

agreement with Seitz and Koehler's theoretical estimate⁶ of 1.9×10^3 .

The strain gauge data confirm the existence of a neutron induced expansion in LiF. It also indicates that

⁶ F. Seitz and J. S. Koehler, *Solid State Physics* (Academic Press, Inc., New York, 1956), Vol. 2, p. 445.

there is a strong saturation of the effect at 6×10^{16} *nv*, where the expansion² averages 2×10^{-5} per 10^{15} thermal neutrons/cm². The results of Mayer *et al.*,⁴ previously cited, show a continuation of this saturation at 3×10^{17} *nv*, where the expansion averages 5×10^{-6} per 10^{15} thermal neutrons/cm².

Stark Effect on Cesium-133 Hyperfine Structure*

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The change of the hyperfine-structure separation energy caused by an electric field has been measured in Cs¹³³ by the atomic-beam magnetic-resonance method. The shift of the ($F=4, m_F=0$) \leftrightarrow ($F=3, m_F=0$) transition frequency in the electric field \mathcal{E} is given by

$$\Delta(\mathcal{E}) = -2.29 \times 10^{-6} (1 \pm 0.03) \mathcal{E}^2 \text{ cps,}$$

where \mathcal{E} is in volts/cm. This number is 2.8 times the value predicted from the atomic polarizability measurements of Scheffers and Stark if a simplified theory which neglects hyperfine-structure perturbations of the ground-state wave functions is used.

AS a continuation of the high-precision atomic hyperfine-structure measurements made in this laboratory in recent years, the effect of an electric field on the hyperfine-structure energy levels of Cs¹³³ has been examined.

The increase $\Delta(\mathcal{E})$ of the

$$(F=4, m_F=0) \leftrightarrow (F=3, m_F=0)$$

transition frequency (for the ground state of Cs¹³³) has been measured in electric fields between zero and 6.7×10^4 volts/cm by the atomic-beam magnetic-resonance technique. The transition frequency for zero electric and magnetic fields is $9\,192\,631\,830 \pm 10$ cps. The maximum observed shift of this transition frequency caused by the electric field was approximately 4×10^3 cps. The line widths for the resonance curves obtained in these measurements were approximately 120 cps.

The ($4, 0$) \leftrightarrow ($3, 0$) transition was induced by using the Ramsey two-transition-region method,² with two microwave cavities separated by a distance $L_{rf} = 78.4$ cm along the beam. The electric field was obtained with a parallel-plate condenser of length $L_s = 66.0$ cm, located between the two cavities. A crystal-stabilized klystron was used as the microwave source.

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¹ L. Essen and J. V. L. Parry, *Nature* **176**, 280 (1955).

² N. F. Ramsey, *Phys. Rev.* **78**, 695 (1950).

Figure 1 is a typical record of the data. With the frequency increasing linearly (toward the right in the figure) a resonance curve was first traced out with the electric field $\mathcal{E} = 3.48 \times 10^4$ volts/cm; then, without changing either the rate or direction of the frequency variation, the electric field was turned off and a resonance curve for $\mathcal{E} = 0$ was traced out. The upper horizontal straight line on the record indicates that the electric field is on; the lower line, that the field is off. The markers on the lower margin of the data record were obtained from a "frequency indexer" which put out a pulse each time the frequency changed by 60 cps.

The observed shift $\bar{\Delta}(\mathcal{E})$ of the resonance frequency can be obtained directly from the recorded data by counting the number of 60-cps markers between the two resonance peaks. For the data shown, $\bar{\Delta}(\mathcal{E}) = 2260$ cps.

Two corrections must be applied to $\bar{\Delta}(\mathcal{E})$ before obtaining the change $\Delta(\mathcal{E})$ of the atomic energy level separation caused by the electric field:

(1) The observed shift $\bar{\Delta}(\mathcal{E})$ of the resonance frequency must be increased by a small factor because the resonance pattern will be asymmetrical when the average energy level separation of an atom in the region between the two transition cavities is not equal to the energy level separation in the cavities. Detailed line-shape calculations (which have been compared with the experimentally observed resonance shapes) indicate that this asymmetry decreases the apparent resonance frequency by $\bar{\Delta}(\mathcal{E})(l_{rf}/L_{rf}) + K(B_{rf})$, where l_{rf} is the length of each cavity (1.27 cm), and $K(B_{rf})$ is a correction factor which depends on the magnitude of the

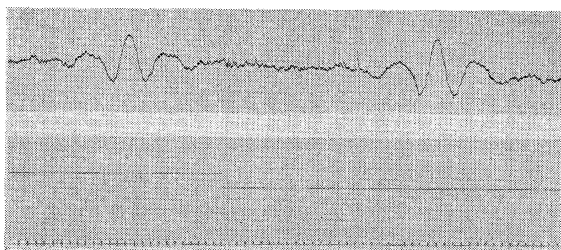


FIG. 1. Typical recording of data. $\mathcal{E}=3.48 \times 10^4$ volts/cm. The frequency was increasing approximately linearly from left to right. The evenly spaced marks on the lower margin occur every 60 cps.

microwave power used to induce the transitions. Both parts of this correction are of approximately the same magnitude for the power levels used in this experiment. The total asymmetry correction amounts to about 2% of $\bar{\Delta}(\mathcal{E})$.

(2) The observed shift, corrected for asymmetry, must be multiplied by L_{rt}/L_s because the Ramsey two-transition-region method yields resonances at the frequency that corresponds to the average energy level separation in the region between the two transition regions.

In Fig. 2 the observed shift corrected for asymmetry is plotted as a function of the square of the voltage applied to the Stark field electrodes. The solid line in this figure was found by computing the weighted mean ratio of the corrected shift to the square of the applied voltage, the ratio for each value of voltage being weighted with the reciprocal square of the error.

When the correction factors are applied to $\bar{\Delta}(\mathcal{E})$, the value for the shift of the $(4, 0) \leftrightarrow (3, 0)$ energy level separation is

$$\Delta(\mathcal{E}) = -2.29 \times 10^{-6} (1 \pm 0.03) \mathcal{E}^2 \text{ cps,}$$

where \mathcal{E} is in volts/cm. The principal sources of the quoted error are the uncertainty in the distance between the Stark field electrodes and the inhomogeneity of the electric field. The other experimental errors and the asymmetry correction errors are believed to contribute only slightly to this error. Approximately 200 observations (about ten at each value of \mathcal{E}) were used in obtaining this value of $\Delta(\mathcal{E})$.

The dependence of $\bar{\Delta}(\mathcal{E})$ on magnetic field was examined over a range of approximately 4 gauss, with the electric field at 3.14×10^4 volts/cm. The observed frequency shift did not vary by more than 1% over this range of magnetic field. The observations used to obtain the value of $\Delta(\mathcal{E})$ given above were all taken in fields of less than 0.5 gauss.

The following order-of-magnitude calculation, is perhaps helpful in understanding the Stark effect on hyperfine structure. It was suggested by Schwartz³ that this

³ C. Schwartz (private communication).

simple calculation might be a reasonable first approximation.

Let us consider the electric field interaction as a perturbation on hyperfine structure. If e is the electronic charge, \mathcal{E} is the electric field, and z is the component along \mathcal{E} of the vector connecting the center of the nucleus with the electron, then the electric field interaction is given by $ez\mathcal{E}$. Since this is an odd-parity operator, and since the atomic ground state has well-defined parity, the diagonal matrix elements of the electric field perturbation are zero; i.e., the atom has no permanent dipole moment.

Proceeding to second order we obtain a nonzero energy shift Δ_F for each of the F levels:

$$\Delta_F = \sum_{n', F'} \frac{|\langle n', L'=1, F' | ez\mathcal{E} | n=6, L=0, F \rangle|^2}{W^0(6, 0, F) - W^0(n', 1, F')}.$$

Now $W^0(6, 0, 4) = W^0(6, 0, 3) + h\Delta\nu$, where $h\Delta\nu$ is the zero-field separation of the two F levels in Cs^{133} (the "hyperfine-structure separation"). Since $h\Delta\nu \ll W^0(n, 1) - W^0(6, 0)$, we can rewrite the shift of the upper level ($F=4$) as

$$\Delta_4 = - \sum_{n', F'} \frac{|\langle n', 1, F | ez\mathcal{E} | 6, 0, F \rangle|^2}{W^0(n', 1, F') - W^0(6, 0, 3)} \times \left(1 + \frac{h\Delta\nu}{W^0(n', 1, F') - W^0(6, 0, 3)} \right).$$

The shift of the lower level ($F=3$) is

$$\Delta_3 = - \sum_{n', F'} \frac{|\langle n', 1, F' | ez\mathcal{E} | 6, 0, F \rangle|^2}{W^0(n', 1, F') - W^0(6, 0, 3)}.$$

The shift of the atomic hyperfine-structure energy level separation attributable to the electric field is given by $\Delta(\mathcal{E}) = \Delta_4 - \Delta_3$. The mixing of all states except the $6p$ state can be neglected. As a first approximation let us neglect the sum over the F levels in the $6p$ state and let us also assume that the $F=4$ and the $F=3$ wave functions are the same. We then obtain

$$\Delta(\mathcal{E}) \doteq - \frac{|\langle 6, 1 | ez\mathcal{E} | 6, 0 \rangle|^2}{[W^0(6, 1) - W^0(6, 0)]^2} h\Delta\nu.$$

The usual atomic polarizability calculation⁴ gives the energy of a cesium atom with polarizability α to be

$$\frac{1}{2} \alpha \mathcal{E}^2 = \frac{|\langle 6, 1 | ez\mathcal{E} | 6, 0 \rangle|^2}{W^0(6, 1) - W^0(6, 0)}.$$

Scheffers and Stark⁵ measured the polarizability of cesium by electrostatic deflection of a beam of cesium

⁴ J. C. Slater, *Quantum Theory of Matter* (McGraw-Hill Book Company, Inc., New York, 1951), p. 388.

⁵ H. Scheffers and J. Stark, *Physik. Z.* **35**, 625 (1934).

atoms. They obtained $\alpha = (42 \pm 2) \times 10^{-24} \text{ cm}^3$. Inserting this value in the preceding calculation, we obtain

$$\begin{aligned} \Delta(\mathcal{E}) &\doteq -\frac{1}{2}\alpha\mathcal{E}^2 \frac{\hbar\Delta\nu}{W^0(6, 1) - W^0(6, 0)} \\ &\doteq -0.82 \times 10^{-6} \mathcal{E}^2 \text{ cps.} \end{aligned}$$

We therefore see that the order-of-magnitude calculation predicts the sign and the quadratic field dependence of the Stark effect on hyperfine structure, but the magnitude of the experimentally observed effect is 2.8 times the value predicted by this calculation.

The disagreement between theory and experiment is possibly the result of having neglected the fact that the $F=4$ and the $F=3$ wave functions differ by quantities of order $\hbar\Delta\nu/[W^0(6, 1) - W^0(6, 0)]$ in the order-of-magnitude calculation.

Schwartz³ has recently attempted a complete calculation of the Stark effect on Cs¹³³ hyperfine structure, including these wave-function differences; but, since values for some of the more critical matrix elements are extremely difficult to obtain, this attempt was unsuccessful.

However, Schwartz³ has succeeded in completing the same calculation for the Stark effect on hyperfine structure in the $1s$ state of hydrogen. He finds

$$\Delta(\mathcal{E}) = -\frac{1193 a_0^4}{80 e^2} \mathcal{E}^2 \Delta\nu,$$

where a_0 is the radius of the first Bohr orbit. Only 36% of this result comes from the difference of the energy denominators due to the hyperfine structure of the ground state (i.e., from a calculation exactly similar to the order-of-magnitude calculation outlined in the preceding portions of this paper), while 65.5% comes from

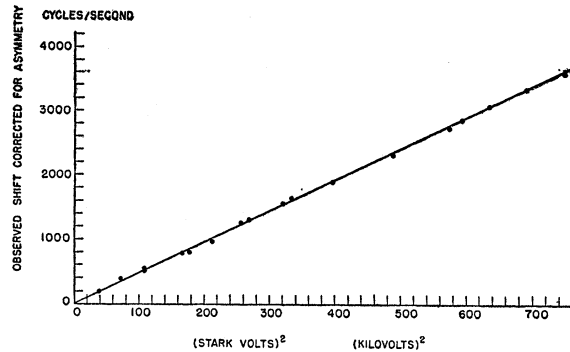


FIG. 2. Corrected observed shift of the $(4, 0) \leftrightarrow (3, 0)$ transition frequency as a function of the square of the voltage applied to the Stark electrodes.

the difference of the ground-state wave functions, and a negative contribution of 1.5% comes from the hyperfine structure of the p states. The complete theory of the Stark effect on hydrogen hyperfine structure is therefore seen to give a result which is 2.8 times the value predicted by the order-of-magnitude calculation which neglects wave-function differences due to hyperfine structure.

Consequently, if one extrapolates from hydrogen to cesium, and assumes that the complete calculation for the hydrogen effect is correct, the experimentally observed value of $\Delta(\mathcal{E})$ for cesium is seen to be in good agreement with theory. However, since such an extrapolation is of doubtful validity, the agreement should be considered as only approximate.

A more complete description of this experiment will appear soon.

The authors wish to thank J. G. King and I. I. Rabi for many helpful discussions and suggestions, and C. Schwartz for making available the results of his calculations before publication.

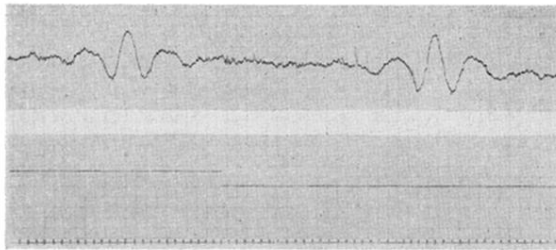


FIG. 1. Typical recording of data. $\mathcal{E}=3.48 \times 10^4$ volts/cm. The frequency was increasing approximately linearly from left to right. The evenly spaced marks on the lower margin occur every 60 cps.