case of violation of invariance under charge conjugation. The imaginary parts of the products of coupling constants appear in the case of noninvariance under time-reversal. As the Coulomb correction terms may be of the order of $\alpha Z/p$ $(\sim\frac{1}{10})$ smaller than the main terms, the interesting asymmetry which appears in the case of noninvariance under time-reversal is

$$-2\,\mathrm{Im}(C_T^*C_S'+C_T'^*C_S-C_A^*C_V'-C_A'^*C_V)\times M_{\mathrm{GT}}^*M_{\mathrm{F}}(p/W\sqrt{6})\sin\Theta\sin\varphi\sin\theta\times(3B_1\cos\theta+2B_2\cos^3\theta), \quad (2)$$

which is related to the terms $\mathbf{J}\cdot(\mathbf{p}\times\mathbf{k})(\mathbf{J}\cdot\mathbf{k})^n$ with $n=1$ and 3, and B_1 and B_2 are given in Appendix I. As B_1 and B_2 have nonzero values in both polarized and aligned nuclei, these two kinds of oriented nuclei may be used. $M_{\mathrm{GT}}^*M_{\mathrm{F}}$ is real and it may be plus or minus.

In order to test the invariance of beta interactions under time-reversal, it is sufficient to know the difference between $W(\Theta,\theta,\varphi)$ and $W(\Theta,\theta,-\varphi)$. Such a difference is possible only in the case of violation of invariance under time-reversal, if one assumes the Coulomb correction for Eq. (2), $\mathrm{Re}(C_A^*C_S'+\cdots)\times(\alpha Z/p)$, to be very small.

To maximize the asymmetry given in Eq. (2), we

TABLE I. Calculated anisotropy and degree of orientation for Co^{58} (CeMgCo nitrate, magnetic field strength for nuclear polarization=800 gauss) with the assumptions $C_i=-C_i'$, $|M_{\mathrm{GT}}|^2=8|M_{\mathrm{F}}|^2$, and $C_V\approx C_A\approx 0$, and the definition of (3′).[a]

Absolute temperature T°K	0.1	0.05	0.03	0.015	0.01	0.005	0.001
Degree of orientation $\langle J_z\rangle/J=\Sigma ma_m/J\Sigma a_m$	0.127	0.262	0.400	0.615	0.737	0.899	1.000
Anisotropy defined[b] in (3′), $-\alpha'$	0.004	0.015	0.037	0.097	0.145	0.226	0.333

[a] Calculated by Miss Hilda Oberthal. The authors are very grateful to her.
[b] *Note added in proof.*—In experiments, it is much easier to normalize the anisotropy in (3′) by $W(\pi/2,\pi/6,\pi/2)$ instead of $W(\pi/2,\pi/2,\pi/2)$. For this case, the third line of Table I should be read as 0.004, 0.015, 0.038, 0.115, 0.183, 0.325, 0.550.

put $\Theta=\varphi=\pi/2$. In this case, most of irrelevant asymmetries in $W(\Theta,\theta,\varphi)$ vanish.

Let us define an anisotropy as follows:

$$\text{Anisotropy at }\theta=\frac{W(\pi/2,\theta,\pi/2)-W(\pi/2,\theta,-\pi/2)}{W(\pi/2,\pi/2,\pi/2)}, \quad (3)$$

where

$$\text{numerator}=-(4/\sqrt{6})(p/W)\times\mathrm{Im}(C_T^*C_S'+C_T'^*C_S-C_A^*C_V'-C_A'^*C_V)\times M_{\mathrm{GT}}^*M_{\mathrm{F}}\sin\Theta\sin\varphi\sin\theta\times(3B_1\cos\theta+2B_2\cos^3\theta),$$

$$\text{denominator}=\{|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2\}|M_{\mathrm{F}}|^2A_1+\{|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2\}|M_{\mathrm{GT}}|^2(A_2/2).$$

This anisotropy has a nonzero value if beta interactions are not invariant under time-reversal. Conversely, if the beta interactions are invariant under time-reversal, it vanishes.

The value of θ, which makes (3) a maximum, cannot be determined without specifying the nuclear orientation. In the case of complete orientation, $a_m=0$ except a_J (and $a_{-J})\neq 0$, (3) becomes

$$\text{Anisotropy at }\theta=(8/\sqrt{6})(p/W)\times\mathrm{Im}(C_T^*C_S'+C_T'^*C_S-C_A^*C_V'-C_A'^*C_V)\times M_{\mathrm{GT}}^*M_{\mathrm{F}}\sin\theta\cos^3\theta\div\{(|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2)|M_{\mathrm{F}}|^2+(|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2)|M_{\mathrm{GT}}|^2\}, \quad (4)$$

where the Fierz terms are set equal to zero in conformity with experimental allowed beta spectra. The anisotropy (4) has its maximum value at $\theta=\pi/6$, namely,

$$\text{Anisotropy at }\pi/6=(3/2\sqrt{2})(p/W)\,\mathrm{Im}(C_T^*C_S'+C_T'^*C_S-C_A^*C_V'-C_A'^*C_V)M_{\mathrm{GT}}^*M_{\mathrm{F}}\div[\text{denominator of (4)}]. \quad (5)$$

The numerical values of the anisotropy are given, for example, for Co^{58} (cerium magnesium cobalt nitrate which was used in the experiments[2] of Wu *et al.*) in Table I, with the assumptions $C_i=-C_i'$, $|M_{GT}|^2=8|M_F|^2$, and $C_V\approx C_A\approx 0$, and the definition

Anisotropy at $\pi/6$

$$=\frac{W(\pi/2,\pi/6,\pi/2)-W(\pi/2,\pi/6,-\pi/2)}{W(\pi/2,\pi/2,\pi/2)}$$

$$=\alpha'(p/W)\,\mathrm{Im}(C_T{}^*C_S'). \quad (3')$$

III. BETA-CIRCULARLY POLARIZED GAMMA ANGULAR CORRELATION FROM ORIENTED NUCLEI

The asymmetry, (2), in $W(\Theta,\theta,\varphi)$ vanishes at the following special values of angular variables: $\Theta=0,\pi$; $\theta=0,\pi/2,\pi;3\pi/2$; or $\varphi=0,\pi$. As we can see in Eq. (A1) of Appendix I, where the beta-circularly polarized gamma angular correlation from oriented nuclei is given for $2-^{\beta}\!\!\rightarrow 2-^{\gamma}\!\!\rightarrow 0$ transitions in the most general geometry, we have no good term to test invariance of beta interactions under time-reversal at $\Theta=0$, π, $\theta=0$, π, or $\varphi=0$, π. At $\theta=\pi/2$ (or $3\pi/2$), however, the beta-gamma angular correlation can still have an asymmetry which appears in the case of violation of invariance under time-reversal. This can be seen by observing the circular polarization of the gamma ray and using polarized nuclei.

For example, we obtain from (A1) of Appendix I an angular correlation function between beta and circularly polarized gamma rays for a special geometry (see Fig. 2):

$$\begin{aligned}W(\pi/2,\pi/2,\varphi,P)=&[|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2\pm 2\,\mathrm{Re}(C_S{}^*C_V+C_S'{}^*C_V')(\gamma/W)]|M_F|^2A_1\\&+[|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2\pm 2\,\mathrm{Re}(C_T{}^*C_A+C_T'{}^*C_A')(\gamma/W)]|M_{GT}|^2(A_2/2)\\&-[2\,\mathrm{Re}(C_T{}^*C_S'+C_T'{}^*C_S-C_A{}^*C_V'-C_A'{}^*C_V)\pm 2\,\mathrm{Im}(C_A{}^*C_S'+C_A'{}^*C_S-C_T{}^*C_V'-C_T'{}^*C_V)(\alpha Z/p)]\\&\times M_{GT}{}^*M_F(p/W\sqrt{6})PA_1\cos\varphi+[-2\,\mathrm{Im}(C_T{}^*C_S'+C_T'{}^*C_S-C_A{}^*C_V'-C_A'{}^*C_V)\\&\pm 2\,\mathrm{Re}(C_A{}^*C_S'+C_A'{}^*C_S-C_T{}^*C_V'-C_T'{}^*C_V)(\alpha Z/p)]M_{GT}{}^*M_F(p/W\sqrt{6})PC_1\sin\varphi\\&\mp[2\,\mathrm{Re}(C_T{}^*C_T'-C_A{}^*C_A')\mp 2\,\mathrm{Im}(C_T{}^*C_A'+C_T'{}^*C_A)(\alpha Z/p)]|M_{GT}|^2(p/3W)PA_3\cos\varphi, \quad (6)\end{aligned}$$

where

$P=+1$ for left-circularly polarized gamma rays
$\;\;=-1$ for right-circularly polarized gamma rays,

and $A_{1,2,3}$ and C_1 are given in Appendix I. The upper (lower) sign refers to the electron (positron). This angular correlation function is also given for the allowed beta ray and quadrupole radiation for $2-^{\beta}\!\!\rightarrow 2-^{\gamma}\!\!\rightarrow 0$ transitions.

This angular correlation function will have the following two asymmetries:

(1) $W(\pi/2,\pi/2,\varphi,P)\neq W(\pi/2,\pi/2,-\varphi,P)$,

(2) $W(\pi/2,\pi/2,\pi/2,P)\neq W(\pi/2,\pi/2,\pi/2,-P)$,

with the assumption of noninvariance of beta interactions under time-reversal.

The anisotropy is defined by

$$\text{Anisotropy}=\frac{W(\pi/2,\pi/2,\varphi,P)-W(\pi/2,\pi/2,-\varphi,P)}{W(\pi/2,\pi/2,\pi/2)}$$

$$\begin{aligned}&=-(4/\sqrt{6})(p/W)\,\mathrm{Im}(C_T{}^*C_S'+C_T'{}^*C_S\\&-C_A{}^*C_V'-C_A'{}^*C_V)M_{GT}{}^*M_FPC_1\sin\varphi\\&\div[\text{denominator of (3)}], \quad (7)\end{aligned}$$

where we also neglect the Coulomb correction. This anisotropy takes its maximum value at $\varphi=\pm\pi/2$.

In the case of complete polarization:

$$\begin{aligned}\text{Anisotropy}=&(4/\sqrt{6})(p/W)\\&\times\mathrm{Im}(C_T{}^*C_S'+C_T'{}^*C_S-C_A{}^*C_V'-C_A'{}^*C_V)\\&\times M_{GT}{}^*M_FP\sin\varphi\div[\text{denominator of (4)}]. \quad (8)\end{aligned}$$

The value of (8) at $\varphi=\pi/2$ is $8/3\sqrt{3}(\approx 1.5)$ times larger than that of (5).

IV. RELATION TO OTHER ANGULAR CORRELATIONS

When one integrates over the direction of emission of the gamma ray, $W(\Theta,\theta,\varphi)$ becomes the angular distribution function of beta rays from polarized nuclei which is given by many authors.[9,10,12]

$$\int W(\Theta,\theta,\varphi)\sin\theta d\theta d\varphi=W(\Theta)\text{ in }2-^{\beta}\!\!\rightarrow 2. \quad (9)$$

Integrating over the direction of emission of the beta ray and the azimuthal angle φ of the gamma ray and also over the electron energy, $W(\Theta,\theta,\varphi)$ becomes the gamma-ray angular distribution function from oriented nuclei.

$$\int d\varphi\int\sin\Theta d\Theta d\Phi\int L_0F(Z,W)K^2pWdWW(\Theta,\theta,\varphi)$$
$$=W(\theta)\text{ in }2-^{\beta}\!\!\rightarrow 2-^{\gamma}\!\!\rightarrow 0, \quad (10)$$

which also agrees with the gamma-ray angular distribution function given by Tolhoek *et al.*[18]

If one puts $a_m = a$ in (6), then $A_1 = (A_2/2) = -2A_3 = 4a$, and $C_1 = 0$; consequently (6) becomes the beta-circularly polarized gamma angular correlation in unoriented nuclei[19]:

$$[(6) \text{ with } a_m = a] = W(\varphi, P), \tag{11}$$

where φ is an angle between $\mathbf{p}$ and $\mathbf{k}$.

This is also obtained from (A1) by putting $a_m = a$ and $\Theta = 0$.

$$[W(\Theta,\theta,\varphi,P) \text{ with } a_m = a \text{ and } \Theta = 0] = W(\theta, P), \tag{12}$$

where θ is an angle between $\mathbf{p}$ and $\mathbf{k}$.

The general formula for the beta-circularly polarized gamma angular correlation is given in Appendix II. It has also been given in reference 12, and (12) can be derived from Eq. (7) of this reference, where the beta interaction is assumed to be *STP*.

As we can see immediately from (A3), the beta-circularly polarized gamma angular correlation in unoriented nuclei does not offer a clear-cut experiment to test invariance of beta interactions under time-reversal, just as in the case of the angular distribution of beta rays from polarized nuclei. The terms which appear due to the violation of invariance under time-reversal are of the order of $(\alpha Z/p)$ smaller than main asymmetry terms even in the forbidden beta transitions.

V. CONCLUDING REMARKS

In the successive decays of oriented nuclei, the angular correlation function between beta and gamma rays (with or without observing circular polarization) has strong asymmetries which appear in the case of non-invariance of beta interactions under time-reversal, namely, $W(\Theta,\theta,\varphi) \neq W(\Theta,\theta,-\varphi)$ and $W(\Theta,\theta,\varphi,P) \neq W(\Theta,\theta,-\varphi,P)$.

When the circular polarization of the gamma rays is not observed, the most convenient geometry is that in which the beta particle is emitted perpendicular to the plane which is formed by $\mathbf{J}$ and $\mathbf{k}$. In this case both aligned and polarized nuclei may be used.

When the circular polarization of the gamma rays is observed, the most convenient geometry is that with $\mathbf{J}$, $\mathbf{p}$, and $\mathbf{k}$ perpendicular to each other. In this geometry, only polarized nuclei should be used. The maximum value of the anisotropy defined by (8) is almost 1.5 times larger than that described in the preceding paragraph and defined by (5).

All of the explicit formulas given in the present paper (except Appendix II) are for the special transition scheme, $2 \xrightarrow{\beta} 2 \xrightarrow{\gamma} 0$ with allowed beta rays and quadrupole gamma rays. The above-described situations are, however, still valid [except for some algebraic factors and the energy dependence of the beta rays for forbidden transitions in $W(\Theta,\theta,\varphi)$ and $W(\Theta,\theta,\varphi,P)$] for other transition schemes including forbidden beta transitions, for which angular correlation functions can be easily obtained from equations given in an earlier paper[20] with slight modifications. It is advisable to choose nuclei which are easily oriented and have $\Delta J = 0$ for allowed beta decay, or $\Delta J = \pm n$ for nth-forbidden beta decay, and M_{GT} and M_{F} (or the corresponding matrix elements for the forbidden cases) of the same order of magnitude.

Further investigation of this subject is now being made.

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APPENDIX I. ANGULAR CORRELATION FUNCTION BETWEEN BETA AND GAMMA RAYS FROM ORIENTED NUCLEI (WITH OR WITHOUT POLARIZATION)

The angular correlation function between beta and gamma rays from oriented nuclei (with or without circular polarization) is easily obtained by slight modifications of the calculation in reference 20.

The angular correlation of the allowed beta ray and the quadrupole gamma ray is given below for the $2 \xrightarrow{\beta} 2 \xrightarrow{\gamma} 0$ transition. For the geometry, see Fig. 1 in Sec. II.

$$\begin{aligned}
W(\Theta,\theta,\varphi,P) = {} & [|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2 \pm 2\,\mathrm{Re}(C_S^*C_V+C_S'^*C_V')(\gamma/W)]|M_{\mathrm{F}}|^2 \\
& \times\{(A_1+3B_1\cos^2\theta+B_2\cos^4\theta)+2P(C_1\cos\theta+C_2\cos^3\theta)\}+[|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2 \\
& \pm 2\,\mathrm{Re}(C_T^*C_A+C_T'^*C_A')(\gamma/W)]|M_{\mathrm{GT}}|^2\tfrac{1}{6}\{(3A_2+3B_3\cos^2\theta-4B_2\cos^4\theta)+2PC_3\cos\theta\} \\
& +[2\,\mathrm{Re}(C_T^*C_S'+C_T'^*C_S-C_A^*C_V'-C_A'^*C_V)\pm 2\,\mathrm{Im}(C_A^*C_S'+C_A'^*C_S-C_T^*C_V'-C_T'^*C_V)(\alpha Z/p)] \\
& \times M_{\mathrm{GT}}^*M_{\mathrm{F}}(p/W\sqrt{6})[\cos\Theta\{(-C_3+3C_4\cos^2\theta-2C_2\cos^4\theta)+2P(A_3\cos\theta+2B_4\cos^3\theta) \\
& +\sin\Theta\cos\varphi\sin\theta\{(3C_4\cos\theta-2C_2\cos^3\theta)+P(-A_1+3B_5\cos^2\theta)\}]+[-2\,\mathrm{Im}(C_T^*C_S'+C_T'^*C_S-C_A^*C_V'-C_A'^*C_V) \\
& \pm 2\,\mathrm{Re}(C_A^*C_S'+C_A'^*C_S-C_T^*C_V'-C_T'^*C_V)(\alpha Z/p)]M_{\mathrm{GT}}^*M_{\mathrm{F}}(p/W\sqrt{6})\sin\Theta\sin\varphi\sin\theta \\
& \times\{(3B_1\cos\theta+2B_2\cos^3\theta)+P(C_1+3C_2\cos^2\theta)\}\pm[2\,\mathrm{Re}(C_T^*C_T'-C_A^*C_A') \\
& \mp 2\,\mathrm{Im}(C_T^*C_A'+C_T'^*C_A)(\alpha Z/p)]|M_{\mathrm{GT}}|^2(p/3W)[\cos\Theta\{(-C_1-3C_5\cos^2\theta+4C_2\cos^4\theta)+2P(A_4\cos\theta+B_6\cos^3\theta)\} \\
& +\sin\Theta\cos\varphi\sin\theta\{(-3C_4\cos\theta+4C_2\cos^3\theta)-P(A_3+6B_4\cos^2\theta)\}],
\end{aligned} \tag{A1}$$

[18] H. A. Tolhoek; see, for example, *Beta- and Gamma Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 613. Morita, Ogata, and Sakai, Physica **22**, 915 (1956); and also Bull. Kobayasi Inst. Phys. Research **6**, 69 (1956).

[19] See Appendix II or reference 12.

[20] M. Morita, Progr. Theoret. Phys. (Japan) **15**, 445 (1956).

with the upper (lower) sign for the electron (positron), $P=+1$ for left-circularly polarized gamma rays, and $P=-1$ for right-circularly polarized gamma rays. $M_{GT}{}^*M_F$ is real and it may be plus or minus.

Without observing the circular polarization of the gamma ray, all of the terms which are multiplied by P vanish.

A_μ, B_μ, and C_μ indicate the dependence of each term on the nuclear orientation. They are introduced only for abbreviation and are not completely independent, but five of them are independent. Their explicit forms are given as follows:

$$\begin{aligned}
&A_1=a_2+a_1+a_{-1}+a_{-2}, && B_1=-a_1+2a_0-a_{-1}, && C_1=-a_2+a_1-a_{-1}+a_{-2},\\
&A_2=2a_2+a_1+2a_0+a_{-1}+2a_{-2}, && B_2=-a_2+4a_1-6a_0+4a_{-1}-a_{-2}, && C_2=a_2-2a_1+2a_{-1}-a_{-2},\\
&A_3=-2a_2+a_1+a_{-1}-2a_{-2}, && B_3=-2a_2+5a_1-6a_0+5a_{-1}-2a_{-2}, && C_3=-2a_2-a_1+a_{-1}+2a_{-2},\\
&A_4=a_2+a_1-3a_0+a_{-1}+a_{-2}, && B_4=a_2-a_1-a_{-1}+a_{-2}, && C_4=-a_1+a_{-1},\\
& && B_5=a_2-2a_0+a_{-2}, && C_5=a_2-3a_1+3a_{-1}-a_{-2},\\
& && B_6=-2a_2-a_1+6a_0-a_{-1}-2a_{-2}, &&
\end{aligned} \tag{A2}$$

with a_m = relative population of initial magnetic substate.

$A_\mu \neq 0$ for both oriented and unoriented nuclei.

$B_\mu \neq 0$ for oriented nuclei,
$=0$ for unoriented nuclei.

$C_\mu \neq 0$ for polarized nuclei,
$=0$ for both aligned and unoriented nuclei.

APPENDIX II. ANGULAR CORRELATION FUNCTION BETWEEN BETA AND GAMMA RAYS FROM UNORIENTED NUCLEI (WITH OR WITHOUT CIRCULAR POLARIZATION)

The angular correlation function between beta rays and circularly polarized gamma rays from unoriented nuclei can be derived from reference 20 with slight modifications. In $M(4)$ and (30) through (32) of reference 20, $2n$ should be replaced by n. Equation (31) and the second line of (32) should be multiplied by $p_1{}^{\delta_1+\delta_1'+L_1+L_1'+n}$. Then the beta–circularly polarized gamma angular correlation function is obtained as follows:

$$W(\theta,P)=\sum_n\Big[\Big\{\sum_{L\leq L'}(-)^{j_1-j}b_{LL'}{}^{(n)}W(j_1j_1LL';nj)(2j_1+1)^{\frac{1}{2}}\Big\}\\
\times\Big\{\sum_{L_1L_1'}(-)^{L_1+L_1'}P^{\delta_1+\delta_1'+L_1+L_1'+n}(j_1\|L_1\|j_2)(j_1\|L_1'\|j_2)F_n(L_1L_1'j_2j_1)\Big\}\Big]P_n(\cos\theta), \tag{A3}$$

for $j-{}^{\beta}\!\!\rightarrow j_1-{}^{\gamma}\!\!\rightarrow j_2$ transitions. P is the circular polarization as given in Appendix I. δ is equal to $0(+1)$ for magnetic (electric) radiation.

The $b_{LL'}{}^{(n)}$'s are given for the allowed beta transition as follows:

$$\begin{aligned}
b_{00}{}^{(0)}&=[|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2\pm2\,\mathrm{Re}(C_S{}^*C_V+C_S'{}^*C_V')(\gamma/W)]|M_F|^2,\\
b_{11}{}^{(0)}&=-\sqrt{3}[|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2\pm2\,\mathrm{Re}(C_T{}^*C_A+C_T'{}^*C_A')(\gamma/W)]|M_{GT}|^2,\\
b_{01}{}^{(1)}&=[2\,\mathrm{Re}(C_T{}^*C_S'+C_T'{}^*C_S-C_A{}^*C_V'-C_A'{}^*C_V)\\
&\qquad\pm2\,\mathrm{Im}(C_A{}^*C_S'+C_A'{}^*C_S-C_T{}^*C_V'-C_T'{}^*C_V)(\alpha Z/p)]M_{GT}{}^*M_F(p/W),\\
b_{11}{}^{(1)}&=\mp\sqrt{2}[2\,\mathrm{Re}(C_T{}^*C_T'-C_A{}^*C_A')\mp2\,\mathrm{Im}(C_T{}^*C_A'+C_T'{}^*C_A)(\alpha Z/p)]|M_{GT}|^2(p/W).
\end{aligned} \tag{A4}$$

Here, the upper (lower) sigh refers to the electron (positron) decay. $M_{GT}{}^*M_F$ is real.

When the circular polarization of the gamma ray is not observed, the beta-gamma angular correlation function is

$$W(\theta)=\tfrac{1}{2}\{W(\theta,P=1)+W(\theta,P=-1)\}, \tag{A5}$$

for which the Legendre polynomials of odd order vanish.

The angular correlation functions between beta rays and circularly polarized gamma rays in the successive triple cascade transitions of unoriented nuclei will be given later (Phys. Rev., to be published).