Parity Nonconservation and the Theory of the Neutrino

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The two-component neutrino theory of Lee and Yang and of Landau is discussed. It is shown that, for the free neutrino, their equations are equivalent to the Majorana equations, and thus are invariant under the full Lorentz group.

RECENTLY Lee and Yang[1] and Landau[2] have independently proposed a theory of the neutrino which seems to offer a satisfactory explanation of the observed left-right asymmetries in various decay processes.[3] However, it has apparently escaped attention that their equations for the free neutrino are equivalent to the Majorana[4] equations. This may be seen in detail as follows. The Dirac equations for zero rest mass are

$$\gamma_\mu p_\mu \psi = 0, \quad p_\mu \equiv -i\partial/\partial x_\mu, \tag{1}$$

where the γ-matrices may be taken to be

$$\gamma_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}, \; j=1,2,3; \quad \gamma_4 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{2}$$

The σ_j are the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

Charge conjugation[5] is defined by

$$\psi^C = C\bar{\psi}^T, \tag{4}$$

where, in the representation (2), the matrix C is given by

$$C = i\gamma_1\gamma_3. \tag{5}$$

The Majorana field is determined by the condition

$$\psi = \psi^C, \tag{6}$$

which, by using (4) and (5), can be reduced to

$$\psi_3 = \psi_2^*, \quad \psi_4 = -\psi_1^*. \tag{7}$$

If we define the two-component field

$$\varphi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \tag{8}$$

then, on taking into account Eq. (7), we can write ψ as

$$\psi = \begin{pmatrix} \varphi \\ i\sigma_2\varphi^* \end{pmatrix}. \tag{9}$$

The Dirac Hamiltonian for zero rest mass is[5]

$$H = i\gamma_4\gamma_j p_j = i\gamma_4 \boldsymbol{\gamma}\cdot\mathbf{p},$$

or

$$H = \begin{pmatrix} \boldsymbol{\sigma}\cdot\mathbf{p} & 0 \\ 0 & -\boldsymbol{\sigma}\cdot\mathbf{p} \end{pmatrix}. \tag{10}$$

Equations (1) can then be written

$$H\psi = i\partial\psi/\partial t. \tag{11}$$

This splits up into the two equations

$$\begin{gathered} \boldsymbol{\sigma}\cdot\mathbf{p}\varphi = i\partial\varphi/\partial t \\ -\boldsymbol{\sigma}\cdot\mathbf{p}(i\sigma_2\varphi^*) = i(\partial/\partial t)(i\sigma_2\varphi^*). \end{gathered} \tag{12}$$

The first of these is just the Lee-Yang equation. On using the explicit form given in the foregoing for the Pauli matrices, and taking into account that $p_j^* = -p_j$, one can reduce the second equation to

$$\boldsymbol{\sigma}^*\cdot\mathbf{p}^*\varphi^* = -i(\partial\varphi^*/\partial t),$$

which is just the complex conjugate of the Lee-Yang equation.

On the surface, the Lee-Yang equations appear not to be invariant under reflections of the coordinates, whereas the Majorana equations are known to be invariant under the full Lorentz group. The reason for this apparent contradiction may be explained as follows. If one writes the Lee-Yang equations in the van der Waerden[6] spinor notation, one gets

$$p^{\dot\beta}{}_{\alpha}\varphi^{\alpha} = 0, \tag{13}$$

the complex conjugate of which is

$$p^{\alpha}{}_{\dot\beta}\varphi^{\dot\beta} = 0. \tag{14}$$

Equation (13) is generally considered not to be invariant under improper Lorentz transformations. However, this is meant only in the conventional sense that there is no transformation of the φ^α alone which will leave Eq. (13) invariant under reflections. If one allows

[1] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[2] L. Landau, Nuclear Physics (to be published).
[3] E. Ambler, Bull. Am. Phys. Soc. Ser. II, **2**, 65 (1957); Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957); Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957). The latter three experiments were reported as post-deadline papers to the New York meeting of the American Physical Society (February 2, 1957).
[4] E. Majorana, Nuovo cimento **14**, 171 (1937). Also W. H. Furry, Phys. Rev. **54**, 56 (1938), and **56**, 1184 (1939).
[5] H. Umezawa, *Quantum Field Theory* (Interscience Publishers, Inc., New York, 1956).
[6] B. L. van der Waerden, *Gruppentheoretische Methoden in der Quantenmechanik*, (Verlag Julius Springer, Berlin, 1932).

transformations of the φ^{α} together with their complex conjugates $\varphi^{\dot{\beta}}$, then Eqs. (13) and (14) are together invariant under the full Lorentz group. There seems to be no reason for eliminating this type of transformation; in fact it is just the transformation which is used in the Majorana case, as well as in the Wigner representation of time reversals.[7] Thus we believe that the Lee-Yang equations for the free neutrino must be considered to be invariant under the full Lorentz group.

Another apparent discrepancy is the following: In the Majorana theory, one sets the particle equal to its charge conjugate, and this is generally interpreted as meaning that there is no distinction between particles and antiparticles. On the other hand, in the Lee-Yang theory, a distinction has been made between the neutrino and antineutrino. However, this difference is due to the use of two different definitions of the antiparticle. One may define it as that particle which interacts with the electromagnetic field with the charge and magnetic moment reversed from that of the particle. Clearly, in this sense neither the Majorana nor Lee-Yang equations have antiparticles different from particles. On the other hand, one may consider the antiparticle as resulting from negative energy solutions of the one-particle equation. In this sense, both the Lee-Yang and Majorana equations have antiparticles, both having negative energy solutions. The former alternative is perhaps more generally accepted. Nevertheless, it is convenient to use the latter definition for neutrinos, since one then has a law of conservation of light particles, as pointed out by Lee and Yang. However, this can be done equally well in the usual Majorana theory as in the two-component formalism.

An additional remark concerns the role of the mass in the Lee-Yang (or Majorana) theory. It is possible to include what appears to be a mass term as follows:

$$\boldsymbol{\sigma}\cdot\mathbf{p}\varphi+\kappa\sigma_2\varphi^*=i\partial\varphi/\partial t. \tag{15}$$

This equation has been considered by Jehle.[8] It is equivalent to the Majorana theory with a mass term. However, it does not appear possible to derive Eq. (15) from a Lagrangian, since the scalar $\bar{\psi}\psi$ of the Majorana theory vanishes identically, in the one-particle theory. Thus one may be justified in requiring that the mass be zero.

Of course, there is no question that the *interactions* proposed by Lee and Yang do not preserve parity. In fact, as pointed out by these authors, their theory of β decay is equivalent to the conventional theory, with the interactions

$$H'=\sum_i C_i(\bar{\psi}_p O_i\psi_n)(\bar{\psi}_e O_i P_{\pm}\psi_\nu),\quad P_{\pm}=\tfrac{1}{2}(1\pm\gamma_5). \tag{16}$$

Now it is generally assumed that the Majorana theory leads to double β decay, without emission of neutrinos. However, this conclusion can be reached only if one is restricted to parity-conserving interactions; either of the Lee-Yang interactions alone will not lead to such β-decay. Thus it is interesting that, if one uses the Majorana theory, the absence of double β decay requires the use of a parity-nonconserving interaction.

[7] E. Wigner, Gött. Nachr. **31**, 546 (1932).

[8] H. Jehle, Phys. Rev. **75**, 1609 (1949).