Experiments with other sources which should permit separation of effects due to internal and external bremsstrahlung are being prepared.

The bremsstrahlung method of analyzing the longitudinal polarization of β rays may prove of special value for high-energy β rays, and might be applicable to electrons from μ-meson decay.

Dr. K. W. McVoy of this Laboratory gives a partial theoretical interpretation of the effect reported here in an accompanying Letter. Dr. R. E. Cutkosky of the Carnegie Institute of Technology has kindly informed us of calculations based on the two-component neutrino theory which indicate that circular polarization should also exist for internal bremsstrahlung.

* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[2] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
[3] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).
[4] J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).
[5] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[6] A. Salam, Nuovo cimento **5**, 299 (1957).
[7] L. Landau, Nuclear Phys. **3**, 127 (1957).
[8] Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957).
[9] R. B. Curtis and R. R. Lewis (to be published).
[10] Frauenfelder, Babone, von Goller, Levine, Lewis, Peacock, Rossi, and Pasquali, Phys. Rev. **106**, 386 (1957).
[11] L. A. Page and M. Heinberg, Phys. Rev. (to be published).
[12] Way, King, McGinnis, and van Lieshout, *Nuclear Level Schemes, A=40–A=92*, U. S. Atomic Energy Commission Report TID-5300 (U. S. Government Printing Office, Washington, D. C., 1955).
[13] See C. S. Wu, *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), pp. 649–659.
[14] S. B. Gunst and L. A. Page, Phys. Rev. **92**, 970 (1953); H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956); G. Trumpy, Nuclear Phys. **2**, 664 (1957).
[15] de Benedetti, Grodzins, Madey, and Sunyar (unpublished).
[16] K. W. McVoy, following letter [Phys. Rev. **106**, 828 (1957)].

Circular Polarization of Bremsstrahlung from Polarized Electrons in Born Approximation*

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The Bethe-Heitler bremsstrahlung cross section has been re-derived for transitions from specific initial to specific final electron spin states, in order to compare the calculated circular polarization with the experimental results obtained by Goldhaber, Grodzins, and Sunyar.[1] Although the matrix elements are simple to derive, the cross section for bremsstrahlung produced in an arbitrary direction appears to be very complicated, and only the cross section in the directly forward direction is presented in detail here.

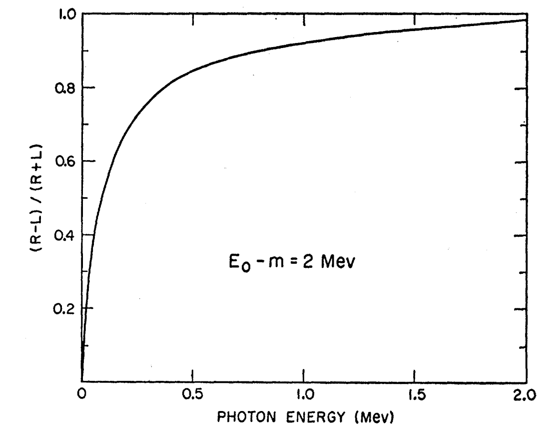

Fig. 1. Circular polarization of the forward bremsstrahlung from 2-Mev "spin-forward" electrons.

In Heitler's notation,[2] the matrix element for the process, after summing over intermediate states, is

$$M=\frac{1}{q^2}\left\{\frac{(u_0^*,(\boldsymbol{\alpha}\cdot\mathbf{e})(E+H_{p'})u)}{2(E_0k-\mathbf{p}_0\cdot\mathbf{k})}-\frac{(u_0^*,(E_0+H_{p''})(\boldsymbol{\alpha}\cdot\mathbf{e})u)}{2(Ek-\mathbf{p}\cdot\mathbf{k})}\right\}, \quad (1)$$

with $\mathbf{p}'=\mathbf{p}_0-\mathbf{k}$, $\mathbf{p}''=\mathbf{p}+\mathbf{k}$, and $\mathbf{q}=\mathbf{p}_0-\mathbf{p}-\mathbf{k}$; $\mathbf{e}$ is the polarization vector of the photon. We shall choose the z axis along the $\mathbf{k}$ direction throughout and write $\mathbf{e}=(\mathbf{e}_x+i\delta\mathbf{e}_y)/\sqrt{2}$ for circularly-polarized light. The state, $\delta=+1$, corresponding to one positive unit of angular momentum being carried in the propagation direction, we shall call "right-circularly-polarized," and $\delta=-1$ "left-circularly-polarized." The recoil energy of the nucleus will be neglected throughout.

The Dirac-matrix commutation rules permit us to write

$$\begin{aligned}(u_0^*,\ (\boldsymbol{\alpha}\cdot\mathbf{e})(E+H_{p'})u)&=(u_0^*,\ [2(\mathbf{e}\cdot\mathbf{p}_0)-k(\boldsymbol{\alpha}\cdot\mathbf{e})+\delta k(\boldsymbol{\sigma}\cdot\mathbf{e})]u),\\(u_0^*,\ (E_0+H_{p''})(\boldsymbol{\alpha}\cdot\mathbf{e})u)&=(u_0^*,\ [2(\mathbf{e}\cdot\mathbf{p})+k(\boldsymbol{\alpha}\cdot\mathbf{e})+\delta k(\boldsymbol{\sigma}\cdot\mathbf{e})]u).\end{aligned} \quad (2)$$

In order to evaluate the matrix elements between specific spin states, we shall choose u's which simultaneously satisfy

$$\begin{aligned}(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)u&=Eu,\\(\boldsymbol{\sigma}\cdot\mathbf{p})u&=\epsilon p u,\end{aligned} \quad (3)$$

which is possible since the operators commute. $\epsilon=+1$ defines the state in which the electron's spin is parallel to its momentum; $\epsilon=-1$, the state in which it is antiparallel. We shall use ϵ_f for the final electron spin state and ϵ_0 for the initial electron state. If we use the usual "low-energy" representation for the Dirac matrices, the spinors are

$$u_\epsilon(\mathbf{p})=2[pE(p+\epsilon p_z)(E+m)]^{-\frac{1}{2}} \times \begin{bmatrix} (E+m)(\epsilon p+p_z) \\ (E+m)p_+ \\ \epsilon p(\epsilon p+p_z) \\ \epsilon p p_+ \end{bmatrix}. \quad (4)$$

Matrix elements which enter the problem can then be written as

$$(u_{\epsilon 0}^*(\mathbf{p}_0),u_{\epsilon f}(\mathbf{p}))=AB;\quad (u_{\epsilon 0}^*(\mathbf{p}_0),(\boldsymbol{\alpha}\cdot\mathbf{e})u_{\epsilon f}(\mathbf{p}))=CD;$$
$$(u_{\epsilon 0}^*(\mathbf{p}_0),(\boldsymbol{\sigma}\cdot\mathbf{e})u_{\epsilon f}(\mathbf{p}))=AD, \quad (5)$$

where

$$\begin{aligned} NA&=(E_0+m)(E+m)+\epsilon_0\epsilon_f p_0 p,\\ B&=p_+(p_0)_-+(\epsilon_0 p_0+p_{0z})(\epsilon_f p+p_z),\\ NC&=\epsilon_f p(E_0+m)+\epsilon_0 p_0(E+m), \qquad (6)\\ D&=(1+\delta)(\mathbf{e}\cdot\mathbf{p})(\epsilon_0 p_0+p_{0z})\\ &\quad+(1-\delta)(\mathbf{e}\cdot\mathbf{p}_0)(\epsilon_f p+p_z),\\ N&=4[pp_0EE_0(p+\xi_f p_z)\\ &\quad\times(p_0+\xi_0 p_{0z})(E+m)(E_0+m)]^{\frac{1}{2}}. \end{aligned}$$

If we consider the special case of bremsstrahlung emitted exactly parallel to the incoming electron's momentum (which is representative for radiation from relativistic electrons), then for $\epsilon_0=+1$ (i.e., "spin-forward" electrons), the matrix element summed over the two final electron spin states is

$$\sum_{\epsilon f}|M|^2=\frac{p^2\sin^2\theta}{4EE_0k^2q^4(\theta)} \times\left(\frac{(1+\delta)k(E_0+p_0+E-p\cos\theta)}{(E-p\cos\theta)^2}+\frac{(1+\delta)k^2}{(E_0-p_0)(E-p\cos\theta)}+\frac{EE_0+m^2+p_0p\cos\theta}{(E-p\cos\theta)^2}\right), \quad (7)$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{p}$. When summed over both photon polarizations, this reduces to the Bethe-Heitler expression. It is clear from this expression that at $k=0$, the lower end of the bremsstrahlung spectrum, the polarization disappears.

When integrated over θ, the outgoing electron's direction, (7) yields a somewhat complicated logarithmic function of p_0, p, and k. If R and L are the integrated cross sections for right and left circularly-polarized radiation, the polarization $(R-L)/(R+L)$, for any incident electron energy, is zero at the lower end of the bremsstrahlung spectrum and rises to its maximum value at the upper end, $k=E_0-m$. This maximum polarization is a simple function of E_0:

$$\frac{R-L}{R+L}\bigg|_{\max}=\left[1+\frac{(E_0-p_0)(E_0+m)}{(2E_0-p_0)(E_0-m)}\right]^{-1}\approx 1-\frac{1}{2}\frac{m^2}{E_0^2}, \quad (8)$$

where the second form is valid for relativistic incoming electrons. The factor $E_0-p_0=E_0(1-v_0/c)$ is the one principally responsible for high polarization, and the largest polarizations are found in bremsstrahlung from relativistic electrons.

The full expression for $(R-L)/(R+L)$ at all photon energies will be given in a subsequent paper; in its stead, we have evaluated it in Fig. 1 for the special case of 2-Mev electrons, which corresponds roughly to the experimental arrangement of Goldhaber, Grodzins, and Sunyar. At this electron energy, the polarization rises very rapidly with photon energy, to a maximum of 97% at the upper end of the bremsstrahlung spectrum. Even at as low an energy as 200 kev, (8) gives 43% for the maximum polarization. Although we have calculated the right-circular polarization from forward-spin electrons, this is equal to the left-circular polarization from backward-spin electrons.

Thanks are due to Dr. Goldhaber, Dr. Grodzins, and Dr. Sunyar for suggesting the calculation and for informing me of their experimental results before publication. I also wish to express my appreciation to Professor F. J. Dyson and to Dr. Adam Bincer for many helpful discussions and suggestions.

* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] Goldhaber, Grodzins, and Sunyar [Phys. Rev. **106**, 826 (1957)], preceeding letter.

[2] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1954), third edition, p. 242.

Production of Heavy Mesons and Hyperons by Protons on Deuterium via Secondary Pions*

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It is clear that some of the heavy mesons and hyperons which emerge when protons strike deuterium arise from a process in which a pion produced in a primary collision interacts with the second nucleon of the deuterium nucleus to produce the so-called "strange" particles. Up to the present time, the experimental data[1] are not sufficiently precise to measure the magnitude of the contribution of this two-step process. However, since there are indications that the cross section for strange-particle production in p-p collisions is appreciably smaller than that in p-d collisions at Cosmotron energies, we have been encouraged to estimate the magnitude of the contribution of this two-step process.