collection of the data presented here make it impossible to decide whether this was the case.

Until adequate data are available from high-counting-rate equipment now operating, it appears to be premature to attempt to proceed further in the discussion of possible mechanisms that could produce the observed long-term change in the asymmetry. However, the effect is likely to be linked with the mechanism that produces the long-term change in the position of the "knee" of the latitude-intensity curve, whatever this may be.[6] In any event, it seems that the systematic study of the east-west asymmetry using high-counting-rate equipment with large counter trays[7] over a period of several years will be worthwhile, since by this means we have a very sensitive method for detecting changes in the relative intensities in these directions, while virtually cancelling out otherwise troublesome atmospheric effects.

This project was assisted by the Australian National Antarctic Research Expedition and the Commonwealth Scientific and Industrial Research Organization.

[6] P. Meyer and J. A. Simpson, Phys. Rev. **99**, 1517 (1955).

[7] N. R. Parsons, Australian National Antarctic Research Expedition Interim Report No. 17 (Antarctic Division, Department of External Affairs, Melbourne).

PHYSICAL REVIEW VOLUME 106, NUMBER 4 MAY 15, 1957

Elastic and Inelastic Scattering of High-Energy Electrons by Some $(2p)$-Shell Nuclei on the Intermediate-Coupling Model

M. K. PAL AND S. MUKHERJEE
Institute of Nuclear Physics, Calcutta, India

(Received November 21, 1956)

The Møller potential has been used to describe the electron-nucleon interaction and a formula has been worked out in the Born approximation for the differential cross sections of elastic and inelastic scattering of high-energy electrons by $2p$-shell nuclei on the intermediate-coupling model with neglect of configuration mixing. Preliminary calculations have been made for Be^9 and C^{12} to test some of the qualitative features of the formula deduced, especially the θ dependence of the $\sigma(\theta)$ *vs* θ curve for inelastic scattering with change of total angular momentum J.

I. INTRODUCTION

DATA on elastic and inelastic scattering of high-energy electrons by various nuclei have been compiled through the experiments of Hofstadter and his associates.[1–3] With the nucleus taken as a continuous charge distribution of various shapes, theoretical calculations for elastic scattering have been made by Schiff[4] in the Born approximation and by Yennie *et al.*[5] through an exact phase-shift analysis. Schiff[6] was the first to deduce the cross section for inelastic scattering from the collective nuclear model. Recently, however, Tassie[7] has attempted an explanation of both elastic and inelastic scattering data from the shell-structure model of the nucleus by utilizing the results of Amaldi *et al.*[8] for μ-meson scattering by nuclei. This treatment considers the nucleus in the independent-particle model where the nucleons are assumed to move independently of one another in a strong central field. The drawback is that the low-lying levels of Be^9, to which Tassie has applied his calculations, cannot be reproduced in their known positions. Figure 2 of his paper shows how his theoretical curves differ from the experimental ones (reference 2) even in qualitative features.

At the present time the work of Jahn and his group,[9–14] following the intermediate coupling model suggested by Inglis,[15] has indicated that the low-lying levels of $2p$-shell nuclei, with the exception of a few at the end of this shell [Be^9 is a $2p$-shell nucleus with the configuration $(1s)^4(2p)^5$], all belong to the same configuration and are split up through the interactions among the nucleons. For the $2p$-shell nuclei the central and noncentral interactions are assumed to be present in comparable proportions since neither the pure LS- nor the pure jj-coupling model holds within this shell.

We have worked out the general theory of electron

[1] Hofstadter, Fechter, and McIntyre, Phys. Rev. **91**, 422 (1953); **92**, 978 (1953); Hofstadter, Hahn, Knudsen, and McIntyre, Phys. Rev. **95**, 512 (1954); J. A. McIntyre and R. Hofstadter, Phys. Rev. **98**, 158 (1955); R. Hofstadter and R. W. McAllister, Phys. Rev. **98**, 217 (1955).

[2] McIntyre, Hahn, and Hofstadter, Phys. Rev. **94**, 1084 (1954).

[3] J. H. Fregeau and R. Hofstadter, Phys. Rev. **99**, 1503 (1955).

[4] L. I. Schiff, Phys. Rev. **92**, 988 (1953).

[5] D. R. Yennie *et al.*, Phys. Rev. **95**, 500 (1954).

[6] L. I. Schiff, Phys. Rev. **96**, 765 (1954).

[7] L. J. Tassie, Proc. Phys. Soc. (London) **A69**, 205 (1955).

[8] Amaldi, Fidecaro, and Mariani, Nuovo cimento **7**, 553 (1950); **7**, 758 (1950).

[9] H. A. Jahn, Proc. Roy. Soc. (London) **A201**, 516 (1950).

[10] H. A. Jahn, Proc. Roy. Soc. (London) **A205**, 192 (1951).

[11] H. A. Jahn, cyclostyled notes of three lectures held in the Institut Henri Poincaré, Paris, in May, 1952 (unpublished).

[12] Elliott, Hope, and Jahn, Trans. Roy. Soc. (London) **A246**, 241–279 (1953).

[13] H. A. Jahn and H. Van Wieringen, Proc. Roy. Soc. (London) **A209**, 502 (1951).

[14] J. P. Elliott, Proc. Roy. Soc. (London) **A218**, 345 (1953).

[15] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).

scattering by $2p$-shell nuclei, using nuclear wave functions calculated with the above-mentioned intermediate-coupling model[16] and a Møller potential for the interaction between a nucleon and the high-energy electron. This latter has also been used by Amaldi *et al.*[8]

In working out the general formulas, we have adopted the first-order perturbation method. A better refinement would seem to be a useless luxury in view of the inaccuracy of the nuclear wave functions arising out of the uncertainty in the nucleonic interactions. Moreover, we have used the Dirac wave function for a free particle to describe both the incoming and outgoing electron but have adopted only nonrelativistic wave functions for the nucleus. This presumes small recoil velocity of the nucleus under consideration. For the same reason no distinction has been made between laboratory and center-of-mass energies and angles.

Among the $2p$-shell nuclei the work of Hofstadter and his group makes available elastic and inelastic scattering data in the case of Be^9 (reference 2) and C^{12} (reference 3). We have assumed, for Be^9, a simplified nuclear Hamiltonian having a strong Majorana term which justifies the consideration of LS-coupling states as belonging to the [41] symmetry alone so far as the first few energy levels are concerned. The results are discussed later under the proper heading. In the case of C^{12}, we have made a choice of the harmonic oscillator well-parameter by fitting the two inelastic curves corresponding to the excitation of the 4.43-Mev and 9.61-Mev levels in Fig. 10 of reference 3. With the same value of this parameter, we have tested the fit of the curves B and D of Fig. 4 with regard to their slopes and also the fit of the curve in Fig. 9 of the same reference.

It may not be out of place to mention here that in conjunction with the magnetic moment and ground-state quadrupole moment which are customarily used to test the ground-state wave functions of nuclei, the inelastic scattering data lend themselves to the testing of wave functions belonging to higher energy levels. Calculations like the ones reported here will, therefore, provide an additional check on the validity of a certain form of nuclear Hamiltonian and thus help in exploring a pertinent problem in nuclear physics, i.e., the description of nuclear forces in terms of a two-body type potential summed over all nucleon pairs.

II. SKETCH OF THE THEORY

Our treatment differs from those of references 7 and 8 in that we have used intermediate-coupling nuclear wave functions instead of their independent-particle ones. It can easily be seen that Eq. (1) of reference 7 applies to our case, when one attaches the following meaning to ρ_{if} and $\mathbf{J}_{if}$:

$$\rho_{if}=\langle\Psi_f|\textstyle\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}|\Psi_i\rangle, \tag{1a}$$

$$\mathbf{J}_{if}=\left\langle\Psi_f\left|\frac{i\hbar}{2Mc}\textstyle\sum_j[\exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\boldsymbol{\nabla}^{(j)} +\boldsymbol{\nabla}^{(j)}\exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})]\tau_-^{(j)}\right|\Psi_i\right\rangle. \tag{1b}$$

Here i and f denote initial and final states, Ψ denotes the nuclear wave function, (j) denotes the jth nucleon and the isotopic spin operator $\tau_-^{(j)}=\frac{1}{2}(1-\tau_3^{(j)})$ picks out the protonic part out of a nuclear wave function involving both neutron and proton parts. Other symbols are as used in reference 7. We have neglected the interaction of the electron with the nucleons brought about by the magnetic moments.

If one writes Ψ_i and Ψ_f in the manner of reference 14, the general matrix elements that appear in ρ_{if} and $\mathbf{J}_{if}$ are of the form

$$\langle(1s)^4(2p)^n[\lambda]LTS,JM_TM|\textstyle\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}|(1s)^4(2p)^n[\lambda']L'T'S',J'M_TM'\rangle \tag{2a}$$

and

$$(i\hbar/2Mc)\langle(1s)^4(2p)^n[\lambda]LTS,JM_TM|\textstyle\sum_j[\exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\boldsymbol{\nabla}^{(j)} +\boldsymbol{\nabla}^{(j)}\exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})]\tau_-^{(j)}|(1s)^4(2p)^n[\lambda']L'T'S',J'M_TM'\rangle, \tag{2b}$$

where the state on the right containing dashed quantum numbers is any of the LS-coupling states whose superposition is required to construct the initial nuclear state, i.e., the ground state, while the state on the left is a similar one for the final state. There will be two types of matrix elements appearing in each of (2a) and (2b) corresponding to the decoupling of an s particle or a p particle. For (2a) these two types are

$$\begin{aligned}&\langle(1s)^4(2p)^n[\lambda]LTS,JM_TM|\textstyle\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}|(1s)^4(2p)^n[\lambda']L'T'S',J'M_TM'\rangle_{ss}\\ &=\delta_{LL'}\delta_{SS'}\delta_{JJ'}\delta_{MM'}\mathfrak{A}(-)^{T+T'+2S}\times\frac{1}{[(2T+1)(2T'+1)]^{\frac{1}{2}}(2S+1)}\\ &\times\sum_{\tilde{T}\tilde{S}}(2\tilde{T}+1)(2\tilde{S}+1)\left\{\delta_{TT'}-\sqrt{3}\begin{bmatrix}T' & 1 & T\\ M_T & 0 & M_T\end{bmatrix}U(\tilde{T}T\tfrac{1}{2}1;\tfrac{1}{2}T')\right\}\end{aligned} \tag{3a}$$

[16] At the time of communicating this work for publication we have come across a paper containing intermediate-coupling model calculations by B. F. Sherman and D. G. Ravenhall [Phys. Rev. 103, 949 (1956)] for ground-state 0^+ to 7.68-Mev 0^+ inelastic scattering on C^{12}, and another report by R. A. Ferrell and W. M. Visscher, Bull. Am. Phys. Soc. Ser. II, 1, 17 (1956).

and

$$\langle (1s)^4(2p)^n[\lambda]LTS,JM_TM|\textstyle\sum_j \exp(-i\mathbf{K}\cdot\mathbf{R}^{(j)})\tau_-^{(j)}|(1s)^4(2p)^n[\lambda']L'T'S',J'M_TM'\rangle_{pp}$$

$$=\delta_{SS'}\delta_{MM'}\frac{n}{2}\sum\Bigg[a(\Psi,\bar{\Psi}p)a(\Psi',\bar{\Psi}p)\Bigg\{\delta_{TT'}-\sqrt{3}\begin{bmatrix}T' & 1 & T\\ M_T & 0 & M_T\end{bmatrix}U(\bar{T}T\tfrac{1}{2}1;\tfrac{1}{2}T')\Bigg\}$$

$$\times\Bigg\{\delta_{LL'}\delta_{JJ'}\mathcal{B}+(\sqrt{10})\begin{bmatrix}J' & 2 & J\\ M' & 0 & M\end{bmatrix}U(SJL'2;LJ')U(\bar{L}L12;1L')\mathcal{C}\Bigg\}\Bigg].\quad (3b)$$

In (3a), $\tilde{T}$ can have the common values of $T\pm\frac{1}{2}$ and $T'\pm\frac{1}{2}$. A similar remark holds for $\tilde{S}$. In (3b), $a(\Psi,\bar{\Psi}p)$ and $a(\Psi',\bar{\Psi}p)$ are single-particle-type fractional parentage coefficients tabulated in reference 13. $\begin{bmatrix}j_1 & j_2 & j\\ m_1 & m_2 & m\end{bmatrix}$ stands for the Clebsch-Gordan coefficient corresponding to the coupling of $\mathbf{j}_1$ and $\mathbf{j}_2$ to form $\mathbf{j}$; m_1, m_2, m denote the magnetic quantum numbers going with j_1, j_2, j, respectively. The function U is the same as that defined and tabulated in reference 10. The radial integrals $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ for an isotropic oscillator potential of well-parameter a_0 are given by

$$\mathcal{A}=\int_0^\infty \psi^*(1s)j_0(KR)\psi(1s)\rho^2d\rho,$$

$$\mathcal{B}=\int_0^\infty \psi^*(2p)j_0(KR)\psi(2p)\rho^2d\rho,\quad (4)$$

$$\mathcal{C}=\int_0^\infty \psi^*(2p)j_2(KR)\psi(2p)\rho^2d\rho,\quad \rho=R/a_0,$$

where $\psi(1s)$ and $\psi(2p)$ are the oscillator wave functions belonging to the $1s$ and $2p$ shells given by Swiatecki[17] and $j_0(KR)$, $j_2(KR)$ are the spherical Bessel functions given by Schiff.[18] Using explicit forms of these functions, one gets

$$\begin{aligned}\mathcal{A}&=\exp(-\tfrac{1}{4}K'^2),\\ \mathcal{B}&=(1-\tfrac{1}{6}K'^2)\exp(-\tfrac{1}{4}K'^2),\quad (5)\\ \mathcal{C}&=\tfrac{1}{6}K'^2\exp(-\tfrac{1}{4}K'^2),\quad K'=a_0K.\end{aligned}$$

To evaluate (2b), we first obtain the commutator of the operator ∇ and $e^{-i\mathbf{K}\cdot\mathbf{R}}$ using the familiar commutation rules between the components of $\mathbf{R}$ and momentum $\mathbf{P}(=-i\hbar\nabla)$. We get

$$[\nabla,e^{-i\mathbf{K}\cdot\mathbf{R}}]=-i\mathbf{K}e^{-i\mathbf{K}\cdot\mathbf{R}},\quad (6a)$$

or

$$\nabla e^{-i\mathbf{K}\cdot\mathbf{R}}=e^{-i\mathbf{K}\cdot\mathbf{R}}\nabla-i\mathbf{K}e^{-i\mathbf{K}\cdot\mathbf{R}}.\quad (6b)$$

Since the single-particle operator ∇ connects the single-particle state l with the state $l\pm1$, $e^{-i\mathbf{K}\cdot\mathbf{R}}\nabla$ will have all the matrix elements vanishing, and we obtain finally

$$\text{expression (2b)}=(\hbar/2Mc)\mathbf{K}\cdot\text{expression (2a)}.\quad (7)$$

[17] W. J. Swiatecki, Proc. Roy. Soc. (London) **A205**, 238 (1951).
[18] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 77.

With the help of these general matrix elements, one can write (1a) and (1b) for given M and M'. The averaging over M' and summation over M, as required by Eq. (1) of reference 7, can be carried out in the following way:

$$\sigma_{if}(\theta)=\frac{1}{2J'+1}\sum_M\sum_{M'}\sigma_{if}{}^{M'M}(\theta).\quad (8)$$

$\sigma(\theta)$ denotes the differential cross section per unit solid angle (sometime written $d\sigma/d\omega$) about angle θ.

III. APPLICATION TO Be^9 AND C^{12}

For Be^9 we have assumed the nuclear Hamiltonian to be of the form

$$H=V_0\sum_{i<j}\Bigg[P_{ij}{}^M\{1+g_tS_{ij}\}\frac{e^{-r_{ij}/r_0}}{r_{ij}/r_0}+g_s\frac{-r_0^2}{r_{ij}}\frac{d}{dr_{ij}}\frac{e^{-r_{ij}/r_0}}{r_{ij}/r_0}$$

$$\times(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot(\mathbf{r}_i-\mathbf{r}_j)\times(\mathbf{p}_i-\mathbf{p}_j)\Bigg],\quad (9)$$

where i, j denote any two of the nucleons; V_0 denotes the depth of the central potential; $P_{ij}{}^M$ is the Majorana exchange operator; g_t and g_s are the depth parameters for tensor and spin-orbit interactions, respectively; r_0 is the range parameter assumed to be equal for central, tensor, and spin-orbit terms; S_{ij} is the tensor interaction operator as given in reference 14.

With the strong Majorana term in the Hamiltonian (9), it may be assumed that the lowest few energy states can be constructed by superposing LS-coupling states belonging to the [41] symmetry alone (see the calculation for B^{10} in reference 14). The LS-coupling states belonging to this symmetry are taken from reference 9. The two-particle-type and single-particle-type fractional parentage coefficients have been obtained from the tables given in references 12 and 13, respectively. With their help, energy matrices have been constructed corresponding to different values of J and T, which are good quantum numbers, by using Eq. (37) of reference 12 and Eqs. (11), (13), and (17) of reference 14.

Figure 1 shows the calculated energy level sequence for two different sets of potential parameters. Only one value of r_0 has been used and this was chosen equal to that assumed by Elliott.[14] The value of the oscillator well-parameter a_0 has been determined by the formula

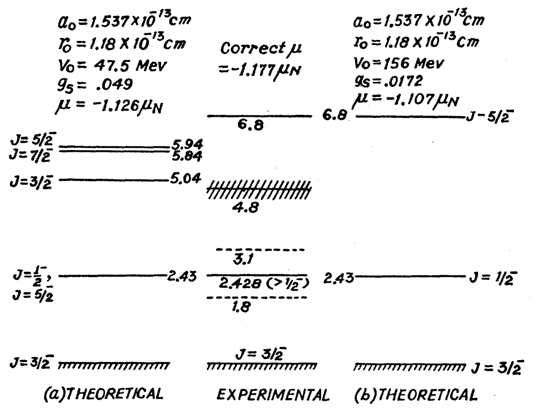

FIG. 1. Calculated and observed energy level diagrams of Be^9. The experimental values are taken from F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

given by Swiatecki.[17] In the level diagram (b), we tried to reproduce the excited levels $J=\frac{1}{2}^-$ and $J=\frac{5}{2}^-$, 2.43 Mev and 6.8 Mev, respectively, above the ground level. This made V_0 extremely high. The wave functions corresponding to this choice of parameters gave a larger cross section for inelastic scattering from the ground level to the 6.8-Mev level than for inelastic scattering from the ground level to the 2.43-Mev level. In obtaining the level sequence (a), we have chosen V_0 in conformity with low-energy n-p and p-p scattering data to correspond to the value assumed for r_0. This, with the specified value of g_s, produces the $J=\frac{1}{2}^-$ and $J=\frac{5}{2}^-$ levels at 2.43 Mev, but fails to give any level at 6.8 Mev. Since the energy of the level involved in inelastic scattering is very small compared to the energy of the electrons (190 Mev), in the formula for $\sigma_{if}(\theta)$ the energy loss of the electron can be neglected to a first approximation.[7] Consequently it seems plausible that the displacement of a level by a few Mev will not matter much in scattering calculations if its spin identification is correct. We have, therefore, used the level scheme (a) to calculate the scattering cross sections for Be^9. The results are shown in Fig. 2 together with the experimental curves of McIntyre *et al.*[2]

To the first approximation our theory predicts that the inelastic scattering cross section for levels having a J value different from that of the ground state will have the form

$$\sigma_{if}(\theta)=\frac{Z^2e^4\cos^2(\theta/2)}{4E_i^2\sin^4(\theta/2)}N\mathcal{C}^2(E_i,\theta), \qquad (10)$$

where $N\mathcal{C}^2(E_i,\theta)$ arises from the square modulus of the nuclear matrix element ρ_{if}. N is a numerical constant whose value differs for inelastic scattering corresponding to the excitation of the nucleus to its different levels. On a logarithmic plot, therefore, for all such inelastic scattering, the $\sigma(\theta)$ *vs* θ curves should run parallel, the constant N determining only the vertical position of the particular curve on the diagram. It may be mentioned here that the curves B and D of Fig. 4 in Fregeau and Hofstadter's paper[3] corroborate this conclusion. In the same figure, curve C corresponds to the excitation of the nucleus to the 7.68-Mev $J=0^+$ level, which has the same J value as the ground level. In this case the term in the nuclear matrix elements involving the radial integrals $\mathcal{A}$ and $\mathcal{B}$ also contribute to $\sigma(\theta)$, and therefore the above simple consideration does not hold.

In Be^9, however, the same conclusion leads us to expect parallelism of the two inelastic curves corresponding to the excitation of Be^9 to the 2.43- and 6.8-Mev levels. This is contrary to the experimental curves shown by McIntyre *et al.*[2] To test this point a little more closely, we have shifted our inelastic curves vertically (solid curve with filled-in triangles) to match the lower inelastic curve of McIntyre *et al.*[2] at 90°. It will be seen from our Fig. 2 that this procedure causes our theoretical curve to coincide exactly with the experimental one. However, the upper experimental inelastic curve, having a more rapid fall with increasing θ, will not match simultaneously. We tried a different value of a_0 to match the theoretical curve with the upper inelastic curve. The value required for this was 2.0×10^{-13} cm. However, with the same value the elastic curve deviated considerably from the experimental one. Lastly we also observe that since the experimental curves do not show errors, we could not

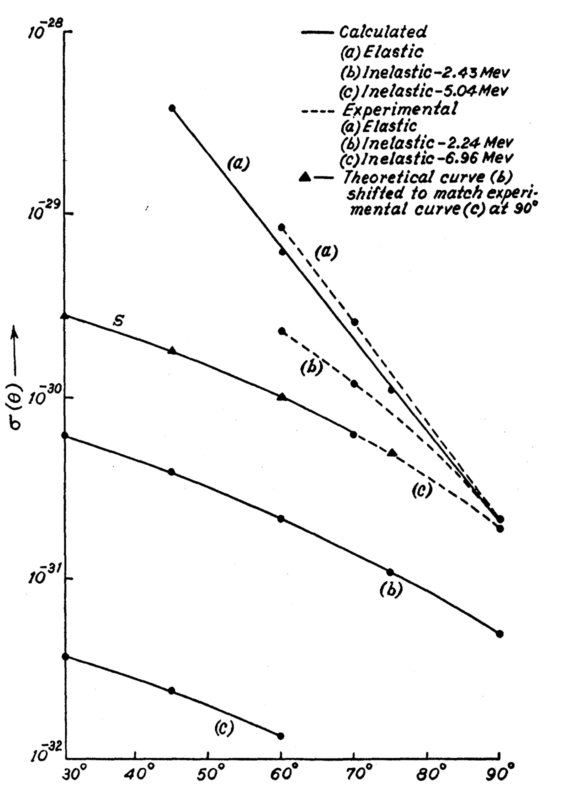

FIG. 2. Elastic and inelastic differential cross-section curves for Be^9. The experimental curves are those of McIntyre *et al.*[2]

try to make an accurate determination of a_0 from them in the above-mentioned way.

We do not ascribe any importance to the vertical position of our inelastic curves on the logarithmic plot. This is because the simplified approach of our calculations, i.e., the consideration of only the [41] symmetry states, does not allow us to claim any accuracy for the multiplying factor N. That the states belonging to [41] symmetry are inadequate can be inferred from the following qualitative arguments. It is known at the present time[19] that both the tensor and spin-orbit interaction should be present simultaneously in the nuclear Hamiltonian. But the states of [41] symmetry of Be^9 have only one value, i.e., $\frac{1}{2}$, of S and hence a spin tensor of second rank, like the one appearing in the tensor interaction S_{ij}, will have vanishing matrix elements between any two of these states. This means a total absence of the tensor interaction, though we have included it formally in the Hamiltonian (9).

As regards C^{12}, we have already discussed the parallelism of two of the inelastic curves. In this case we have made a choice of a_0 by trying to fit the two curves, parallel to the abscissa, in Fig. 9 of reference 3. Though the errors indicated in this figure give some freedom in this choice, it seems that $a_0=1.582\times10^{-13}$ cm gives a better fit than the value $a_0=1.692\times10^{-23}$ cm obtained from Swiatecki's formulas. In this connection we observe that Swiatecki's criterion for the

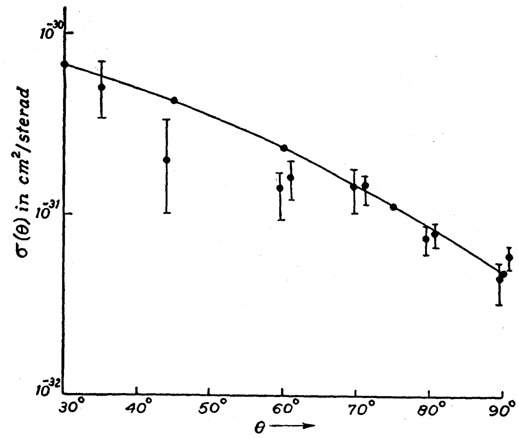

FIG. 4. Comparison of the inelastic ($J'\neq J$) $\sigma(\theta)$ *vs* θ curve for C^{12} with the experimental data of Fregeau and Hofstadter.[3]

determination of a_0, i.e., that the value of r at which $|u_{2p}(r)|^2$ reduces to half of its maximum value is the nuclear radius, is to some extent arbitrary.

With $a_0=1.582\times10^{-13}$ cm, we have plotted $\mathcal{C}^2$ as a function of K. From Eq. (10) it can be easily seen that the points on this curve will differ by a constant factor from the corresponding points on the curve in Fig. 8 of reference 3. To show this, we have given our Fig. 3 which shows side by side the $\mathcal{C}^2$ plot and the above-mentioned experimental curve. Below the point $K=1.2\times10^{13}$ cm^{-1}, the ordinates on our curve can be divided by the same constant factor 1.8 to get the corresponding ordinates of the experimental curve. As regards the drop in the $\mathcal{C}^2$ curve beyond $K=1.2\times10^{13}$ cm^{-1}, we cannot say anything conclusively because the experimental errors permit one to alter the trend of the smooth curve drawn by Fregeau and Hofstadter.[3]

With the same value of a_0, we have plotted the inelastic ($J'\neq J$) $\sigma(\theta)$ *vs* θ curve for C^{12} in Fig. 4. The constant N, which should be determined by the level concerned, has been arbitrarily chosen to fit the experimental value at 90° on curve D of Fig. 4 of reference 3. It can be seen that the experimental points at other angles agree fairly well with the trend of our curve.

Exact calculations for C^{12} are in progress and we reserve any quantitative conclusions until these are completed.

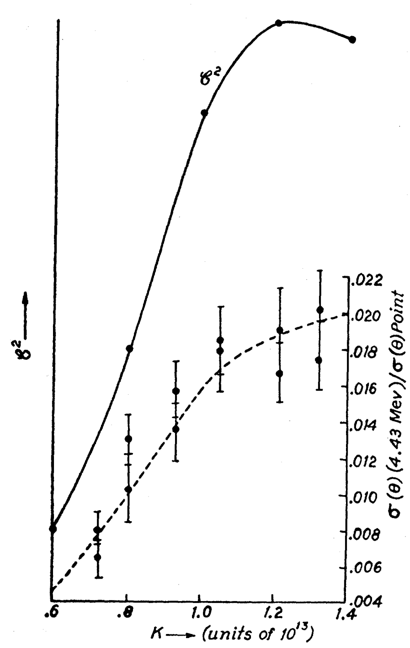

FIG. 3. Comparison of the $\mathcal{C}^2$ *vs* K curve for C^{12} with the experimental $\sigma(\theta)_{4.43\ \mathrm{Mev}}/\sigma(\theta)_{\mathrm{point}}$ *vs* K curve of Fregeau and Hofstadter.[3]

IV. SUMMARY OF RESULTS AND DISCUSSIONS

In the case of Be^9 the wave function of the ground state within the [41] symmetry alone, as found by us, has reproduced the observed elastic scattering to a tolerable extent. But the wave function for the level $J=\frac{1}{2}^-$, not to mention the next higher level, has produced much lower scattering than that observed. Moreover, our formulation predicts a parallelism of the two inelastic curves, which does not agree with the experimental data. By trying to match the θ dependence of the inelastic ($J'\neq J$) $\sigma(\theta)$ *vs* θ curve by a vertical

[19] M. K. Banerjee, Ph.D. dissertation, Calcutta University (unpublished); W. M. Visscher and R. A. Ferrell, University of Maryland, Physics Department, Technical Report No. 19 (September, 1955) (unpublished).

shift of our inelastic curves, we have seen that our calculations agree with the lower inelastic curve for a value of a_0 equal to 1.537×10^{-13} cm calculated from Swiatecki's formula. For the same a_0 the elastic curve agrees tolerably well with the experimental one. For a choice of 2.0×10^{-13} cm for a_0, the θ dependence of our inelastic curve can be matched with that of the upper inelastic curve. For the same a_0 the elastic cross-section curve gives poorer agreement. Finally, since the errors in the experimental curves are not indicated, it seems that we cannot determine a_0 very accurately in the above way.

For C^{12} we have determined $a_0=1.582\times10^{-13}$ cm, which is considerably lower than the value $a_0=1.692\times10^{-13}$ cm obtained from Swiatecki's formula. We observe that this determination of a_0 is not unique in view of the experimental errors. The above choice of a_0 has reproduced the experimentally observed trend of $\mathcal{C}^2$ *vs* K curve below $K=1.2\times10^{13}$ cm^{-1}; above this value the theoretical curve drops while the smooth curve drawn in reference 3 does not show any such trend. We remark that the large errors in the experimental data allow much freedom in altering the experimental curve in this region, and hence no definite conclusions can be made. With the same value of a_0, we have found the trend of our inelastic $(J'\neq J)$ $\sigma(\theta)$ *vs* θ curve to agree fairly well with the observed data within experimental errors.

We emphasize that all our calculations for C^{12} have been directed to testing the qualitative features of the inelastic curves, and the deliberate simplifications we have made in finding the wave functions of Be^9 do not permit us to claim anything quantitatively there also. Further limitations of our method of calculation have been pointed out in the introductory section. The neglect of interaction through the magnetic moment, which will increase the cross section, has also been mentioned in an earlier section.

The calculations reported here have been made principally with a view to testing whether, without going into the detailed procedure of wave-function calculation, we get any evidence from the experimental data contradicting some qualitative feature of our formula. Fortunately we have not obtained any such evidence. To test quantitatively the cross-section formulas, especially those for inelastic scattering, we are at present determining exact wave functions for C^{12} by reproducing the known energy levels after an exact diagonalization of the energy matrices. The nuclear Hamiltonian assumed is in conformity with two-body binding and scattering data. The scattering cross section obtained with such wave functions will be reported elsewhere in due time.

Pending such detailed quantitative testing, we make a passing observation that the inelastic $(J'\neq J)$ $\sigma(\theta)$ *vs* θ curve calculated by our formula allows a more satisfactory determination of a_0 than is obtained by the method of Swiatecki.[17]

ACKNOWLEDGMENTS

We are indebted to Professor A. K. Saha for his unceasing interest in the present work and his many helpful discussions during the period it has been carried out.

PHYSICAL REVIEW VOLUME 106, NUMBER 4 MAY 15, 1957

Interference Phenomena of K^+-Meson Scattering by Nuclei*

D. FOURNET DAVIS†
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received January 29, 1957)

An analysis of the elastic scatters of K^+ mesons in emulsion shows that the data favor a repulsive nuclear potential. A description of the analysis of the experimental results is given.

An analysis of the elastic scattering of K^+ mesons in emulsion has been made using the Born approximation. It is found that the data favor a repulsive nuclear potential when compared with the theoretically predicted curves.[1]

The Born approximation gives the differential cross section for K^+ mesons as

$$d\sigma/d\Omega=[Zf_p+(A-Z)f_n\pm Zf_C]^2F^2 \text{ (elastic or coherent)}$$
$$+[Z(f_C\pm f_p)^2+(A-Z)f_n^2][1-F^2] \text{ (inelastic or incoherent)},$$

where f_C, f_p, and f_n are the Coulomb, proton, and neutron scattering amplitudes and F is the nuclear form factor. The plus or minus sign is chosen depending on whether the nuclear and Coulomb forces are of the same or opposite sign, i.e., whether the nuclear potential

* This work was supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

† American Association of University Women Fellow, now at the University of Rochester, Rochester, New York.

[1] L. S. Osborne, Phys. Rev. 102, 296 (1956).