Photodisintegration of the Lightest Nuclei*

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It is shown that for nuclei in which the ground state wave function is symmetric in the space coordinates of all the nucleons, such as H^2, H^3, He^3, He^4, the integrated bremsstrahlung-weighted cross section for electric dipole absorption is simply related to the mean-square radius of the nucleus, independent of the existence of correlations between the motions of the nucleons. This relationship was first derived by Levinger and Bethe on the assumption of absence of correlation, and was found to be at variance with known experimental results in heavy nuclei. Experimentally, the relationship is well verified by data on H^2 and He^4. The relationship of the integrated bremsstrahlung-weighted cross section to certain nuclear parameters in the case of Li^6 is briefly discussed.

LET the electric dipole absorption cross section, neglecting retardation, of a nucleus for photons of energy E, averaged over all orientations of the nucleus, be denoted by $\sigma(E)$. The integrated bremsstrahlung-weighted cross section σ_b is then defined by

$$\sigma_b=\int_0^\infty \frac{\sigma(E)}{E}dE. \tag{1}$$

In a now classic paper, Levinger and Bethe[1] showed that under certain assumptions, σ_b is related to the mean-square-radius $\mathcal{R}^2$ of the nucleus in its ground state. The assumption involved was essentially a lack of correlation between the motion of individual nucleons in the nucleus (statistical model). A comparison of the formula which they obtained,[2]

$$\sigma_b=\frac{4\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\frac{ZN}{A-1}\mathcal{R}^2 \tag{2}$$

(where N, Z, and A are, respectively, the neutron number, atomic number, and mass number of the nucleus), with available experimental data showed that the above assumption is not justified in heavy nuclei. These authors suggested that better agreement with experiment would follow if one assumes a crude α-particle model of the nucleus and takes $\mathcal{R}^2$ in Eq. (1) to represent the mean-square-radius of the α-particle units.

Recently, Rustgi, and Levinger[3] have studied the photoeffect in the lightest nuclei H^2, H^3, He^3, and He^4, and among other matters have investigated the cross section σ_b. The entire analysis has been based primarily on wave functions for these nuclei obtained by variational techniques. In the case of He^4, where recent experimental data allow the determination of an experimental value for σ_b, they find that this is nearly twice as large as the theoretical values obtained from the wave functions. They attribute the difficulty to the fact that the wave functions give too small a mean square radius for He^4, as is also indicated by the fact that the mean square radius of He^4 as measured by scattering of fast electrons is also in disagreement with the theoretical values. On the other hand, they note that if the observed σ_b is used to calculate a mean square radius from Eq. (1), this value is in agreement with the experimental value measured by electron scattering.

The present author has noted that the validity of Eq. (1) for these four nuclei does not depend at all on the presence or absence of correlations in the motion of the nucleons in these nuclei but follows from the single assumption that the ground-state wave function is symmetric in the space coordinates of the nucleons involved. Such symmetry can exist even in the presence of intense correlations in the nucleonic motion.

We are thus faced with the question as to the extent that this assumption of space symmetry is justified. In the case of H^2, this assumption is rigorously correct on the basis of parity considerations. It would also be rigorously correct for H^3, He^3, and He^4 if the forces between nucleons are of an attractive Wigner and Majorana character only, and nearly correct if forces of this character strongly dominate the coupling scheme in these nuclei (Wigner supermultiplet approximation). Even in the presence of some Bartlett, Heisenberg, and tensor forces, variational calculations have indicated that the dominant term in the ground state of these nuclei is still the completely symmetric S state with relatively small admixtures of D, P, and asymmetric S states. This last statement, in itself, is not sufficient to justify Eq. (1) since even if the percentages of asymmetric states are small (of the order of a few percent), their amplitudes may still be quite large and there can be contributions from cross-terms between symmetric and asymmetric states. However, a fortunate circumstance intervenes: After the symmetric S state, the most important terms which appear to be present in the ground states are D terms, and cross terms between S and D terms do not enter into the derivation

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[1] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).
[2] In the original formula of Bethe and Levinger, which was correct only to terms of order $1/A$, the factor A replaced the factor $A-1$.
[3] M. L. Rustgi and J. S. Levinger, Bull. Am. Phys. Soc. Ser. II, **2**, 62 (1957); M. L. Rustgi, doctoral dissertation, Louisiana State University (unpublished); M. L. Rustgi and J. S. Levinger, Phys. Rev. **106**, 530 (1957).

of (2). The only cross-terms which could be important are those between the dominant symmetric S state and another asymmetric S state. The amplitudes of the latter seem to be very small.

Our conclusion is therefore that Eq. (2) should be quite accurate for these nuclei, and its failure to be verified would raise serious doubts concerning our present picture of the ground-state wave functions for these nuclei.

To derive Eq. (2) on our stated assumptions, we begin with some general considerations. Let $\mathbf{r}_p$ ($p=1, 2, \cdots Z$) be the position vectors of the protons, $\mathbf{r}_n$ ($n=1, 2, \cdots N$) be the position vectors of the neutrons in a nucleus of mass number $A=Z+N$. We let $\mathbf{R}$ represent the center of mass of the nucleus, $\mathbf{R}_P$ the center of mass of the protons in the nucleus, and $\mathbf{R}_N$ the center of mass of the neutrons in the nucleus. The electric dipole moment of the nucleus, relative to its center of mass, is then

$$\mathfrak{P}=(ZN/A)e\mathbf{R}_{PN}, \quad \mathbf{R}_{PN}=\mathbf{R}_P-\mathbf{R}_N.$$

Now employing the familiar expression for $\sigma(E)$ and the closure relation in the usual way,[1] one obtains

$$\sigma_b=\frac{4\pi^2}{3\hbar c}\langle\mathfrak{P}^2\rangle=\frac{4\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\frac{Z^2N^2}{A^2}\langle R_{PN}^2\rangle, \tag{3}$$

where the angular brackets mean the expectation value in the ground state of the nucleus. We have further that

$$\langle R_{PN}^2\rangle=\left\langle\left\{\frac{1}{Z}\sum_p \mathbf{r}_p-\frac{1}{N}\sum_n \mathbf{r}_n\right\}^2\right\rangle$$

$$=\left\langle\frac{1}{Z^2}\sum_p r_p^2+\frac{1}{N^2}\sum_n r_n^2+\frac{1}{Z^2}\sum_{p\neq p'}\mathbf{r}_p\cdot\mathbf{r}_{p'}\right.$$

$$\left.+\frac{1}{N^2}\sum_{n\neq n'}\mathbf{r}_n\cdot\mathbf{r}_{n'}+\frac{1}{NZ}\sum_{p,n}\mathbf{r}_p\cdot\mathbf{r}_n\right\rangle.$$

By the exclusion principle, the wave function is antisymmetric under exchange of space and spin coordinates of any pair of like nucleons. Since R_{PN}^2 does not involve the spin operators, it readily follows that $\langle r_p^2\rangle$ has the same value α_P for all protons, $\langle r_n^2\rangle$ has the same value α_N for all neutrons, $\langle\mathbf{r}_p\cdot\mathbf{r}_{p'}\rangle$ has the same value β_{PP} for all distinct pairs of protons, $\langle\mathbf{r}_n\cdot\mathbf{r}_{n'}\rangle$ has the same value β_{NN} for all distinct pairs of neutrons, and $\langle\mathbf{r}_p\cdot\mathbf{r}_n\rangle$ has the same value β_{PN} for all proton-neutron pairs. Hence

$$\langle R_{PN}^2\rangle=\frac{1}{Z}\alpha_P+\frac{1}{N}\alpha_N+\frac{Z-1}{Z}\beta_{PP}+\frac{N-1}{N}\beta_{NN}-2\beta_{PN}.$$

We can derive similar expressions for the mean square radius $\mathfrak{R}^2$ of the nucleus, and for the mean square radius $\mathfrak{R}_c^2$ of the nuclear charge distribution:

$$\mathfrak{R}^2=\frac{1}{A}\langle\sum_p(\mathbf{r}_p-\mathbf{R})^2+\sum_n(\mathbf{r}_n-\mathbf{R})^2\rangle$$

$$=\frac{Z(A-1)}{A^2}\alpha_P+\frac{N(A-1)}{A^2}\alpha_N-\frac{Z(Z-1)}{A^2}\beta_{PP}$$

$$-\frac{N(N-1)}{A^2}\beta_{NN}-\frac{2ZN}{A^2}\beta_{PN},$$

$$\mathfrak{R}_c^2=\frac{1}{Z}\langle\sum_p(\mathbf{r}_p-\mathbf{R})^2\rangle$$

$$=\left(1-\frac{2}{A}+\frac{Z}{A^2}\right)\alpha_P+\frac{N}{A^2}\alpha_N-\frac{(2A-Z)(Z-1)}{A^2}\beta_{PP}$$

$$+\frac{N(N-1)}{A^2}\beta_{NN}-\frac{2N^2}{A^2}\beta_{PN}.$$

Now, if the ground-state wave function is further assumed to be completely symmetric in the space coordinates of all nucleons, one has[4]

$$\alpha_P=\alpha_N=\alpha, \quad \beta_{PP}=\beta_{NN}=\beta_{PN}=\beta.$$

Thereupon

$$\mathfrak{R}^2=\mathfrak{R}_c^2=\frac{A-1}{A}(\alpha-\beta),$$

$$\langle R_{PN}^2\rangle=\frac{A}{ZN}(\alpha-\beta)=\frac{A^2}{ZN(A-1)}\mathfrak{R}^2,$$

and

$$\sigma_b=\frac{4\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\frac{ZN}{A-1}\mathfrak{R}_c^2. \tag{4}$$

It should be remarked that the complete spatial symmetry of the wave function is a sufficient but not a necessary condition for the validity of (4).

The best experimental determinations of the mean square radius of the charge distribution are those obtained by high-energy electron scattering.[5] However, care must be exercised in employing these in the above equation since these experiments have also shown that the free proton itself has a root-mean-square radius $\mathfrak{R}_P=0.77\times10^{-13}$ cm, while the above derivation is correct only for a proton with a point charge. The electron-neutron interaction experiments also indicate the existence of a finite second radial moment of the charge density distribution of a neutron, but this is sufficiently small to be neglected in the above calculation. The formula (4) can be corrected for the finite size of the proton charge distribution, provided one assumes that this charge distribution is not appreciably polarized when the proton is in the close proximity of other nucleons in the nucleus. In this case, we may

[4] For a charge-symmetric (self-conjugate) nucleus, $\alpha_P=\alpha_N$ and $\beta_{PP}=\beta_{NN}$.

[5] See R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).

write in place of (4)

$$\sigma_b=\frac{4\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\frac{ZN}{A-1}(\mathcal{R}_c{}^2-\mathcal{R}_P{}^2), \tag{5}$$

where $\mathcal{R}_c{}^2$ is now the directly measured mean square radius of the nucleus involved.

If one applies Eq. (5) to deuterium and inserts the values obtained from high-energy electron scattering experiments by Hofstadter and his collaborators:[5–7] $\mathcal{R}_c=2.10\times10^{-13}$ cm, $\mathcal{R}_P=0.77\times10^{-13}$ cm, one obtains $\sigma_b=3.70$ millibarns. From the experimental data on photodisintegration of the deuteron, Rustgi and Levinger[3] obtain a value for σ_b (after correction for photomagnetic contributions to the cross section) of 3.7 millibarns. The agreement is excellent. The situation in He^4 is also satisfactory. The measured value of $\mathcal{R}_c$ by electron scattering[5,8] is 1.61×10^{-13} cm, with an accuracy of probably better than 5%. This yields from Eq. (5) the value $\sigma_b=2.53$ millibarns. Rustgi and Levinger[3] obtain for a direct experimental value 2.7 millibarns with an accuracy of 10%. The good agreement suggests (1) that the symmetry assumption is reasonably well fulfilled, and (2) that the charge distribution measured by high-energy electron scattering is the same charge distribution which gives rise to electric dipole absorption of radiation at moderate energies.

The failure of variational wave functions to yield satisfactory values of either $\mathcal{R}_c$ or σ_b for He^4 suggests that the assumptions concerning nuclear forces contained in these calculations may be severely in question. Whether repulsive cores, many-body forces, or nonlocal forces are required to rectify the situation is not clear. Unfortunately, no information is available concerning H^3 and He^3 and it would clearly be of interest to have determinations of $\mathcal{R}_c$ and σ_b for these nuclei.

When one goes to nuclei heavier than He^4, σ_b is no longer related to any simple parameters of the nucleus and it therefore can yield useful information only on the basis of further assumptions concerning the ground-state wave function which may have little substantial basis in fact. The case of Li^6 shows some promise, however. Some valuable information may be obtained if one is willing to admit the following assumptions[9]:

(1) Li^6 has a structure consisting of an alpha particle and two loosely bound nucleons, the alpha particle being unpolarized by the orbital nucleons.

(2) One can neglect the Pauli principle in requiring that the wave function be antisymmetric with respect to an orbital nucleon and a like nucleon bound in the alpha particle. In this case one can show that

$$\sigma_b(\mathrm{Li}^6)=\sigma_b(\mathrm{He}^4)+\frac{\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\langle r^2\rangle, \tag{6}$$

where $\langle r^2\rangle$ is the mean square separation of the orbital proton and neutron. Under the same assumptions, knowledge of the mean square radius of the charge distribution in Li^6 (which is available)[5] would allow one further to determine a second important parameter of the wave function, namely $\langle\rho^2\rangle$, the mean square separation of the center of mass of the alpha particle and the center of mass of the two orbital nucleons, through the formula

$$\mathcal{R}_c{}^2(\mathrm{Li}^6)=\tfrac{2}{9}\langle\rho^2\rangle+\tfrac{1}{12}\langle r^2\rangle+\tfrac{2}{3}\mathcal{R}_c{}^2(\mathrm{He}^4)+\tfrac{1}{3}\mathcal{R}_P{}^2. \tag{7}$$

Knowledge of the two parameters $\langle r^2\rangle$ and $\langle\rho^2\rangle$ would be quite valuable in setting up reasonable wave functions to represent the ground state of Li^6. The validity of the above assumptions is not free from question, however.

[6] J. A. McIntyre and R. Hofstadter, Phys. Rev. **98**, 158 (1955); J. A. McIntyre, Phys. Rev. **103**, 1464 (1956).

[7] Professor Levinger has kindly informed us that he and M. L. Rustgi have reanalyzed the data of McIntyre by a somewhat different method. They obtain a value for $\mathcal{R}_c(\mathrm{H}^2)=2.14\times10^{-13}$ cm in essential agreement with McIntyre. We are indebted to Professor Levinger for this information and for general comments on this manuscript.

[8] R. W. MacAllister and R. Hofstadter, Phys. Rev. **102**, 851 (1956); R. Blankenbecler and R. Hofstadter, Bull. Am. Phys. Soc. Ser. II, **1**, 10 (1956).

[9] These assumptions are equivalent to assuming that the ground-state wave function is a simple product of a function of the internal coordinates of the α particle by a function of the coordinates of the orbital nucleons relative to the center of mass of the α particle.