Kirensky type. Preliminary measurements by McGuire[9] of K_1 for this crystal by the microwave resonance method, indicate that the major part, if not all, of the first term represents the difference between the values of K_1 obtained by a static method and that obtained by a microwave method.

Measurements were also made on the crystal after it had been heat treated by cooling slowly in a magnetic field parallel to each of the principal crystallographic directions from 700°C. Generally this heat treatment caused a departure from cubic symmetry of the torque curves and a maximum change in the torque values of about 15%.

Since the theories of ferromagnetic anisotropy have not been explicitly extended to ferrites, it is of interest then that the temperature dependence has the form expected of a ferromagnetic substance. If the experimental values of K_1 and M for the nickel cobalt ferrite crystal are substituted in Eq. (2) the following values of n are obtained: 8 ± 4 at 100°K 8 ± 2 at 200°K, and 8 ± 1 at 300°K.

V. ACKNOWLEDGMENTS

The writer wishes to express his sincere appreciation to Dr. T. R. McGuire, Professor R. K. Wangsness, his faculty adviser, and Dr. L. R. Maxwell for their encouragement and very substantial assistance throughout this work. He also wishes to thank Dr. R. M. Bozorth, Dr. A. J. Williams, and Dr. E. A. Nesbitt of the Bell Telephone Laboratories for acquainting him with their work.

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New Type of Oscillatory Magnetoresistance in Metals*

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The magnetoresistive properties of a thin sodium wire have been studied at 1°K in transverse magnetic fields (H_T) up to 60 000 gauss. This study was undertaken in order to determine whether magnetoresistive oscillations of the de Haas-van Alphen type could be detected in the vicinity of 60 000 gauss. Although no such oscillations were found, the magnetoresistance for H_T below 15 000 gauss exhibited a completely new type of oscillatory phenomena. These new oscillations are periodic in H with a decreasing amplitude in increasing magnetic fields, whereas the de Haas-van Alphen oscillations would be periodic in H^{-1} with an increasing amplitude in increasing magnetic fields. The period of these new oscillations is in excellent agreement with the period of the oscillatory behavior predicted theoretically by Sondheimer for the magnetoresistance due to surface scattering of thin metallic films in H_T. From this period, a value for the electronic momentum was obtained. The significance of these new oscillations is discussed.

INTRODUCTION

IN the study of the electronic energy-band structure of metallic media, de Haas-van Alphen and cyclotron resonance phenomena[1] have proved to be important tools for obtaining some information concerning various electronic parameters and the nature of some parts of the Fermi surface. However, there exists a disappointing gap between the successful experiments which have been performed and tractable theory. Whereas the detailed theory on the one hand is based upon a free-electron model employing simple Fermi surfaces, so that it can best be applied to the Group I metals, the experiments on the other hand have revealed these phenomena only in non-Group I metallic conductors with complicated Fermi surfaces. These phenomena have not as yet been observed in the Group I metals because of the inherent difficulties in establishing the necessary experimental conditions. Since the Group I metal sodium is the closest known approximation to an isotropic free-electron metal, a very careful search was made in this experiment to find the characteristic H^{-1} oscillations of the de Haas-van Alphen[2] type in the magnetoresistance[3] of sodium under reasonably favorable experimental conditions. While no H^{-1} oscillations were found,[4] a new type of magneto-oscillatory behavior stemming from magnetic "size effects" was observed.

The first theoretical analysis using Fermi statistics

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* For a preliminary report, see J. Babiskin and P. G. Siebenmann, Bull. Am. Phys. Soc. Ser. II, 2, 140 (1957).

[1] R. G. Chambers, Can. J. Phys. 34, 1395 (1956). A recent review on cyclotron resonance and de Haas-van Alphen phenomena as related to the Fermi surface.

[2] For a recent review article on the de Haas-van Alphen effect, see D. Shoenberg, *Progress in Low Temperature Physics* (North Holland Publishing Company, Amsterdam, 1957), Vol. II, Chap. VIII.

[3] A one-to-one correspondence has been experimentally established between the periods in H^{-1} of both the de Haas-van Alphen effect and of the oscillatory galvanomagnetic and thermomagnetic effects for various non-Group I metals; e.g., see M. C. Steele and J. Babiskin, Phys. Rev. 98, 359 (1955).

[4] Previous unsuccessful attempts to find H^{-1} oscillations in Group I metals have been made in steady magnetic fields up to 32 000 gauss [J. S. Dhillon and D. Shoenberg, Trans. Roy. Soc. (London) 248, 1 (1955)] and in pulsed magnetic fields up to 100 000 gauss [D. Shoenberg, Physica 19, 791 (1953)]; see also reference 2.

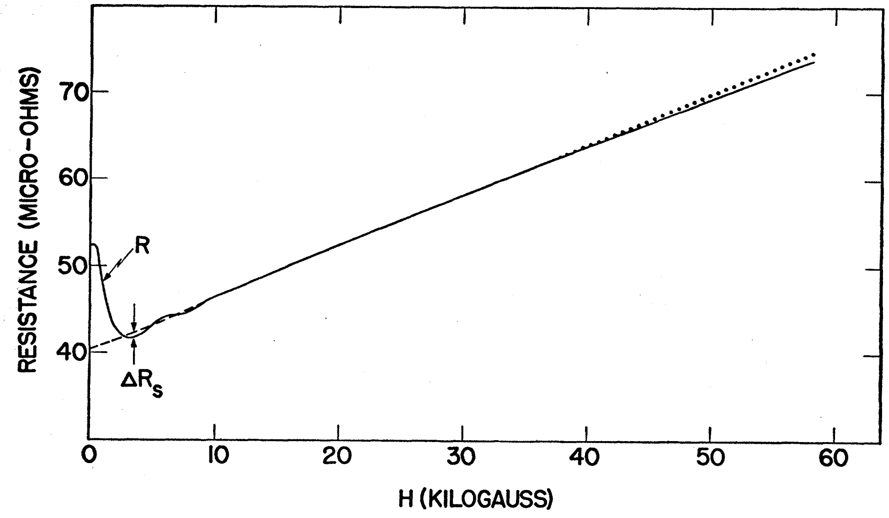

FIG. 1. The transverse magnetoresistance of an 80 μ sodium wire at 1°K.

of the "size effects"[5] (the increase in the resistivity of a conductor as the distance between the external boundaries is diminished) for an isotropic conductor was given by Fuchs[6] for a thin film in the absence of an applied magnetic field (H). This analysis was extended to the two-dimensional case of thin wires by Dingle[7] and by MacDonald and Sarginson.[8] Further statistical analyses of the size effects in H (i.e., magnetic size effects) have been given for thin films by Sondheimer[9] and by MacDonald and Sarginson[8] in a transverse magnetic field (H_T) and by Azbel'[10] in a longitudinal magnetic field (H_L). Magnetic size effects have also been treated using a kinetic approach by Chambers[11] for a thin wire in H_L and for a thin film in H_T, and by Koenigsberg[12] for a thin film in H_L. Experimentally, magnetic size effects were first discovered in the magnetoresistance of thin sodium wires by MacDonald,[13] who observed negative magnetoresistances ($dR/dH<0$) in H_L and H_T. The negative magnetoresistance was attributed to the decreased surface scattering caused by H. Subsequent experiments on the magnetoresistance of thin sodium wires by MacDonald and Sarginson,[8] by Chambers,[11] and by White and Woods[14] have also exhibited similar magnetic size effects. Although all of these theoretical analyses yield a negative magnetoresistance as observed, the analysis of Sondheimer was the first to also predict an oscillatory behavior which had never been observed until now.

EXPERIMENTAL DETAILS

The high-purity ($R_{295}/R_{4.2}=2000$) sodium specimen was obtained through the kindness of S. B. Woods of the National Research Council of Canada and was prepared by methods described by MacDonald.[15] The sodium was contained in a soft-glass capillary with bulbous ends through which two current and two potential probes of platinum were sealed. The sodium capillary was 80 μ (microns) in diameter and 1.1 cm long. This diameter was determined from the resistivity and measured resistance at 295°K, as well as from direct optical measurements. Since the sodium solidified slowly from one end during its preparation, it is believed to be a single crystal or nearly so.

The magnetic field was produced by a Bitter solenoid[16] and was smoothly and slowly varied to 60 000 gauss during the continuous recordings. The magnetic field was known to 1% and was uniform over the specimen to better than 0.1%. The specimen length was aligned perpendicular to H to within 1°.

The measurements were made at 1°K and 4.2°K and were recorded continuously on a 10-millivolt X-Y recorder. The magnetic field which was directly proportional to the solenoid current was recorded on the X axis, while the voltage across the potential probes in the sodium specimen was amplified by a Leeds and Northrup dc microvolt amplifier and then recorded on the Y axis. The specimen current was 10 amperes and was constant to better than 0.1%. By setting both the X and Y voltages to $\sim$90 millivolts at 60 000 gauss, and by using standard biasing techniques in conjunction with the 10 millivolt scales of the X-Y recorder, the data were recorded on the equivalent of a 90 in.×90 in. chart. The recorded measurements have a relative accuracy of better than 0.03% over any 10-millivolt

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[5] For relevant reviews on size effects, see E. H. Sondheimer, *Advances in Physics* (Taylor and Francis, Ltd., London, 1952), Vol. 1, p. 1; reference 15, p. 183; reference 26, p. 242.

[6] K. Fuchs, Proc. Cambridge Phil. Soc. **34**, 100 (1938).

[7] R. B. Dingle, Proc. Roy. Soc. (London) **A201**, 545 (1950).

[8] D. K. C. MacDonald and K. Sarginson, Proc. Roy. Soc. (London) **A203**, 223 (1950).

[9] E. H. Sondheimer, Nature **164**, 920 (1949); also Phys. Rev. **80**, 401 (1950).

[10] M. Ya. Azbel', Doklady. Akad. Nauk S.S.S.R. **99**, 519 (1954).

[11] R. G. Chambers, Proc. Roy. Soc. (London) **A202**, 378 (1950).

[12] E. Koenigsberg, Phys. Rev. **91**, 8 (1953).

[13] D. K. C. MacDonald, Nature **163**, 637 (1949).

[14] G. K. White and S. B. Woods, Phil. Mag. **1**, 846 (1956).

[15] D. K. C. MacDonald, *Handbuch der Physik* (Springer-Verlag, Berlin, 1956), Vol. 14, p. 165.

[16] F. Bitter, Rev. Sci. Instr. **10**, 373 (1939).

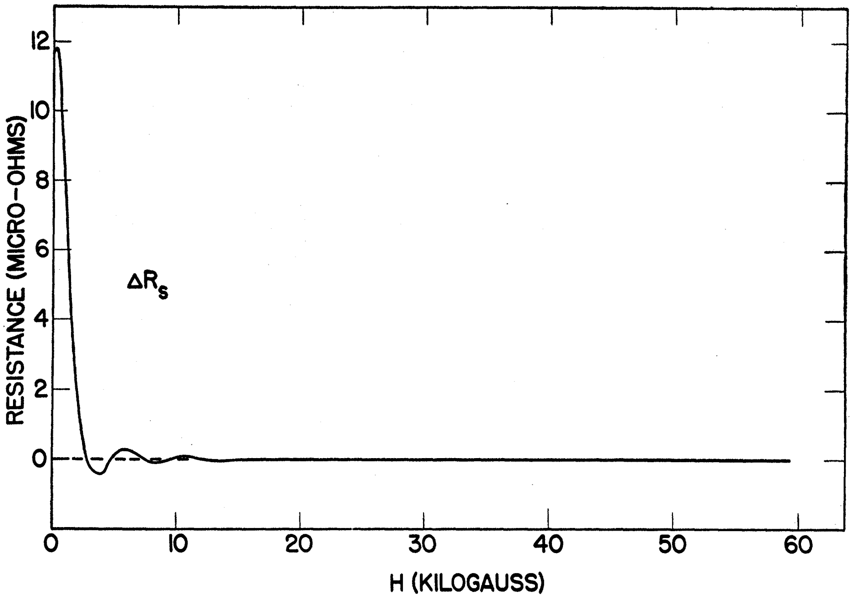

FIG. 2. ΔR_s *vs* H for the 80 μ sodium wire at 1°K on an expanded ΔR_s scale.

interval ($\sim$7000 gauss on the X axis) and closer to 0.01% for smaller intervals.

RESULTS AND DISCUSSION

The solid curve of Fig. 1 shows a continuous plot of the measured total resistance (R) against H at 1°K for the 80 μ sodium wire, where R consists of both the bulk resistance (R_B) and the resistance due to surface scattering (R_s). Since the resistance of bulk sodium normally rises slowly in increasing magnetic fields, the departures from this behavior as reported here and in previous experiments[8,11,13,14] are attributed to the variation of R_s with H (the surface magnetoresistance) or, in other words, to magnetic size effects. Thus in Fig. 1, the surface magnetoresistance exhibited an initial increase to a maximum at $\sim$300 gauss and then exhibited a strong negative magnetoresistance. This general behavior is very similar to that previously observed by White and Woods.[14] However, in the present experiment the surface magnetoresistance exhibited a damped oscillatory behavior as shown in Fig. 1 before becoming completely damped out at $\sim$15 000 gauss. Thus the observed magnetoresistive maxima of previous experiments[8,13,14] as well as the initial maximum of the present experiment can now be recognized to be part of an oscillatory behavior which had been predicted,[9] but not previously observed.

From $\sim$15 000 gauss to $\sim$35 000 gauss, the magnetoresistance exhibited a linear dependence, which is in agreement with previous observations[17] on the Group I metals in H_T. The dotted straight line at the high fields is an extension of this linear region. However, above $\sim$35 000 gauss, the magnetoresistance falls below this linear extrapolation in such a manner as to suggest the beginning of a saturation effect. Although saturation is predicted by the theory[18] of magnetoresistance for Group I metals, no indications of a saturation effect have been previously observed in H_T. No evidence for an oscillatory magnetoresistance of the de Haas-van Alphen type was found at the high fields in the vicinity of 60 000 gauss, even with a resolution of $\sim$0.01% for both R and H over small intervals of H. The calculated spacing in H ($\Delta H = \beta^* H^2/E_0$) from Table I is $\sim$14

TABLE I. Electronic parameters[a] of sodium at 1°K.

(A) Intrinsic parameters	Theoretical[b]	Experimental[c]
$m^*\bar{v}$	9.71×10^{-20}	9.27×10^{-20}
$n=8\pi(m^*\bar{v})^3/3h^3$	2.64×10^{22}	2.30×10^{22}
$A_B=1/ne$ (cgs emu)	2.37×10^{-3}	2.72×10^{-3}
$rH=m^*\bar{v}c/e$	6.06	5.79
$\beta^*/E_0=2e\hbar/c(m^*\bar{v})^2$ (gauss^{-1})	3.59×10^{-9}	3.93×10^{-9}
$\sigma_B/l_B=ne^2/m^*\bar{v}$ (ohm^{-1} cm^{-2})	6.97×10^{10}	6.36×10^{10}
$\mu_B/l_B=e/m^*\bar{v}$	4.95×10^{9}	5.18×10^{9}
$\bar{v}$	1.07×10^{8} [d]	1.02×10^{8} [d]
$E_0=m^*\bar{v}^2/2$ (ev)	3.23[d]	2.95[d]
(B) Structure-sensitive parameters[e]	Theoretical	Experimental
σ_B (ohm^{-1} cm^{-1})	1.53×10^{9}	1.40×10^{9}
μ_B	1.09×10^{8}	1.14×10^{8}
σ_B/σ	3.71	3.39
p	$\sim$0.05	$\sim$0.10
$\tau=l_B/\bar{v}$	2.06×10^{-10} [d]	2.16×10^{-10} [d]

[a] All units are in cgs esu unless otherwise specified.
[b] These values are based on $m^*\bar{v}=9.71\times10^{-20}$ which was obtained as explained in the text.
[c] These values are based on $m^*\bar{v}=9.27\times10^{-20}$ from the experimentally determined period of the oscillations.
[d] These parameters are calculated for $m^*=m_0$.
[e] These parameters are based on $l_B=220$ μ at $H=0$ for this 80 μ sample.

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[17] For the most recent reference, see R. G. Chambers, Proc. Roy. Soc. (London) **A238**, 344 (1957). Also, R. T. Webber and J. R. de Launay have privately reported to us that the transverse magnetoresistance of high purity copper at 4.2°K exhibits a linear dependence up to 100 000 gauss.

[18] M. Kohler, Ann. Physik **5**, 99 (1949).

gauss at 60 000 gauss. Thus, it is seen that the de Haas-van Alphen oscillations would have been just detectable, if they had sufficient amplitude. The above measurements were not appreciably affected upon varying the temperature from 1°K to 4.2°K.

In order to show the new oscillatory behavior more clearly, Fig. 2 shows a plot of ΔR_s against H. As shown in Fig. 1, ΔR_s is the difference between the experimental data for R and the dashed curve which approximately represents the mean of the oscillatory data. This dashed curve is included for illustrative purposes only and for reasons to be presented shortly should not be taken to represent the bulk magnetoresistance. Figure 2 exhibits clearly the first three maxima and the first two minima of the oscillations, while a third minimum can be seen by sighting along the ΔR_s curve. In the actual continuous recording on the equivalent of a 90 in.×90 in. chart, this third minimum was more clearly visible and a fourth maximum was also discernible.

This discovery of a magneto-oscillatory size effect has finally opened up the possibility of making definitive studies on the Group I metals with much greater ease than previously envisioned for the de Haas-van Alphen or cyclotron resonance phenomena. This is so, since these oscillations occur at comparatively low magnetic fields with measurable amplitudes and, as will be shown, they yield significant information concerning electronic parameters and the nature of the Fermi surface. In order to observe these effects, it is desirable to satisfy three conditions: (a) the bulk mean free path (l_B) should be of the order of, or preferably larger than, the smallest specimen dimension; (b) the radius of the electron orbits (r) as varied by H should range from $r=\infty$ at $H=0$ to r of the order of 10 to 20 times smaller than the smallest specimen dimensions; (c) the bulk magnetoresistance should not be so large as to overwhelm the surface magnetoresistance. All these conditions were satisfied in these experiments.

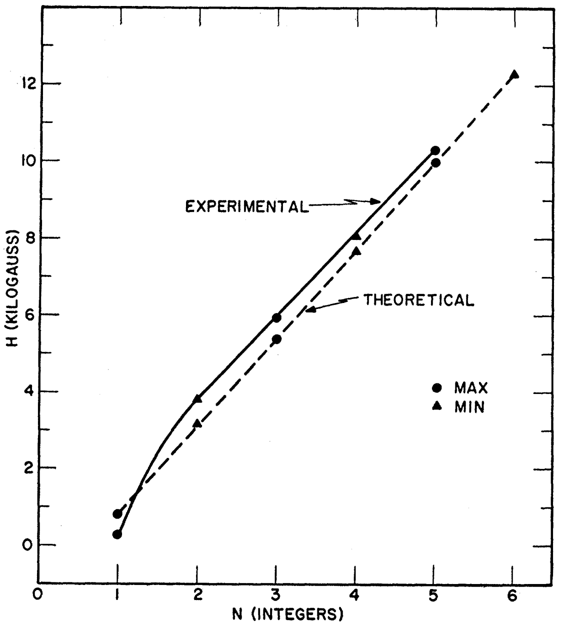

FIG. 3. Values of H at which the magnetoresistive maxima and minima occur for both the experimental and theoretical oscillations plotted against corresponding integers.

The characteristics of this new type of oscillatory behavior differ completely from those of the de Haas-van Alphen oscillations in that they exhibit a damped periodicity as a function of H rather than of H^{-1}. However, the period and behavior of these new oscillations are in excellent agreement with the oscillatory behavior, which was predicted theoretically by Sondheimer[9] for the surface magnetoresistance of thin films in H_T. A similar behavior, which appears to be oscillatory, was also predicted[19] by Azbel'[10] for thin films in H_L. In a kinetic treatment, Chambers[11] also predicts a similar oscillatory behavior for thin films in H_T and gives physical reasons as well for the existence of such an oscillatory behavior.

In Sondheimer's theory, the variation of σ_B/σ is plotted as a function of a dimensionless parameter[20] $\gamma=a/r=aeH/m^*\bar{v}c$ for fixed values of another dimensionless parameter $\kappa=a/l_B$ and of the surface reflection coefficient (p). The quantity σ_B is the bulk conductivity; σ is the total conductivity; a is the thickness of the film; m^* is the effective mass; $\bar{v}$ is the Fermi velocity. The coefficient p ranges from $p=0$ for completely diffuse scattering of the electrons at the specimen boundaries to $p=1$ for completely specular reflection. Sondheimer finds that this theoretical plot of σ_B/σ *vs* γ (for constant κ and p) exhibits a damped oscillatory behavior. The position of the maxima and minima of σ_B/σ occur periodically in γ with a period of $\Delta\gamma\cong 6$ and with the first maximum occurring at $\gamma\cong 1$. Sondheimer also finds that the amplitude of the oscillations is strongly affected by the choice of the values for κ and p, but that the periodicity and the positions of the maxima and minima are not appreciably affected.

The nature of the agreement between Sondheimer's theory and the present experiment is shown in Fig. 3. The values of H at which the maxima and minima occur for both the experimental and theoretical oscillations are plotted against successive integers (N) which are assigned to the successive maxima and minima. The theoretical values of H at $\gamma=1$, 4, 7, 10, 13, 16 were obtained by substituting $a=80\ \mu$ and $m^*\bar{v}=9.71\times10^{-20}$ into γ. This value of $m^*\bar{v}$ was obtained for sodium from the relation $n=(8\pi/3)(m^*\bar{v}/h)^3$ assuming one conduction electron per atom, where n is the number of conduction electrons per cm^3 using the density[21] of 1.0066

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[19] Azbel' uses an ellipsoidal Fermi surface of constant energy and shows that the anisotropy of masses does not change the nature of the results.

[20] In Sondheimer's notation, β is used instead of γ. This change in notation was made, since β^* is used here as an effective double Bohr magneton.

[21] *Handbook of Chemistry and Physics* (Chemical Rubber Publishing Company, Cleveland, 1952), thirty-fourth edition.

g/cm^3 at 85°K. The experimental values of H are plotted in Fig. 3 only for the first three maxima and the first two minima. The observed third minimum and fourth maximum are not included, because the damping made it difficult to locate their positions with sufficient accuracy. Since the respective theoretical and experimental points of Fig. 3 fall on straight lines (except for the first experimental maximum at $N=1$), these oscillations are periodic in H. The value of the period in H is obtained from the slopes of the lines. The experimental slope is 4.5% smaller than the theoretical slope, so that the experimental value of $m^*\bar{v}=9.27\times10^{-20}$ is obtained. This pleasant agreement between Sondheimer's statistical theory and the present experiment not only shows that sodium does closely approximate the free-electron model, but more importantly shows that a high degree of confidence in Sondheimer's theory is justifiable.

Although this agreement is rather good, the validity of a comparison between a thin-film theory and a thin-wire experiment is naturally open to question, because the magnetic size effects depend upon the specimen geometry. In a later article by one of us (J.B.), an analysis which extends some aspects of Sondheimer's thin-film theory in H_T to Dingle's[7] thin-wire theory at $H=0$ will be presented. The pertinent results of this analysis relative to the present discussion indicate (a) that σ_B/σ of a thin wire in H does exhibit an oscillatory behavior and (b) that the periodic nature of the oscillatory behavior of thin films is relatively unchanged in going to thin wires, so that the good agreement in the above comparison is not too surprising.

From a phenomenological standpoint, even the relatively small differences in Fig. 3 between the results of the thin-film theory and the thin-wire experiment can be reasonably explained by the following arguments. When the curvature of the electron orbits becomes large compared to the curvature of a regular bounding surface, one would reasonably expect that the magnetic size effects will become less dependent upon the geometry of the bounding surface. Thus one would expect the oscillatory behaviors of thin films and thin wires to approach one another, when H is such that $r<a/2$ or $d/2$, where a is the thickness of the film and d is the diameter of the wire. Since $r>a/2$ or $d/2$ at $N=1$, the large deviation between the results of the thin-wire experiment and the thin-film theory near $N=1$ is not unexpected from the above arguments. However, at $N=2$, $r<a/2$ or $d/2$ and becomes progressively smaller at higher N. For $N=2$ or higher, it is seen in Fig. 3 that the experimental and theoretical behaviors are in good agreement. It is also seen that the phase difference between the experimental and theoretical oscillations decreases continuously as N increases, so that they appear to be merging at the higher N (or H). These observations are again in accord with the above arguments. The phase difference is equal to the horizontal distance between the experimental and theoretical curves, and is equal to π for $\Delta N=1$.

On the basis of this apparent merger at higher H, the choice of the theoretical value of $m^*\bar{v}=9.71\times10^{-20}$ for the ensuing calculations would appear to have some merit. However, the alternate choice of the experimental value of $m^*\bar{v}=9.27\times10^{-20}$ may be preferable, since the following evidence from other sources indicates a somewhat smaller value than the theoretical value: (a) Chambers[11] obtains a value of $m^*\bar{v}=9.1\times10^{-20}$ from measurements on the longitudinal magnetoresistance of 30 μ sodium wire at 4.2°K; (b) from x-ray measurements, Barrett[22] finds that the crystal structure of sodium partially transforms from body-centered cubic to close-packed hexagonal at low temperatures, so that the Fermi energy (E_0) probably becomes smaller; (c) MacDonald[23] observes that the Hall voltage of sodium increases in going from 300°K to 4.2°K; (d) Brooks[24] has calculated a value for $m^*=0.98m_0$.

Since both the theoretical and experimental values of $m^*\bar{v}$ have significance, the various parameters given in Table I are calculated for both values. Upon substituting the values of $m^*\bar{v}$ into the free-electron relations as given in Table I, one can calculate directly the values for the following intrinsic parameters: n, r, the bulk Hall coefficient (A_B), and also the period (β^*/E_0) of the carefully sought de Haas-van Alphen oscillations, where $\beta^*=e\hbar/m^*c$ is the effective double Bohr magneton and E_0 is the Fermi energy. One can also calculate the values of the intrinsic parameters σ_B/l_B and μ_B/l_B, which are the ratios of the structure-sensitive parameters σ_B, l_B and the bulk mobility (μ_B).

A bulk mean free path of 220 μ at $H=0$ was determined graphically by comparing the experimental data for R at $H=0$ of three sodium wires of different diameters (our 80 μ wire and the 130 μ and 350 μ wires of White and Woods[14]) with the theoretical curves given by Dingle[7] for the size effects of a thin wire at $H=0$. This determination was based on three assumptions: (a) Dingle's free-electron theory can be applied to sodium; (b) l_B and p are the same for the three specimens, since they were prepared by the same techniques for the same sodium; (c) $p\cong0$ from arguments presented by Chambers[11] on the basis of anomalous skin-effect measurements. This value of $l_B=220\ \mu$ is estimated to be reliable to ~10%. A more direct method of determining l_B would be from the damping of the oscillations in the family of κ curves at $p=0$ given by Sondheimer.[9] Since neither the present thin-wire data nor the thin-film theory were sufficiently detailed, this method could not be employed with accuracy. However, a value for l_B could be bracketed between 160 μ and 300 μ using this method.

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[22] C. S. Barrett, Acta Cryst. **9**, 671 (1956).

[23] D. K. C. MacDonald, Phil. Mag. **2**, 97 (1957).

[24] H. Brooks, private communication quoted by D. Pines, Phys. Rev. **95**, 1090 (1954).

Using this value of l_B, values for σ_B and μ_B can now be calculated. Since R_B at $H=0$ as obtained from σ_B is $\sim$15 micro-ohms, the reasons become obvious as to why the dashed curve of Fig. 1 was not taken to represent the bulk magnetoresistance. From this value of R_B and the value for $R=53.0$ micro-ohms at $H=0$ in Fig. 1, values for σ_B/σ can also be obtained. Upon plotting the values obtained for σ_B/σ at $\kappa=0.364$ on the Dingle curves, approximate values for p were obtained. Values for all these structure-sensitive parameters are given in Table I. Although a value for $m^*\bar{v}$ can be obtained from the period of the oscillations, the values of m^* and $\bar{v}$ cannot be obtained separately. However, if m^* is assumed to be equal to the free-electron mass (m_0), then appropriate values can also be calculated for $\bar{v}$, E_0 and τ as given in Table I, where $\tau=l_B/\bar{v}$ is the collision relaxation time.

In Sondheimer's theory, the Fermi surface was assumed to be spherical so that there is no bulk magnetoresistive effect[25] and l_B is independent of H. In the present experiment, the observed bulk magnetoresistive effect is comparatively small, but it is not zero. Therefore, l_B does depend weakly on H for sodium and the Fermi surface is not perfectly spherical. In view of the difference between Sondheimer's thin-film theory and the present thin-wire experiment, and because the bulk magnetoresistance could not be separated from the surface magnetoresistance, a comparison of the amplitudes of the theoretical and experimental oscillations was not attempted. However, it is interesting to point out that the experimental data would not fit any single Sondheimer curve of constant κ due to the dependence of l_B upon H. Instead, the data will cut across the family of constant κ curves.

From the agreement between Sondheimer's statistical theory and the present experiment, it appears that the quantum restriction[26] of $\omega\tau<1$ on the Boltzmann transport equation in H is not at all severe as long as the Fermi energy $E_0 \gg \beta^*H$, where β^*H is the spacing between the magnetically quantized energy levels and $\omega=eH/m^*c$ is the cyclotron resonance frequency. For example, at $H=10\,000$ gauss where the amplitude of the oscillations still can be observed, $\omega\tau=37$ for $l_B=220\,\mu$, but there still exist 25 400 quantum levels, so that a statistical treatment should still be applicable. This argument is further supported by the infinitesmal amplitude of the de Haas-van Alphen oscillations, which cannot be observed at these magnetic fields.

It is interesting to note that the first observed maximum occurs at $\omega\tau\cong 1$ or $r=l_B$ rather than at $r=a$ (or $\gamma=1$) as predicted in Sondheimer's theory. For $l_B=220\,\mu$ and $H=270$ gauss, β^*H becomes equal to the collision broadening of the quantum levels and $\omega\tau=1$. In previous experiments,[8,14] the first observed maxima also occur at $\omega\tau\cong 1$ when the appropriate value for l_B is taken into account. These observations suggest the possibility that all the observed first maxima might be due to an unexplained bulk effect rather than a size effect.

Magnetic size effects should be observable in Group I metals other than sodium and even in non-Group I metals. For metals having a smaller n than sodium and consequently a smaller $m^*\bar{v}$, the oscillations for specimens with the same sizes should occur at lower H than for sodium but will be complicated[27] because of the anisotropy of their Fermi surfaces. The longitudinal magnetoresistance of antimony[28] and bismuth[29] have both exhibited an anomalous resistance maximum, which has been tentatively attributed to the magnetic size effects due to the scattering of electrons from internal boundaries.[30]

A systematic magnetoresistive study on sodium wires and films with sizes ranging from $\sim 20\,\mu$ to bulk specimens is planned in order to clarify some of the questions which have been suggested by the present experiment. For specimens thinner than $80\,\mu$, the oscillations should occur at higher magnetic fields and with greater amplitudes than those observed in the present experiment. From the variation of the period of the oscillations with orientation for thin metallic single crystals, it should be possible to map out the Fermi surface. Since the oscillations for the thinner sodium specimens would occur principally above 7500 gauss where $\beta^*H > kT$ at $T=1°$K, the effects of anisotropic lattice scattering[31] would be subdued and the Fermi surface could be studied alone. Finally, since no disturbing flaws in the theory were detected in the present experiments, we believe that the de Haas-van Alphen oscillations will be observable at $H>60\,000$ gauss and that the search should continue.

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We wish to thank S. B. Woods, whose kind cooperation in supplying the sodium specimen made this research possible; M. F. M. Osborne and J. R. Clement for helpful discussions; and the members of the Cryogenics Branch for their helpful interest.

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[25] R. Peierls, Ann. Physik **10**, 97 (1931).

[26] A. H. Wilson, *Theory of Metals* (Cambridge University Press, New York, 1953), p. 210.

[27] R. Englman and E. H. Sondheimer, Proc. Phys. Soc. (London) **B69**, 449 (1956).

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[29] J. Babiskin, Ph.D. thesis, The Catholic University of America, 1956 (unpublished); also J. Babiskin, Phys. Rev. **107**, 981 (1957).

[30] D. K. C. MacDonald, Phil. Mag. **62**, 756 (1951); **63**, 124 (1952).

[31] D. K. C. MacDonald and K. Sarginson, Repts. Progr. in Phys. **15**, 249 (1952), see p. 267.