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Radiative Transition Widths in the $1p$ Shell*

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The nuclear wave functions, obtained from a shell model with variable strength of spin-orbit coupling, are used to compute $M1$ and $E2$ transition widths. Comparison with experiment is made for Be^8, B^{10}, B^{11}, and C^{12}. The agreement is not nearly so good as was that obtained for energy level schemes. The pure $M1$ transitions are in good agreement with experiment. The values computed for $E2$ transition strengths are found to be generally low, though about the right order of magnitude. This suggests the need for adding some collective behavior to the model.

I. INTRODUCTION

In an earlier paper,[1] the energy levels and electromagnetic moments of nuclei in the $1p$-shell were studied as a function of the strength of spin-orbit coupling relative to central-force nucleon interaction. The conclusion drawn from comparison of this calculation with experiment was that an intermediate-coupling picture, with the strength of spin-orbit coupling increasing as the shell is filled, gives an encouragingly good representation of the experimental data for the nuclei between He^4 and O^{16}. However, the degree of agreement varied considerably, so further tests seem desirable to seek ways of improving the model. The radiative transition width is a quantity which offers a good test since it is often more sensitive to nuclear wave functions than are the energy levels, and because there is a reasonable amount of experimental data for comparison.

The matrix elements for the $M1$ and $E2$ transitions in the shell have been computed by using the wave functions obtained previously.[1] These were obtained by diagonalizing the energy matrices for a one-particle spin-orbit term and a central nucleon-nucleon interaction with an exchange mixture of 80% space exchange and 20% spin exchange. The ratio of the central integrals, L/K, was kept at a value of 6.8 since changing this ratio to 5.8 did not seriously affect the results. For a few cases the effect of changing L/K is given in the discussion. The parameter a/K which measures the relative strength of spin-orbit and central energies was varied over the range for which the energy-level schemes are reasonable.

II. TRANSITION STRENGTHS

The expressions for the $M1$ and $E2$ transition widths have been put in a convenient form by Lane and Radicati.[2] For $M1$,

$$\Gamma(M1)=2.76\times10^{-3}E^3\Lambda(M1), \tag{1}$$

where Γ is in ev, E in Mev, and

$$\Lambda(M1)=\left(\frac{2J_f+1}{2J_i+1}\right)\frac{|\langle J_fm|\mu|J_im\rangle|^2}{(J_i1m0|J_fm)^2}. \tag{2}$$

The dimensionless quantity defined in Eq. (2) contains the square of the nuclear matrix element in units of nuclear magnetons, between the initial and final states, of the usual magnetic moment operator summed over all nucleons. The denominator involves the square of a vector addition coefficient.

The $E2$ transitions can be expressed in a similar fashion as

$$\Gamma(E2)=8.02\times10^{-6}E^5\Lambda(E2), \tag{3}$$

where Γ is in ev, E is in Mev, and

$$\Lambda(E2)=\left(\frac{2J_f+1}{2J_i+1}\right)\frac{|\langle J_fm|Q/e|J_im\rangle|^2}{(J_i2m0|J_fm)^2}. \tag{4}$$

Q/e is the electric quadrupole operator summed over all protons:

$$Q/e=\textstyle\sum_p(3z_p^2-r_p^2).$$

The matrix element in Λ is in units of $\langle r^2\rangle$. The value of $\langle r^2\rangle$ is somewhat uncertain, but to give a reasonable order of magnitude in this calculation it is assumed to be:

$$\langle r^2\rangle=10^{-25}\ \text{cm}^2.$$

The Λ's, which depend on the size of the nuclear matrix element without the complication of the energy factor in Γ, have been appropriately named "transition strengths" by Lane.[3]

Table I lists the transition strengths, Λ, for those transitions in the shell for which the dominant mode of decay is expected to be $M1$, and for which some experimental evidence is known. They are given as functions of the ratio a/K which indicates the strength of spin-orbit coupling. It is evident that the values of $\Lambda(M1)$ are fairly evenly distributed throughout a range whose limits differ by a factor of about a thousand. This is the sort of spread which Wilkinson[4] has found from the

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] D. Kurath, Phys. Rev. **101**, 216 (1956).
[2] A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954).
[3] A. M. Lane, Proc. Phys. Soc. (London) **A68**, 189, 197 (1955).
[4] D. H. Wilkinson, Phil. Mag. **1**, 127 (1956).

TABLE I. Transition strengths, Λ, for $M1$ transitions as functions of the relative strength of spin-orbit coupling, a/K. The parent level and its daughters are identified by (JT) in the first column.[a]

$(JT)_i \rightarrow (JT)_f$	a/K 0	1.5	3.0	4.5	6.0	7.5
$A=8$						
(11)→(00)	0	0.63	1.25	1.75	2.20	2.59
(20)	0	0.16	0.39	0.49	0.52	0.50
$A=12$						
(11)→(00)	0	0.43	1.45	3.32	5.74	7.96
(20)	0	0.00	0.08	0.30	0.66	1.08
(21)→(20)	0	0.31	1.00	1.70	2.50	3.30
$A=10$						
(21)→(30)	...	9.26	↔[b] 1.19	0.09	↔ 0.15	0.90
(10)	...	0.06	9.96	13.59	14.31	14.67
(10)*	...	8.44	3.03	0.26	0.18	0.20
(20)	...	15.19	7.04	4.63	3.52	2.82
(20)→(30)	...	0.005	0.044	0.062	0.069	0.072
(10)	...	0.000	0.052	0.060	0.051	0.044
(10)*	...	0.115	0.032	0.008	0.007	0.008
(10)*→(01)	...	0.02	↔ 9.22	13.27	12.27	11.31
(10)	...	0.001	0.135	0.046	0.039	0.038
(01)→(10)	...	62.8	↔ 29.7	14.10	15.70	18.40
$A=11$ (A11 $T=\frac{1}{2}$)						
5/2*→3/2	0.00	0.17	0.19	0.64	1.92	2.42
5/2	0.00	1.24	1.78	0.85	0.17	0.53
7/2	15.38	12.70	11.03	8.80	0.65	0.04
3/2*	0.05	0.60	0.60	0.17	0.02	0.02
5/2**→3/2	0.02	0.03	0.32	0.59	0.00	0.02
5/2	0.00	0.11	0.43	1.27	1.58	0.93
7/2	0.00	0.00	0.09	1.88	9.69	9.92
3/2*	0.04	0.08	0.20	0.32	0.17	0.03
3/2*→3/2	0.36	1.46	4.59	5.51	5.63	5.57
1/2	0.00	1.14	2.75	3.02	2.82	2.68
5/2	13.83	14.53	12.78	11.81	11.50	11.43
5/2→3/2	0.90	↔ 0.05	0.97	1.54	1.69	1.76
7/2	0.08	0.10	0.07	0.00	0.11	0.37
1/2→3/2	16.59	14.33	10.12	7.58	6.03	5.02

[a] A state labeled with an asterisk refers to the second lowest state in energy of the specified J and T. A state labeled with a double asterisk refers to the third lowest state in energy of the specified J and T.
[b] The symbol ↔ means that the matrix element changes sign between these entries.

experimentally observed $M1$ transitions in this region, although there are only about ten transitions that are common to his compilation and to Table I. As one would expect, the behavior as a function of a/K is also varied, depending on whether the transition is strongly favored or unfavored at the LS or jj limits. There are also cases of erratic behavior, which can usually be explained by the fact that one of the levels in the transition belongs to a pair of nearly degenerate levels of the same spin, so that at some value of a/K the wave function is perturbed violently. Finally, there are a few transitions that are quite insensitive to variation of a/K over wide ranges. The fairly strong $M1$ transitions, calculated for the three-nucleon configurations found at mass numbers 7 and 13, by Lane[3] and Radicati,[2] are other examples of relatively insensitive behavior.

The $E2$ transition strengths, Λ, are listed in Table II as functions of a/K. The variation in magnitudes is not quite so great as for the $M1$ transitions, but the distributions is again quite flat. The relative importance of the $E2$ and $M1$ transition modes when both are possible is given by Eqs. (1) and (3). It is more readily found from Fig. 1, in which the ratio $\Lambda(E2)/\Lambda(M1)$ is presented as a function of the energy for various values of the ratio of widths, $\Gamma(E2)/\Gamma(M1)$. For a given ratio of transition strengths, one can then easily obtain a rough idea of the mixing ratio knowing the energy. Of the 25 transitions common to Tables I and II, only 5 have $E2$ contributions that are more than a few percent of the $M1$ contributions. They occur either because the $M1$ strength is weak or because of a combination of a strong $E2$ and large energy. These cases will be treated in the discussion of particular transitions.

In some cases the experimental information comes from a lifetime measurement rather than a width, to which it is related by $\Gamma\tau=\hbar$, or

$$\Gamma\tau=6.58\times10^{-16},$$

where Γ is in ev and τ in seconds.

III. PARTICULAR CASES

In making comparisons with experiment one finds that the amount of experimental information varies considerably, reflecting the degree of difficulty of the experiment. The chief sources of information in transition widths are analyses of resonances in reactions, a few lifetime measurements and often only relative widths as indicated by branching ratios in gamma decay. One is rarely able to obtain experimental values of Λ for each single mode of transition, since this requires the branching ratios and the relative strengths of the $E2$ and M1 modes as well as the total width. Therefore, in making comparisons with experiment, the Λ's of Tables I and II are used together with the experimental values for the energy to determine either the transition width, Γ, or the branching ratio:

$$B_x=\Gamma_x/\Gamma_T,$$

where Γ_T is the total width for all radiative decay branches of a given nuclear level.

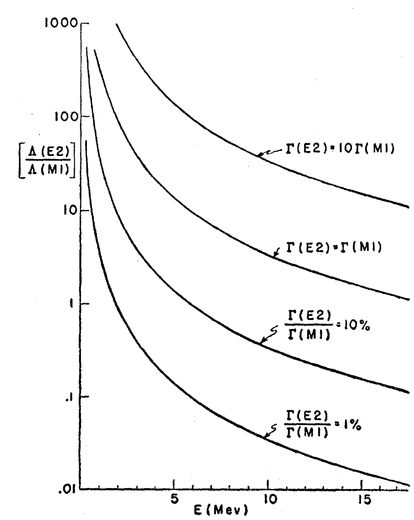

FIG. 1. Curves of relative intensities, $\Gamma(E2)/\Gamma(M1)$, as functions of energy and relative transition strengths, $\Lambda(E2)/\Lambda(M1)$.

TABLE II. Transition strengths, Λ, for $E2$ transitions as functions of the relative strength of spin-orbit coupling, a/K. The parent level and its daughters are identified by (JT) in the first column.[a]

	a/K					
$(JT)_i \to (JT)_f$	0	1.5	3.0	4.5	6.0	7.5
$A=8$						
$(11)\to(20)$	0	0.235	0.362	0.371	0.371	0.360
$A=12$						
$(11)\to(20)$	0	0.046	0.048	0.027	0.010	0.002
$(21)\to(00)$	0	0.004	0.095	0.151	0.205	0.245
(20)	0	0.000	0.048	0.069	0.087	0.102
$(20)\to(00)$	0.895	0.884	0.848	0.778	0.684	0.595
$A=10$						
$(21)\to(30)$	···	0.000	0.005	0.037	0.090	0.149
(10)	···	0.007	0.026	0.066	0.088	0.105
$(10)^*$	···	0.000	0.017	0.005	0.003	0.002
(20)	···	0.001	0.000	0.003	0.006	0.009
$(20)\to(30)$	···	0.032	0.078	0.116	0.140	0.155
(10)	···	0.286	0.152	0.392	0.381	0.350
$(10)^*$	···	0.611	0.612	0.289	0.238	0.219
$(10)^*\to(30)$	···	0.092	0.638	0.503	0.451	0.428
(10)	···	0.510	0.016↔[b]	0.146	0.170	0.165
$(10)\to(30)$	···	0.685↔	0.001 ↔	0.126	0.184	0.212
$A=11$						
$(A11\ T=\frac{1}{2})$						
$\frac{5}{2}^*\to\frac{3}{2}$	0.053	0.155	0.124	0.053↔	0.003	0.016
$\frac{1}{2}$	0.146	0.072	0.056	0.060	0.018	0.010
$\frac{5}{2}$	0.008	0.108	0.123	0.066	0.002	0.000
$\frac{7}{2}$	0.225	0.014↔	0.014	0.099	0.110	0.091
$\frac{3}{2}^*$	0.019	0.077	0.170	0.178	0.019	0.003
$\frac{5}{2}^{**}\to\frac{3}{2}$	0.000	0.001	0.012	0.050	0.075	0.048
$\frac{1}{2}$	0.000	0.000	0.000	0.003	0.060	0.082
$\frac{5}{2}$	0.000	0.007	0.013	0.029	0.056	0.035
$\frac{7}{2}$	0.000	0.016	0.033	0.012↔	0.065	0.135
$\frac{3}{2}^*$	0.001	0.005	0.000 ↔	0.027	0.191	0.202
$\frac{3}{2}^*\to\frac{3}{2}$	0.981	0.615	0.184	0.092	0.073	0.065
$\frac{1}{2}$	0.826	0.905	0.802	0.706	0.637	0.579
$\frac{5}{2}$	0.155	0.004	0.106	0.129	0.096	0.059
$\frac{7}{2}$	0.042	0.080	0.094	0.149	0.209	0.250
$\frac{7}{2}\to\frac{3}{2}$	0.175	0.180	0.167	0.120	0.078	0.050
$\frac{5}{2}\to\frac{3}{2}$	1.365	1.320	1.180	1.120	1.070	1.040
$\frac{1}{2}$	0.416	0.515	0.551	0.558	0.553	0.544
$\frac{7}{2}$	0.047	0.004↔	0.000	0.004	0.016	0.032
$\frac{1}{2}\to\frac{3}{2}$	0.237	0.000	0.118	0.213	0.259	0.289

[a] A state labeled with an asterisk refers to the second lowest state in energy of the specified J and T. A state labeled with a double asterisk refers to the third lowest state in energy of the specified J and T.
[b] The symbol ↔ means that the matrix element changes sign between these entries.

Be^8

The level at 17.63 Mev, reached by the $Li^7(p,\gamma)Be^8$ reaction, decays both to the ground state and to the first excited state at 2.90 Mev. Since the 17.6-Mev level is believed to be the $(J=1^+,\ T=1)$ state, the statistical factor $\omega=\frac{3}{8}$ means that the width, taken from Ajzenberg and Lauritsen,[5] is: $\Gamma_T=25.1$ ev. The branching ratio varies with energy,[6] but right at resonance the greater energy is favored by about a 2 to 1 ratio. The ground state transition to $J=0^+$ is pure $M1$, while the other branch is a mixture of $M1$ and $E2$ since the first excited state is $J=2^+$. No experimental information exists on the mixing ratio of $M1$ and $E2$.

The total width and the separate contributions of the two transitions are plotted as functions of a/K in Fig. 2. The curve of total width crosses the experimental line at $a/K\sim 3$. (For $L/K=5.8$ this crossing is at $a/K\sim 2.5$.) Either of these values is in satisfactory agreement with the a/K value determined by matching the energy level scheme.[1] The experimental widths for each gamma ray, under the assumption that the 2 to 1 branching ratio is correct, are given by the broken lines of Fig. 2. These do not agree with the individual computed widths for $a/K\sim 3$, particularly in the case of the 14.7-Mev transition.

There are two possible explanations of the discrepancy. One is that the 14.7-Mev transition observed at the resonance is not all due to the $J=1^+$ level. There is an indication of this in the fact that off resonance the 14.7-Mev transition becomes much stronger with respect to the 17.6-Mev transition. This explanation implies that the width of the 17.6-Mev level is the only meaningful one for comparison with experiment in Fig. 2.

The other possibility is that the computed $E2$ contribution is not sufficient. The $E2$ contribution to the 14.7-Mev transition is already quite large because of the high energy, being about 40% at $a/K=3$.

If it is necessary to include collective effects in the theory, the $E2$ contribution would be further enchanced, which might bring about agreement with observation.

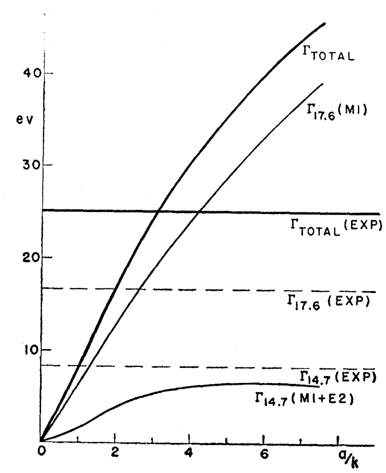

FIG. 2. Transition widths for the decay of the $(J=1^+,\ T=1)$ state at 17.6 Mev in Be^8, both to the ground state and to the excited state at 2.9 Mev. Curves are theoretical Γ's as functions of the relative spin-orbit strength, a/K. Straight lines are experimental values.

[5] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[6] M. B. Stearns and B. D. McDaniel, Phys. Rev. **82**, 450 (1951); J. G. Campbell, Australian J. Phys. **9**, 156 (1956).

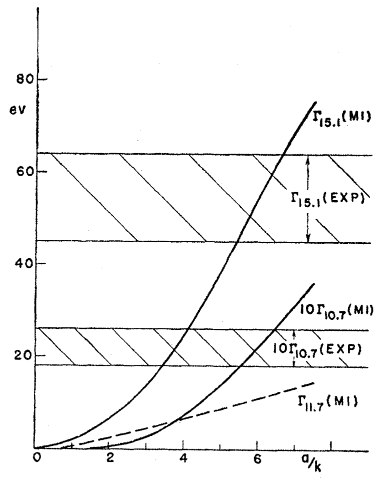

FIG. 3. Transition widths for the decay of the $(J=1^+,\ T=1)$ state at 15.1 Mev in C^{12}, both to the ground state and the excited§ state at 4.4 Mev. Curves are theoretical Γ's as functions of the relative spin-orbit strength, a/K. Straight lines are experimental values. Dotted curve is for the decay of the $(J=2^+,\ T=1)$ state at 16.1 Mev to the 4.4-Mev level.

In this respect it would be of great interest to know the experimental $E2/M1$ ratio for the 14.7-Mev transition.

At any rate the pure $M1$ transition to the ground state, which provides the bulk of the width, seems to require[7] an a/K value consistent with that which is deduced from matching the observed and experimental level schemes.

C^{12}

In C^{12}, the decay of states for which $T=1$ presents a situation very similar to that which exists in Be^8.[8] The case of the $(J=1^+,\ T=1)$ state analogous to the level discussed in Be^8, has been studied, and it is found that the width of the ground state transition is

$$\Gamma_{15.1}=54.5\pm9.3 \text{ ev}.$$

The intensity of the branch[9] to the first excited state§ $(J=2^+)$ is 4% of that to the ground state. Because of the spins involved, the transition to the ground state is a pure $M1$, while for $a/K>4$, the transition to the excited state is computed to be more than 95% $M1$. Therefore only the computed $M1$ widths are plotted in Fig. 3. The theoretical and experimental results agree when a/K lies between 5.5 and 6.5, a quite reasonable size for this mass number.

The $B^{11}(p,\gamma)C^{12}$ reaction produces a pair of high-energy transitions from the $(J=2^+,\ T=1)$ level at 16.1 Mev. Here the experimental results in Ajzenberg and Lauritsen[5] state that the pure $E2$ transition to the ground state has a width of less than 3 ev, while the $(M1+E2)$ transition to the first excited state is given a width of 70 ev. The computed width of the ground-state transition rises from 0 at $a/K=0$ to 1 ev at $a/K=4$ and 2 ev at $a/K=7.5$, so this is consistent with observation. The 11.7-Mev transition to the first excited state $(J=2^+)$ is computed to be about 98% $M1$, so only the $M1$ width is plotted as the dashed curve of Fig. 3. Its value is much lower than the experimental width, being only some 11 ev at $a/K=6$ as compared with the experimental width of 70 ev. While the experimental value is subject to considerable uncertainty, it will take a large change to remove the discrepancy.

The $E2$ transition to the ground state from the 4.43-Mev state $(J=2^+)$ has been investigated by fast electron scattering,[10] and found to have the width

$$\Gamma=0.0125\pm0.0025 \text{ ev}.$$

This value has been confirmed by a measurement using self-absorption of resonance radiation.[11] The computed width goes smoothly from $\Gamma=0.012$ ev at $a/K=0$ to $\Gamma=0.008$ ev at $a/K=7.5$. This width has the right order of magnitude, but is really somewhat too low since the value of $\langle r^2\rangle$ chosen for the $E2$ matrix elements is more likely to be too large than too small, as discussed in Sec. IV.

B^{11}, C^{11}

The experimental information about the mirror nuclei B^{11} and C^{11} is not sufficient to enable one to make definite spin assignments for the excited states, so it would seem that the computed transition probabilities might be useful in choosing among alternative possibilities. The experimental information on transition widths comes almost entirely from the resonances near

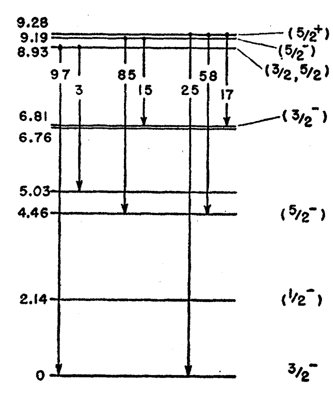

FIG. 4. Branching ratios from the $Li^7(\alpha,\gamma)B^{11}$ resonances. with suggested spins on right. (After Ajzenberg and Lauritsen).

[7] A similar result for the ground state transition has been found by J. B. French and A. Fujii, Phys. Rev. **105**, 652 (1957).

[8] E. G. Fuller and E. Hayward (to be published); Physica **22**, 1138 (1956).

[9] C. N. Waddell, quoted in reference 8.

§ *Note added in proof.*—Waddell, Adelson, Moyer, and Shaw, Bull. Am. Phys. Soc. Ser. II, **2**, 181 (1957) give a revised branching ratio of ~10%. Other sources suggest ~3% so the experimental value for the 10.7-Mev branch is still unsettled.

[10] J. H. Fregeau [Phys. Rev. **104**, 225 (1956)] quotes Ravenhall and Helm for this result on p. 235.

[11] Swann, Metzger, and Rasmussen, Bull. Am. Phys. Soc. Ser. II, **2**, 29 (1957).

9 Mev in the reaction $Li^7(\alpha,\gamma)B^{11}$. The branching ratios for these transitions, taken from Ajzenberg and Lauritsen,[5] are given in Fig. 4. Information on γ-γ and α-γ correlations leads to the tentative spin assignments of the higher excited states. Measurements based on the Doppler shift[12] give limits of $\tau<4\times10^{-14}$ sec for decay of both the first and second excited states to the ground state. This shows both to be predominantly $M1$ transitions and suggests spin assignments. The deutron-stripping experiments[13,14] show that at least one of the two states around 6.8 Mev and all the states below this have negative parity.

The theoretical spin assignments[1] for negative-parity states are, in order of increasing excitation energy: $\frac{3}{2}$, $\frac{1}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$, and $\frac{3}{2}$ for one of the states at 6.8 Mev. In addition there is a $\frac{5}{2}^-$ state somewhere around 11 Mev. Since the experimental resonances near 9 Mev suggest $\frac{5}{2}^-$ as one possible spin, one would like to associate the theoretical $\frac{5}{2}^-$ state with one of these resonances. However, the fact that the excitation is so high means that the wave function is not reliable,[15] since the calculation neglects contributions from possible states arising by excitation of two nucleons from the $1p$ shell or double excitation of one from the $1s$ shell. The effect of interaction with such states of double excitation would be to push down the $\frac{5}{2}^-$ level which was computed to lie too high and mix with it. Admixtures of these states would, however, only serve to depress the absolute transition width provided the lower energy states contain negligible admixtures from double excitation. This is because the gamma transition operator is a one-particle operator. There is another complication for the $\frac{5}{2}^-$ state in that the computed level has a partner which comes within 1 Mev of it between $a/K=4$ and 6, the region in which one would hope for agreement with experiment. This manifests itself in the rather drastic changes in transition strengths in Table I as one goes through this region. Another consequence is that the wave function is much more sensitive to variation of L/K, and to the exact form of the two-body interaction.

Nevertheless, the computed widths for transitions from the second and third $\frac{5}{2}^-$ states, indicated by $\frac{5}{2}$* and $\frac{5}{2}$** in Table I, were evaluated. The computations show all the transitions to be almost entirely $M1$. The experimental energies are used to obtain the Γ's. The results indicate that the only important transitions from $\frac{5}{2}$* and $\frac{5}{2}$** are those to the ground state $\frac{3}{2}$ and to the $\frac{5}{2}$ and $\frac{7}{2}$ pair at 4.46 and 5.03 Mev above[16] the ground state. The computed widths of the transitions to the $\frac{1}{2}$ state at 2.14 Mev and the $\frac{3}{2}$* state at 6.8-Mev account for less than 1% of the total width for all values of a/K. The branching ratios computed for the strong transitions from the $\frac{5}{2}$* state are plotted in Fig. 5. This scheme has similarities to two of the experimental $(\alpha\gamma)$ resonances, but there are always discrepancies so that there is probably no meaningful correlation present.

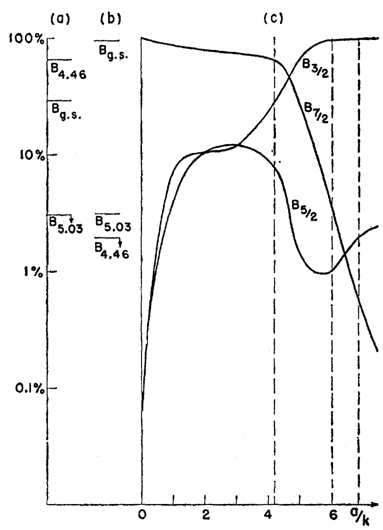

FIG. 5. Branching ratios computed for decay of the $\frac{5}{2}$* state in B^{11} as function of a/K. (a) Experimental branching for 9.28-Mev state omitting branch to 6.8 Mev. (b) Experimental branching for 8.93-Mev state. (c) Theoretical branching with similarities to experiment noted by dotted lines.

One possibility of agreement occurs near $a/K=4$ where the branching resembles that of the 9.28-Mev resonance of Fig. 4 if one omits the branch to the 6.8-Mev doublet. To justify this omission one must assume that this upper branch is an $E1$ transition to a state having positive parity, something which is not included in the computation. The adjusted branching ratios for the 9.28-Mev resonance are given in column (a) of Fig. 5. The observed total width,[17] excluding the branch to the state at 6.8 Mev, is some 2.7 ev compared to the computed total of 3.8 ev. In view of the reduction of the theoretical value that would be produced by mixing in two-nucleon excitation, this is reasonable agreement. The difficulties with this scheme are, first, that since it identifies the 4.46-Mev state as $\frac{7}{2}^-$, the computed $E2$ lifetime of $\tau\sim3\times10^{-13}$ sec for this state is too long for the observed upper limit of 4×10^{-14} sec; and second, that the identification of the 6.8 level as being a positive-parity state runs into difficulties with the α-γ correlation[18] measurements.

Owing to the considerable change in the wave

[12] G. A. Jones and D. H. Wilkinson, Phil. Mag. **43**, 958 (1952); D. H. Wilkinson, Brookhaven National Laboratory Report BNL-2907, 1956 (unpublished).

[13] N. T. S. Evans and W. C. Parkinson, Proc. Phys. Soc. (London) **A67**, 684 (1954).

[14] Maslin, Calvert, and Jaffe, Proc. Phys. Soc. (London) **A69**, 754 (1956).

[15] This objection does not arise for the states of high excitation discussed for Be^8 and C^{12} because these states have $T=1$. Double-excitation states having $T=1$ lie much higher.

[16] Since the order of the $\frac{5}{2}$ and $\frac{7}{2}$ states is unknown, an average gamma energy of 4.5 Mev has been used in computing both Γ's.

[17] Bennett, Roys, and Toppel, Phys. Rev. **82**, 20 (1951); also reference 4 wherein the total radiation widths quoted in Ajzenberg and Lauritsen are corrected.

[18] G. A. Jones and D. H. Wilkinson, Phys. Rev. **88**, 423 (1952).

function of the $\frac{5}{2}^*$ state near $a/K=5$, where the $\frac{5}{2}^{**}$ state has very nearly the same energy, there is another region of a/K that fits the branching of the state at 8.93 Mev. Between $a/K=6$ and 7 the transitions from $\frac{5}{2}^*$ go almost entirely to the ground state, and comparison with the observed branching in column (b) of Fig. 5 allows either the $\frac{5}{2}$ or the $\frac{7}{2}$ state to be identified as the 5.03-Mev level. Furthermore, changing L/K from 6.8 to 5.8 shifts the whole pattern to the left in Fig. 5 so that agreement is obtained for a/K between 5 and 6. However, there is a serious drawback to this identification of the $\frac{5}{2}^*$ state with the 8.93-Mev level, namely that the total width is computed to be some 4.5 ev whereas the observed level width of the 8.93-Mev state is 0.04 ev. Therefore, both possibilities of agreement with experiment present in Fig. 5 contain defects which cast serious doubts on the apparent similarities. This result is probably due to the difficulties in obtaining a reliable wave function for a state of such high excitation as the $\frac{5}{2}^*$.

Among the lower-lying excited states where the wave functions are presumably more trustworthy, the opportunities for experimental investigation of the transition widths are quite limited. The lifetime for the decay of the first excited state is computed to vary smoothly from 2.5×10^{-15} to 5×10^{-15} sec as a/K increases from 3 to 7.5, which is comfortably below the experimental upper limit of 4×10^{-14} sec. The computed lifetime for the transition to the ground state from the $\frac{7}{2}$ state is $\tau\sim3\times10^{-13}$ sec, while from the $\frac{5}{2}$ state, for which the $E2$ contribution lies between 5% and 10%, one computes $\tau\sim1.5\times10^{-15}$ sec. A recent measurement[11] of the lifetime of the 4.46-Mev level obtains $\tau=(1.0\pm0.2)\times10^{-15}$ sec, so that this level seems to be the $\frac{5}{2}$. Probably the best chance for a comparison lies in the branching ratio from the $\frac{3}{2}^-$ excited state which should be one of the states comprising the doublet near 6.8 Mev. This state, listed as $\frac{3}{2}^*$ in Table I, has large transition strengths to the $\frac{3}{2}$, $\frac{1}{2}$, and $\frac{5}{2}$ states, with a branching ratio that is nearly constant between $a/K=3$ and 7.5. The values of these branching ratios are:

$$
\begin{aligned}
\tfrac{3}{2}^*(6.8\text{ Mev}) &\to \tfrac{3}{2}(\text{g.s.}) \quad \sim 80\% \\
&\to \tfrac{1}{2}(2.14\text{ Mev}) \sim 15\% \\
&\to \tfrac{5}{2}(4.46\text{ Mev}) \sim 5\%.
\end{aligned}
$$

B^{10}

The amount of experimental information available for this nucleus is considerably more than for B^{11}, especially for the low-lying excited states. The angular momenta and isotopic spins for the states with excitation of 5.16 Mev or less are fairly well established. With the exception of the 4.77-Mev state, these are in good agreement with the level scheme arising from an intermediate-coupling model.[1] The observed branching ratios for such states are taken from Ajzenberg and Lauritsen,[5] and given in Fig. 6. In addition, the lifetime of the 0.72-Mev state has been measured to be 1×10^{-9} second.

In order to see whether computing the nuclear matrix element gives any improvement in comparing theory to experiment, branching ratios have also been computed using the Weisskopf formula wherein all the nuclear matrix elements are assumed to be alike for transitions of a given multipolarity. Hence:

$$\Gamma_w(M1)=2.1\times10^{-2}E^3;\quad \Gamma_w(E2)=2.6\times10^{-6}E^5.$$

These values, the results computed for $a/K=3.0$ and 4.5, and the observed branching ratios, are listed in Table III. In cases where both $E2$ and $M1$ radiation are possible, inclusion of the computed $E2$ contribution has little effect on the branching ratio, so only the $M1$ contributions are included. The range of a/K is limited to lie between 3 and 6 for these considerations because the computed energy level order is like the experimental one only in this region. The branching ratios for $a/K=6$ are very nearly the same as those at $a/K=4.5$.

The $J=2^+$, $T=1$ level at 5.16 Mev is computed to decay almost entirely by the $M1$ mode. Only the transition to the ground state has an $E2$ contribution above 1%, and even this case has at most 5% (for $a/K=6$); so the $E2$ mode may be neglected. The branching ratios from the Weisskopf formula depend only on the energy factors, so the second column of Table III shows the weighting according to energy. The transition strengths of Table I show that in the a/K region of interest, the transition to the ground state (3,0) is strongly inhibited. The transition to the first excited state (1,0) is very strong while that to the third excited state $(1,0)^*$ is inhibited. The net result is that the computed branching ratios for $a/K=3$ and 4.5 favor the transition to the first excited state (1,0) by a large factor. The observed branching strongly favors the transition to the $(1,0)^*$ state and gives a further 5 to 1 favoring of the transition to (1,0) over that to the ground state (3,0). While the computation does provide a strong inhibition for the transition to the ground state, there is nothing in the model that can provide the $(1,0)^*$ preference. Also the total width is computed to be about 3 ev compared to an experimental[19] $\Gamma\sim0.6$ ev.

The $J=2^+$, $T=0$ level at 3.58 Mev is quite interesting

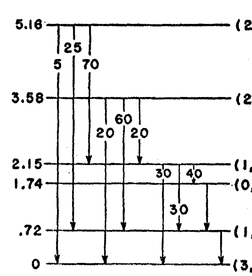

FIG. 6. Branching ratios in B^{10} with probable (J,T) indicated on right. (After Ajzenberg and Lauritsen.)

[19] G. A. Jones and D. H. Wilkinson, Phil. Mag. 45, 703 (1954).

in that all three branches are computed to be very weak $M1$'s as is indicated by the transition strengths of Table I. The $E2$ contribution lies between 10% and 20%, but it does not change the branching ratios seriously. These ratios are computed to be very close to those obtained from the Weisskopf formula as one sees in Table III. They also have no resemblance to the observed branching. It is interesting to note that the Weisskopf formula gives a total width of $\Gamma\sim1.5$ ev while the computation gives $\Gamma\sim0.01$ ev, although both methods predict the same branching ratios. It may be that because the $M1$ transition strengths are so weak, the width of the 3.58-Mev level is really determined by $E2$ contributions arising from collective motion. It would be of great interest to determine the multipolarity of transitions from this state.

The $J=1^+$, $T=0$ level at 2.15 Mev $(1,0)^*$ has competing modes of decay varying among pure $E2$ to the ground state (3,0), mixed $M1$ and $E2$ to the first excited state (1,0), and pure $M1$ to the second excited state (0,1). The Weisskopf formulas say that the decay should go almost entirely to the (1,0) state. However, the transition strengths of Table I show that the transition to (0,1) is quite strong, a factor of some 3000 stronger than the weak $M1$ to (1,0). The $E2$ strength for the transition to the ground state is also strong as shown in Table II. Therefore, the branching ratios computed from the model are much more nearly like the observed ones, than are those obtained from the Weisskopf formulas, although the $E2$ branch is still weak, as is seen in Table III. The total width is computed to be

$$\Gamma\sim3\times10^{-3}\text{ ev},\quad\text{or}\quad\tau\sim2\times10^{-13}\text{ sec}.$$

The $E2$ transition from the first excited state to the ground state has a computed lifetime of 2 to 3×10^{-9} sec for $a/K>4.5$. Below this value the nuclear matrix element varies rapidly, going through zero several times. This behavior has also been noted by French and Fujii[7] who find that the experimental value[20] of

$$\tau=(1.07\pm0.10)\times10^{-9}\text{ sec}$$

can be found in the region of $a/K=4$ as well as for smaller values.

TABLE III. Branching ratios for B^{10}. The parent level and its daughters are identified by (J,T) in column 1.[a] Computed values are given in the next three columns—experimental values in the last.

	Branching ratios (%)			
$(JT)_i\to(JT)_f$	Weisskopf formula	$a/K=3.0$	$a/K=4.5$	Exp.
5.16 Mev				
$(2,1)\to(3,0)$	54	13	1	5
(1,0)	34	78	97	25
(1,0)*	$10\frac{1}{2}$	7	$\frac{1}{2}$	70
(2,0)	$1\frac{1}{2}$	2	$1\frac{1}{2}$	...
3.58 Mev				
$(2,0)\to(3,0)$	$63\frac{1}{2}$	59	66	20
(1,0)	$32\frac{1}{2}$	38	33	60
(1,0)*	4	3	1	20
2.15 Mev				
$(1,0)^*\to(3,0)$	$\frac{1}{2}$	$7\frac{1}{2}$	6	30
(1,0)	$97\frac{1}{2}$	$35\frac{1}{2}$	12	30
(0,1)	2	57	82	40

[a] A state labeled with an asterisk refers to the second lowest state in energy of the specified J and T.

IV. SUMMARY AND CONCLUSIONS

The predictions of this intermediate-coupling model do not give a satisfactory picture of the experimental evidence concerning gamma transitions in the $1p$-shell. There are some cases of good agreement, a few unexplained contradictions, and a number of cases which suggest that introduction of some collective motion might bring them into agreement with observation. In addition, there are transitions observed for which there are not enough data to draw any conclusions. This is particularly true of the low-lying states in B^{11}, but recent experimental evidence[21] may soon make possible a more fruitful comparison with theory.

The cases for which the agreement with observation is good, all involve transitions for which the $M1$ mode is the only one possible. The first of these is the $(11)\to(00)$ transition in Be^8 which is the chief decay mode of the 17.6-Mev level. The analogous $(11)\to(00)$ transition in C^{12} is well described by the model, and even the 4% branch $(11)\to(20)$ is predicted. The third case for which the model offers an explanation concerns the $(10)^*$ level at 2.15 Mev in B^{10}. The reason that the 0.41-Mev $M1$ transition, $(10)^*\to(01)$, competes successfully with the 1.43-Mev $M1$ transition, $(10)^*\to(10)$, is that the computed transition strength, Λ, of the former is very large while Λ for the latter is quite small.

The second category of transitions contains those for which the agreement is often rough or even nonexistent, but which at least give some suggestion of why there is disagreement. They are either pure $E2$ transitions or mixtures of $M1$ and $E2$ for which the computed $M1$ transition strength is very small. There are three pure $E2$'s among the transitions for which there is enough experimental evidence to draw conclusions. These are the transitions to the ground state for the levels at 2.15 Mev and 0.72 Mev in B^{10} and the level at 4.43 Mev in C^{12}. In all of these it appears that the $E2$ transition strengths given by the model are of the right order of magnitude, but too weak. This happens despite the fact that the value of $\langle r^2\rangle=10^{-25}$ cm^2, which is used to obtain numerical values in this calculation, is larger than is indicated by the fast-electron scattering experiments in C^{12}. Ferrell and Visscher[22] treat the 4.43-Mev transition in C^{12} using a value of $\langle r^2\rangle$ which is about $\frac{2}{3}$ of that used here, and obtain the experimental transition strength by mixing in collective excitation.

[20] Bloom, Turner, and Wilkinson, Phys. Rev. **105**, 232 (1957).
[21] L. Meyer-Schützmeister and S. S. Hanna, Bull. Am. Phys. Soc. Ser. II, **2**, 28 (1957); Ferguson, Gove, Litherland, Almquist, and Bromley, Bull. Am. Phys. Soc. Ser. II, **2**, 51 (1957).
[22] R. A. Ferrell and W. M. Visscher, Phys. Rev. **104**, 475 (1956).

Thus it may be that some collective motion which would enhance the $E2$ transition strengths is needed, even for such light nuclei as these in the $1p$ shell. Introducing such an effect would clear up the pure $E2$ transitions and might also explain the observed branching from the 3.58-Mev level, (20), in B^{10}. The $M1$ transitions from this level are all computed to be very weak, and might be masked if the $E2$ enhancement is large enough. In this regard it would be of great interest to know the experimental strength of the $E2$ mode relative to the $M1$ mode in these branches. There is evidence on one branch,[23] which says that the $E2$ contribution is either about 10% or very large.

The third category contains two contradictions between the model and observation which are not readily explained. These are the very large observed width of the $(21)\rightarrow(20)$ transition in C^{12} and the preference for the $(21)\rightarrow(10)^*$ transition in the decay of the 5.16-Mev level in B^{10}, neither of which is given by the model.

The predictions arising from the form of the intermediate-coupling model which was used in these calculations, can be modified without introducing collective motion. The two possible ways are either to change the ratio of central-interaction integrals, L/K, or to vary the exchange mixture in the two-body interaction. The results do not seem to be very sensitive to variation of the L/K ratio. There is also evidence that varying the exchange mixture does not make radical changes.[3] However, the effect would be serious in cases where a level involved in a transition is nearly degenerate in energy with a level of the same (J,T). Among the transitions we have treated, the levels of B^{10} for a/K near 4 are therefore most likely to be affected. The amount of labor involved in varying the exchange mixture is extremely large.

In conclusion, the result of extending the intermediate-coupling model to calculate gamma transition widths, is to indicate that some modification of the model is needed. The $E2$ transitions give an indication that some collective behavior should be included, although one does not need nearly as much enhancement of the $E2$ strength as is found for the strongly deformed nuclei. The level schemes obtained without including extra collective behavior are in rather good agreement with observation. The problem will be to see whether one can add collective effects sufficient to explain the $E2$ transitions without seriously disturbing the energy level schemes.

[23] S. M. Shafroth and S. S. Hanna, Phys. Rev. **104**, 399 (1956).

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