Angular Correlation of Annihilation Radiation from Oriented Graphite*

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THE probability of observing a certain angle between the two photons emitted in an electron-positron annihilation depends on the Fourier transform of the wave function product[1]:

$$\chi(\mathbf{p})=\int e^{-i\mathbf{p}\cdot\mathbf{r}}\psi_+(\mathbf{r})\psi_-(\mathbf{r})d\mathbf{r}. \tag{1}$$

The measurement of the angular correlation when positrons annihilate in a crystal gives, therefore, indirect information on the electron-positron momenta in the crystal. This is to be compared with a measurement of the Compton profile of an x-ray line scattered from the same crystal, which is directly related to[2]

$$\chi_-(\mathbf{p})=\int e^{-i\mathbf{p}\cdot\mathbf{r}}\psi_-(\mathbf{r})d\mathbf{r}. \tag{2}$$

The existing measurements of the angular distribution have been made in polycrystalline samples so that only the spherical average of $|\chi(\mathbf{p})|^2$ is determined.[1] In this letter we describe the experimental results for a single graphite crystal and attempt a theoretical interpretation of them.

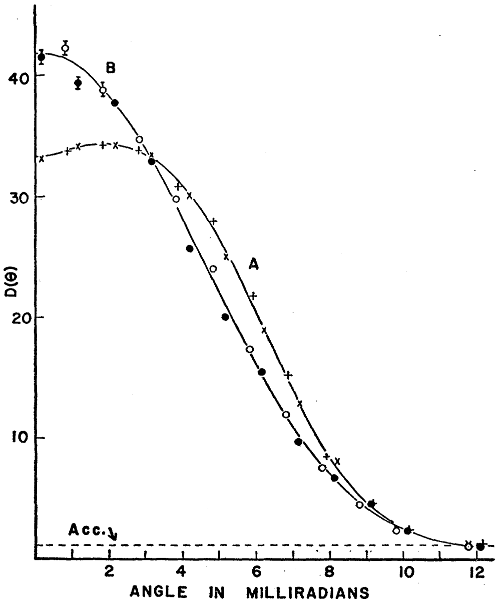

Fig. 1. Experimental correlation curve for (A) oriented graphite crystal with hexagonal planes parallel to collimating slits; (B) polycrystalline graphite. The statistical errors are shown, except where smaller than the points. The dashed line labeled "Acc." represents the accidental counting rate of the coincidence circuit.

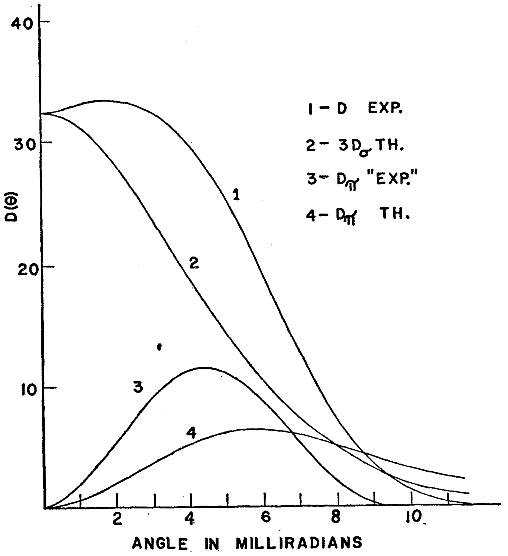

Fig. 2. Curve 1 is curve A of Fig. 1 with the accidental rate subtracted from it. Curve 2 is the theoretical $3D_\sigma(\theta)$ discussed in the text. Curve 3 is obtained by subtracting 2 from 1, and represents the experimental D_π. Curve 4 is the computed D_π assuming single atom wave functions.

The experimental setup was standard: two large NaI crystals measured coincidences between collimated annihilation gamma rays. The lead collimators subtended an angle of 1.5×10^{-3} radian from the sample, which was bombarded on one side by positrons from a Na^{22} source deposited in a lead block. The graphite crystal consisted of purified and annealed natural Madagascar single-crystal flakes, well-oriented, and cold pressed at 4000 psi into a disk 1 inch in diameter by $\frac{1}{8}$ inch thick. The *c* axis was perpendicular to the disk. The crystal was aligned so that the apparatus measured momenta perpendicular to the hexagonal planes (slits parallel to the disk). In order to minimize observed absorption effects in the sample, the disk was rotated by about 3° so as to expose the irradiated face to the movable counter. This also minimized the asymmetry of the effective radiation source resulting from the unidirectional bombardment of the sample with positrons.

The result of a typical set of measurements is shown in Fig. 1, curve A, where the experimental points taken on both sides of 0° are plotted to the same side, in order to exhibit the inherent symmetry of the setup. It is seen that the angular distribution is bimodal. The possibility of explaining this effect by coherent gamma scattering from the crystal planes was ruled out because of the rotated geometry of the sample.[3]

Since it is not easy to obtain large samples with a flat face perpendicular to the hexagonal planes, we have measured, for comparison, the angular correlation from polycrystalline graphite with the identical geometry. Curve B in Fig. 1 shows this distribution, which is in excellent agreement with Page's[4] and Stewart's[5] data. Curves A and B represent experimental

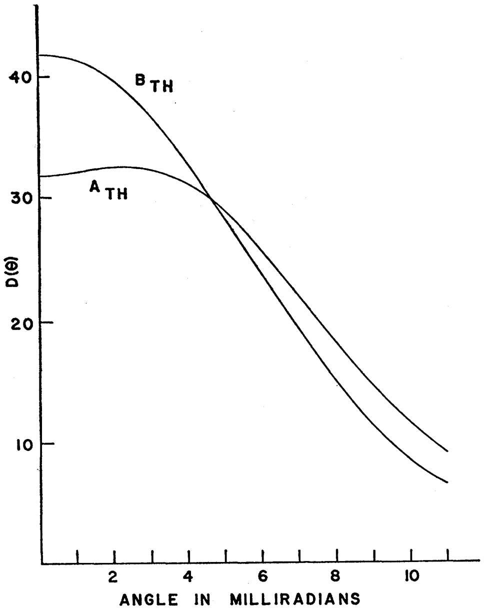

FIG. 3. Theoretical curves representing $D(\theta)$ for oriented single crystal (A_{Th}) and for polycrystalline graphite (B_{Th}).

counting rates and were not normalized in any way; their areas are equal to within 1%.

According to the theory, if the slits are parallel to the z axis and $\pi-\theta$ is the angle between the two slits as observed from the source, the angular distribution should be

$$D(\theta)=\int\int|\chi(p_x,p_y,mc\theta)|^2dp_xdp_y. \qquad (3)$$

To compute this integral, the molecular orbital approximation was made for the trigonal σ bond and the single atom $2p_z$ function was used for the π band.[6] Slater atomic wave functions were used. If the positron wave function is taken to be a constant, but annihilation with the $1s^2$ electrons is neglected, then only the momentum distribution curves for the σ and π electrons are important. That annihilation with the $1s^2$ electrons does not occur is in accord with the good experimental parabola obtained for lithium,[7] corresponding to annihilation with $2s$ electrons only. The distribution, $D_\sigma(\theta)$, of the c-axis component of the momentum for one σ electron (averaged over all directions in the hexagonal plane) is a usual bell-shaped curve. The π electrons, on the other hand, have bimodal distributions D_π, with $D_\pi(0)=0$. This is still true even when Bloch wave functions are used for the π electrons. When the distributions are added ($3D_\sigma+1D_\pi$), the final distribution is broader than for the polycrystal, but not bimodal. Since the σ bond has been computed more accurately, it might be thought that this is due to too broad a D_π. In Fig. 2 the experimental curve is shown; the theoretical $3D_\sigma$ subtracted from it gives an "experimental" D_π which is compared with the theoretical D_π. The area of the experimental D_π is indeed about $\frac{1}{3}$ of the theoretical $3D_\sigma$. This explanation has the disadvantage, however, that it would make the $3D_\sigma+D_\pi$ distribution in the polycrystal even narrower than that predicted by Coulson and Duncanson,[8] and their distribution is already narrower than the experimental Compton profile. (The Compton profile has a contribution from $1s^2$ also, but this does not alter the argument.)

To a certain extent, therefore, the positron must be making both σ and π distributions narrower, and we should not regard the D_σ as much better than the theoretical D_π. The positron may also weight the π electrons preferentially. This is also suggested by the fact that Kirkpatrick and DuMond[9] did not observe a difference in Compton profile between single and polycrystalline graphite. We show in Fig. 3 the theoretical distributions with areas in the ratio $\pi/\sigma=1.82/2.18$. This ratio gives the correct shapes, but the distributions are too broad. A further theoretical study of the correct positron wave function in graphite would therefore be needed before a better fit with experiment could be expected. We thank Dr. D. E. Soule of the National Carbon Company, for kindly supplying us with the graphite crystal.

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[1] DeBenedetti, Cowan, Konneker, and Primakoff, Phys. Rev. **77**, 205 (1950); for review articles see R. A. Ferrell, Revs. Modern Phys. **28**, 308 (1956) and S. Berko and F. L. Hereford, Revs. Modern Phys. **28**, 299 (1956).

[2] G. E. M. Jauncey, Phys. Rev. **25**, 314 (1925); for a review article see J. W. M. DuMond, Revs. Modern Phys. **5**, 1 (1933).

[3] The Bragg angle for 0.5-Mev photons occurs at $\theta\sim\hbar/(mcd)$, where d is the distance between hexagonal planes; it is interesting to note that $\theta\sim\hbar/(mcd)$ is exactly the spread of the angular correlation angle predicted by the uncertainty principle if the electrons are contained within the hexagonal planes.

[4] L. A. Page and M. Heinberg, Phys. Rev. **102**, 1545 (1956).

[5] A. T. Stewart (private communication).

[6] The importance of the bonding on the annihilation correlation can be seen by comparing our unoriented graphite curve with Lang's diamond distribution [L. G. Lang, Ph.D. thesis, Carnegie Institute of Technology, 1956 (unpublished)]; that the diamond curve is broader agrees with Duncanson and Coulson's calculations on the tetrahedral and trigonal C−C bonds [W. E. Duncanson and C. A. Coulson, Proc. Cambridge Phil. Soc. **37**, 406 (1941)].

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[8] W. E. Duncanson and C. A. Coulson, Proc. Phys. Soc. (London) **65A**, 825 (1952).

[9] H. A. Kirkpatrick and J. W. M. DuMond, Phys. Rev. **54**, 802 (1938).

Test of Neutrino—Antineutrino Identity*

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NEW mass spectroscopic data[1] for the atomic masses of Nd^{150} and Sm^{150} bear on the results of a double-beta-decay search using Nd^{150} as it relates