$C^{13}(p,p)C^{13}$: Experiment and Analysis*

D. ZIPOY, G. FREIER, AND K. FAMULARO
University of Minnesota, Minneapolis, Minnesota
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The differential cross sections for the $C^{13}(p,p)C^{13}$ elastic scattering interactions were measured over an energy range from 1.5 to 3.4 Mev (incident proton energy) at scattering angles of 45.0°, 54.4°, 74.1°, 90.0°, 106.4°, 127.8°, 148.9° in the center-of-mass system. Methane gas, enriched to 72.5% in C^{13}, was used as a target material. The absolute differential cross sections computed through this experiment were accurate to about 6% in the nonresonant energy regions. The measured nonresonant cross sections at about 2 Mev were essentially equal to those for Coulomb scattering at $\theta=45°$ and increased to about seven times Coulomb cross section at $\theta=148.9°$. There was also an increase in measured cross section relative to the Rutherford cross section at all scattering angles as the proton energy was increased. Maxima in the cross sections were observed at incident proton energies of 2.00, 2.12, 2.33, 2.90, and 3.12 Mev. A phase-shift analysis of the resonances at 2.00, 2.12, and 2.33 Mev yielded assignments for J and π of 1^-, 3^-, and 1^+, respectively, and reduced widths relative to single-particle reduced widths of 0.10, 0.11, and 0.005, respectively.

INTRODUCTION

A STUDY of the interactions of protons elastically scattered from C^{13} was made in order to investigate N^{14*} in the region from 9.0- to 10.6-Mev excitation. Most of the energy range covered in this experiment has been investigated by means of the $C^{13}(p,\gamma)N^{14}$ reactions[1–3] or through the $N^{14}(\alpha,\alpha')N^{14*}$ reaction[4]; however, it was felt that measurements on the elastically scattered protons should resolve any closely spaced levels in N^{14} and would permit a phase-shift analysis of these levels.

APPARATUS AND EXPERIMENTAL PROCEDURE

The measurements of the differential cross sections were made in a conventional manner with a scattering chamber described in previous papers.[5,6] A gas target was chosen because the number of scattering centers in a well-defined volume could be reliably determined from straightforward measurements of temperature and pressure. The only carbon compound available which was enriched in C^{13} was methyl iodide,[7] a liquid at standard conditions, so a Grignard reagent was prepared, and then by carrying out the associated chemical procedure,[8] methane gas containing 72% C^{13} was produced. This was then separated from the reaction reagents by fractional distillation at liquid nitrogen temperatures and gave a final sample of methane gas with 98% purity as determined by mass spectrometric analysis.[9] By a combination of Toepler pumping and freezing with liquid nitrogen, this gas could be admitted into the scattering chamber and removed for purification many times with negligible losses. The gas was placed in the chamber at a pressure of approximately 100 mm Hg as determined by a Wallace and Tiernan differential manometer which in turn had been calibrated against a mercury manometer. The temperature was determined by a thermometer cemented to the wall of the chamber.

The beam of bombarding protons from the electrostatic accelerator entered the target volume through a 0.015-mil Ni isolation foil and left the target volume through a 0.25-mil Mylar foil, where it was then collected in a well-evacuated Faraday cup which was electrically and magnetically biased so that loss of secondary electrons would not interfere in the measurement of the collected charge.

The slit system of the proportional counter defined a small volume at the center of the chamber which was traversed by the well-collimated incident beam, and the pencils of particle trajectories for any scattering angle from 17° to 163° could, in turn, be studied. Pulses from the proportional counter were amplified and then scaled on a 10-channel pulse-height discriminator.[10] The mechanics of the collisions between incident protons and nuclei in the methane molecule led to energy transfers which allowed easy separation of protons scattered from hydrogen, but the resolution was not sufficiently good to discern whether scattering was from C^{12} or C^{13}. The contribution to scattering from C^{12} in our measured number of counts was determined in a separate experiment by using ordinary methane as a target gas in the chamber and measuring the differential cross section for this competing process. A knowledge of this cross section and the concentration of C^{12} in our enriched sample then allowed us to calculate yields due

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to C^{13} alone. (The C^{12} cross sections at angles of 106° or larger had been measured by Jackson *et al.*[11] and were not remeasured except at a few points to check the validity of our own procedures.)

The final errors in the absolute cross sections were about $\pm 6\%$; the main source of error was from the counting statistics associated with each point. Other sources of error were: uncertainty as to the amount of C^{13} in the methane, $\pm 1\%$; errors in the target pressure readings, $\pm 1\%$; uncertainties in the knowledge of the number of protons that had passed through the target volume, $\pm 0.5\%$; and uncertainties in the geometry of the scattering chamber, $\pm 0.5\%$.

In order to determine the energy of the proton beam at the target volume, corrections for energy loss in intervening materials had to be made. The rate of energy loss, $-dE/dx$, for protons in methane was determined by measuring the apparent shift of a resonance when the target gas pressure was changed. The resonance at 1.75 Mev in the $C^{12}(p,p)C^{12}$ cross section,[11] and the one at 3.1 Mev in the $C^{13}(p,p)C^{13}$ cross section[3] were used with the result that the rate of energy loss was 133 kev/cm at 1.75 Mev and 82 kev/cm at 3.12 Mev for methane at one atmosphere pressure and a temperature of 20°C. (The errors in these numbers are about $\pm 10\%$.) The two points fit the shape of Bethe's formula for the rate of energy loss very well, but the magnitudes are about 30% lower than those given by the formula.[12] The energy loss of the incident protons in the nickel foil placed at the entrance slit of the chamber was obtained by first measuring the neutron threshold energy for the $Li^7(p,n)Be^7$ reaction when the proton beam went through the nickel foil, and then the same threshold when the foil was removed. The energy loss in the foil was then given by the difference of these two threshold energies; on the average the difference was about 20 kev. During the experiment the entrance foil showed an increase in thickness which was found to be proportional to the methane pressure and to the amount of charge that had passed through the chamber; the effect was assumed to be due to carbon (from the methane) being deposited on the foil. The energy loss for protons while passing through the target volume was about 3 kev. The total spread in the proton energy as seen by the detector amounted to about 9 kev, and was due to the finite target thickness and to the straggling of the protons in the methane and isolation foil. The error in the absolute energy was about ± 20 kev; principally, this error came from the measurement of the rate at which protons lost energy in the target gas.

RESULTS

The absolute differential cross sections with the associated probable errors are plotted in Fig. 1. Below 1.8 Mev, the determination of the cross section was very poor due to the fact that there is a very large anomaly in the $C^{12}(p,p)C^{12}$ cross section in this energy region[11] which obscured a resonance that is probably at 1.75 Mev.[1,2] A level at 9.49-Mev excitation (2.10-Mev incident proton energy) has been found in a study of the $C^{13}(p,\gamma)N^{14}$ reaction[2]; it is not certain whether the γ radiation from this reaction came from one or more than one of the three levels near 9.5-Mev excitation found in the present experiment. A level has been reported[4] at 10.05 Mev (2.73-Mev protons); it probably corresponds to a combination of the rather broad anomalies observed at 2.9 Mev and 3.11 Mev (proton energy) in the present experiment. The resonance at 3.11 Mev (proton energy) has been observed previously.[3] There were no noticeable effects near 3.09 or 3.24 Mev where the (p,p')[13] and (p,n)[14,15] reactions begin to occur.

ANALYSIS OF THE DATA

A phase-shift analysis of the resonances at 2.00, 2.12, and 2.33 Mev was performed in order to determine the spins and parities of the corresponding states in N^{14}. There were no reactions competing with elastic scattering in this energy region except radiative capture of the protons; however, this latter process is much more unlikely to occur than elastic scattering, so for simplicity it was neglected. Assuming that the orbital angular momentum quantum number, l, did not change during the interaction, the general formula used was

$$\begin{aligned} k^2 d\sigma/d\Omega = \tfrac{1}{4}\textstyle\sum_S \sum_M \Big| k f_C(\theta)\chi_{SM} + \sum_l \sum_J \left[4\pi(2l+1)\right]^{\frac{1}{2}} \\ \times e^{i[2\eta(l)+\delta(l,J,S)]} \sin[\delta(l,J,S)] C_{lS}(J,M;O,M) \\ \times \textstyle\sum_m C_{lS}(J,M;M-m,m) Y_l^{M-m}(\theta,\varphi)\chi_{Sm}\Big|^2, \quad (1) \end{aligned}$$

where $k=2\pi$ divided by the deBroglie wavelength of the proton in the c.m. system, $d\sigma/d\Omega=$ differential cross section in the c.m. system, $f_C(\theta)=$ Coulomb scattering amplitude, $\chi_{SM}=$ total spin function of the proton and carbon nucleus ($S=0, 1$), $\eta(l)=$ Coulomb phase shift for the lth partial wave, $\delta(l,J,S)=$ nuclear phase shift, $C_{lS}(J,M;M-m,m)=$ Clebsch-Gordan coefficients,[16] and $Y_l^m(\theta,\varphi)=$ spherical harmonic.

The first step in the analysis was to fit the nonresonant cross section (hereafter to be called background). An attempt at fitting the background with S-wave parameters alone failed. When the P-wave phase shifts were included, it was not obvious, because of the large number of parameters involved (6), whether a fit could be obtained. In order to eliminate any doubt on this point, the problem was coded for use in the Remington Rand 1103 digital computer.[17] It was

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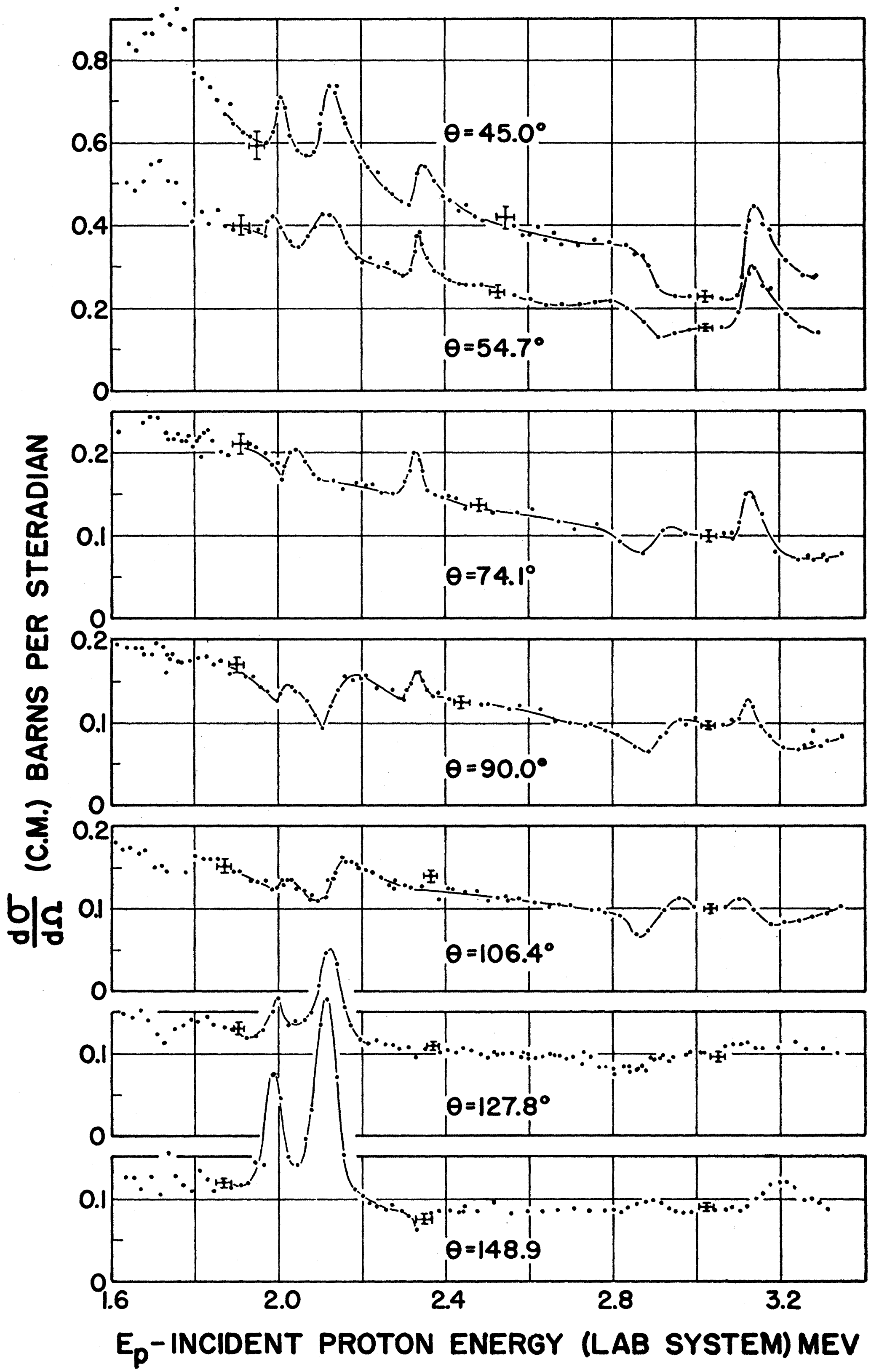

FIG. 1. The absolute differential cross sections for $C^{13}(p,p)C^{13}$ as a function of energy. The errors in cross sections are ±6%. The absolute error in the energy is ±20 kev.

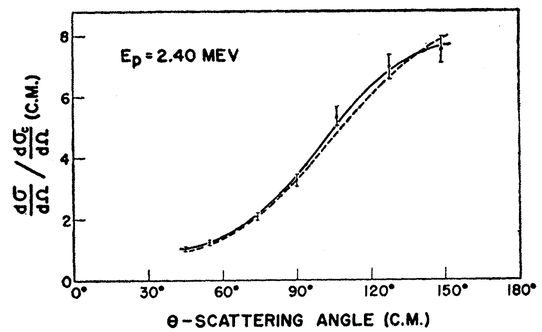

FIG. 2. Plot of fits to the nonresonant cross sections. The dashed line is the best fit using two S-wave and three P-wave phase shifts. The solid line is the best fit using one phase-shift for each of the S, P, and D waves (see text).

found that S and P waves together were not sufficient to fit the experimental angular distribution satisfactorily at 2.4-Mev proton energy.

Because of the necessity of including the D-wave phase shifts, a method of successive approximations was devised in order that the large number of parameters could be handled with some ease. As a first step in this method, the dependence on the total angular momentum, J, was removed; this reduced the number of parameters to three. These three phase shifts were then computed to fit the angular distribution at the roots of the first- and second-order Legendre polynomials (90°; 54.7°; 125.3°). In order to obtain a better fit, the scattering formula was expanded in a Taylor's series around these values of the phase shifts, and a least-squares fit to the data at all seven angles was obtained (only the first order terms in the expansion were retained). It was found that good agreement with the experiment could be attained by using only these three phase shifts. (See Fig. 2.) Later in the calculation the dependence of the phase shifts on J was included; however, before this, a search for the resonant phase shifts was undertaken.

The resonance at 2.33 Mev was considered first. It can be shown that the scattering formula can always be put into the following form:

$$k^2 d\sigma/d\Omega = A + B\sin 2\delta + C\cos 2\delta,\qquad M \equiv (B^2+C^2)^{\frac{1}{2}}, \tag{2}$$

where δ is any one of the phase shifts, and A, B, C are functions of the remaining parameters. In formula (2), δ was chosen as the resonant phase shift, and A, B, C were evaluated at each angle, using the values of the nonresonant parameters computed previously. It can be seen from (2) that the value of the maximum cross section minus the minimum cross section is given by M. This quantity was then computed for different choices of the resonant phase shift, and the result was compared with experiment. The criterion used in this comparison was that the computed value of M had to be as large or larger than the experimental values. Of those that satisfied this condition, some were eliminated because the resonances they gave were much too large (factors of five to ten at some angles). Of the remainder, a further selection could be made on the basis of the shape of the resonance; this selection was made by the following method. It was assumed, for simplicity, that the phase shift changed by 180° on going through resonance, and that the shape of the peak could be given by the usual Breit-Wigner formula:

$$\delta = \delta_0 - \cot^{-1}\left(\frac{E-E_0}{\frac{1}{2}\Gamma}\right),$$

where δ = resonant phase shift, δ_0 = value of phase shift far from resonance, Γ = full width at half-maximum of resonance, E_0 = resonant energy, and E = energy of incident protons. It was also required that the value of the resonant phase shift far away from the resonance had to be comparable to the value computed previously to fit the background. All of these conditions served to determine, quite unambiguously, the parameters for the 2.33-Mev resonance; these parameters were: $J=l=S=1$. It should be noted that for $J=l$, there are two possible values of the channel spin, $S(=0, 1)$. In this specific case, however, a linear combination of the two spin states did not appreciably improve the calculated fit. As a final calculation, the background phase shifts were then changed slightly, by means of a Taylor's expansion which was dependent on J, in order that the absolute magnitude and shape of the cross section would agree as closely as possible with experiment.

TABLE I. Tabulation of the nonresonant phase shifts used to fit the experimental data at two energies.

E_p, Mev \ $\delta(l,J)$	$\delta(0,0)$	$\delta(0,1)$	$\delta(1,0)$	$\delta(1,1)$	$\delta(1,2)$	$\delta(2,1)$	$\delta(2,2)$	$\delta(2,3)$
2.06	136.1°	136.1°	166.3°	171.3°	0.3°	174.8°	178.0°	176.0°
2.33	131.5°	131.5°	165.3°	170.0°	2.0°	176.9°	176.9°	176.9°

The remaining two resonances were fitted in a similar manner; however, the following additional experimental result was used which enabled the two resonances to be fitted simultaneously. At $\theta=90°$, the resonances at 2.00 and 2.12 Mev interfere destructively, whereas at large scattering angles there is either no interference or constructive interference. This meant that the possible resonant phase shifts had to be chosen in pairs which would satisfy these conditions; this greatly reduced the number of possible choices. Another qualitative feature of the experimental results is that the peaks show a marked symmetry around $\theta=90°$. The experimental values, for both resonances, of the maximum minus the minimum cross sections are very nearly equal for equal displacements on either side of $\theta=90°$ (such as 54.7° and 127.8°). This fact would strongly indicate that l would have to be even (S or D waves). It was found that S-wave phase shifts either would not give large enough

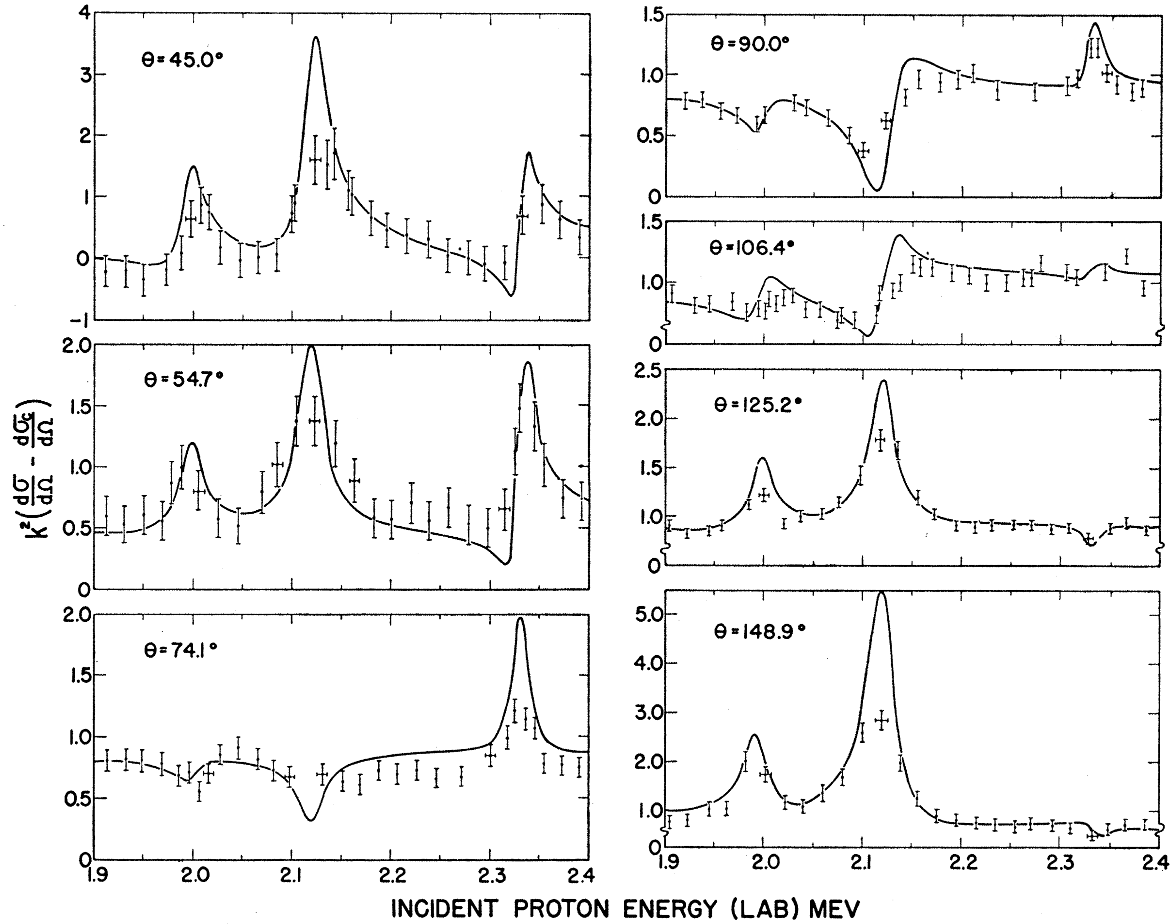

FIG. 3. Plots of the calculated fits to the experimental points. The errors shown in the vertical scale correspond to ±6% errors in the absolute cross sections; the errors in the energy scale denote the total spread in the beam energy. The assignments for the corresponding levels in N^{14} are: 1^- at 2.00 Mev, 3^- at 2.12 Mev, and 1^+ at 2.33 Mev.

resonances or would give the wrong shape. The remaining P-wave resonances were checked out, and it was found that the Coulomb interference terms were not large enough to bring about the required symmetry. Of the allowable pairs of D-wave phase shifts, the best fit was obtained by using $J=1$, $l=2$ for the 2.00-Mev resonance, and $J=3$, $l=2$ for the 2.12-Mev resonance.

RESULTS AND CONCLUSIONS

The calculated curves and the experimental results are shown in Fig. 3. It can be seen in Fig. 3 that the calculated resonance at 2.12 Mev is somewhat larger than the experimental resonance. However, it is quite certain that the assignment for the level is correct since any other assignment does not fit the shapes as well or the absolute sizes as well as the above values for J and π. The discrepancy therefore must be due to the energy resolution of the experimental apparatus because of the fairly thick target and the straggling in the proton beam. This would also require that the half-widths of the resonances be somewhat smaller than those listed in Table II, particularly for the 2.12-Mev resonance. Another possibility could be that there are resonances, in this region (1.5 to 2.4 Mev) which was analyzed, which were not resolved by the experiment.

Table I lists the values of the background phase shifts for two energies. It can be seen in Table I that these phase shifts were quite constant with energy, which should be the case. It can also be seen that, for simplicity in calculations, the dependence of the phase shifts on J and S was neglected whenever possible.

Table II gives the information regarding the level assignments. The intrinsic parity of C^{13} is negative, and thus the parity of a state in N^{14} is opposite to that of the orbital angular momentum of the interaction which formed this state. The last column in Table II gives the reduced widths of the levels compared to that of a single particle interaction. The reduced width, γ^2, is related to Γ by the equation

$$\Gamma=2k\gamma^2/(F^2+G^2),$$

TABLE II. Tabulation of the resonances seen in this experiment and also the various parameters corresponding to the resonances. E_p is the incident proton energy (lab system), E_e is the corresponding excitation in N^{14}, J and π are the total angular momentum and parity of the levels, respectively, Γ is the full width at half-maximum of the resonances, and the last column gives the reduced widths of the levels relative to the reduced width for single-particle type excitation.

E_p, Mev	E_e, Mev	J	π	Γ, kev	$\gamma^2 / \frac{3}{2}(\hbar^2/\mu a)$
2.00±0.02	9.39	1	−	25±3	0.10
2.12±0.02	9.51	3	−	32±6	0.11
2.33±0.02	9.72	1	+	15±3	0.005
2.90±0.04	10.29	...	...	80±30	...
3.12±0.03	10.51	...	...	80±10	...

where F and G are the regular and irregular solutions, respectively, of the Coulomb scattering problem evaluated at the radius of the nucleus, a.[18] The single particle reduced width is $3\hbar^2/2\mu a$, where μ is the reduced mass of the system and $a=(1.33A^{\frac{1}{3}}+0.77)\times10^{-13}$ cm.[19] It is quite apparent from these reduced widths that none of the three levels analyzed are formed through single-particle excitation.

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Capture of Protons in K^{39}

J. DE VEIGA SIMÃO* AND J. P. F. SELLSCHOP
Cavendish Laboratory, Cambridge, England

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Evaporated natural potassium metal targets were bombarded with protons from a Cockcroft-Walton generator. The excitation function was studied from 0.5 to 1.15 Mev. The gamma rays from a level found in calcium 40 at 9.29 Mev above the ground state were studied with single-crystal spectrometers and with a pair spectrometer. Gamma rays of energy 9.29±0.005 Mev, 5.93±0.005 Mev, 5.55±0.005 Mev, 3.73±0.005 Mev, and 0.500±0.005 Mev were found and suggest a decay scheme in agreement with the levels in Ca^{40} found by Braams by inelastic proton scattering.

THE doubly-magic nucleus Ca^{40} is of particular interest, especially in the light of the shell-model picture. One expects the first excited state to be rather high in energy above the ground state. The levels of Ca^{40} have been studied by inelastic scattering of protons by Braams,[1] who by magnetic analysis finds levels at 3.348 (0+), 3.730, 3.900, 4.483, 5.202, 5.241, 5.272, 5.606, 5.621, 5.901, and 6.029 Mev. See Fig. 1.

We have produced excited states in Ca^{40} by the capture reaction $K^{39}(p,\gamma)Ca^{40}$. Protons were accelerated in a conventional Cockcroft-Walton machine, resolved beams of 50–100 microamperes being used. Targets were prepared *in situ* by evaporation in vacuum of freshly cleaned, metallic potassium. Target thicknesses were 20–30 kev to 1-Mev protons. We have used two types of detection equipment. Firstly single-crystal spectrometers with NaI scintillators of cylindrical form of dimensions 2 in. diameter by $2\frac{1}{2}$ in. long, and $1\frac{1}{2}$ in. diameter by $1\frac{1}{2}$ in. long. Secondly a three-crystal pair spectrometer of the design of Bell, Graham, and Petch. Both detection systems were used in conjunction with an 80-channel Hutchinson-Scarrott kicksorter. The equipment was calibrated with the following sources: (i) Cs^{137} (0.627 Mev), (ii) Na^{22} (0.500 and 1.277 Mev), (iii) Rd-Th (2.62 Mev), and (iv) Po-Be (4.43 Mev), and with the reactions: (v) $F^{19}(p,\alpha,\gamma)$ (6.14 Mev), (vi) $C^{13}(p,\gamma)$ (8.06 Mev), and (vii) $Li^7(\alpha,\gamma)$ (9.276 Mev).

The excitation function has been studied from 0.2 to 1.2 Mev, and a number of resonances found. The yield was measured at two bias values, 4 Mev and 6.5 Mev. Resonances were found at incident proton energies of (i) 0.883±0.010 Mev, (ii) 0.925±0.010 Mev, (iii) 0.980 ±0.010 Mev, and (iv) 1.150±0.010 Mev. Relative cross sections are (i) 1.21, (ii) 1, (iii) small, and (iv) 10.1, respectively. These resonances correspond to levels in Ca^{40} at the following energies above the ground state: (i) 9.03±0.010 Mev, (ii) 9.07±0.010 Mev, (iii) 9.13±0.010 Mev, and (iv) 9.29±0.010 Mev.

A study of the γ-ray spectra by single-crystal spec-

* On leave of absence from Coimbra University, Coimbra, Portugal.

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