Noise in a Molecular Amplifier*

M. W. MULLER
Varian Associates, Palo Alto, California and Stanford University, Stanford, California
(Received December 3, 1956)

The quantum theory of noise in lossy electrical circuits is extended to systems which are not in thermal equilibrium. The theory is applied to microwave molecular amplifiers (masers) and predicts a correction to the classically calculated noise figure which can be identified with spontaneous emission from the molecules of the active medium. The correction is small at ordinary temperatures but becomes significant at very low temperatures.

I. INTRODUCTION

THE limiting sensitivity of high-frequency electronic amplifiers is determined primarily by the thermal noise which arises from the uncontrollable motion of charged particles in the dissipative elements of the amplifier, and by shot noise whose source is the finite size of the elementary quantum of electric charge. The invention of the NH_3 "maser,"[1] an amplifier which is based on the interaction of the electromagnetic field with uncharged particles, and its proposed magnetic analogs,[2] provides a means of eliminating shot noise. Moreover, the possibility (or necessity) of employing materials cooled to very low temperatures to interact with radiation in these devices has led to the hope that amplifiers of exceedingly low noise factor might eventually result from this technique.

We propose to examine this idea in the light of the following considerations:

(1). The material which interacts with the radiation in a maser is not in thermal equilibrium[3] and thus caution must be exercised in assigning to it a noise temperature.

(2). Interaction can take place with vacuum fluctuations of the electromagnetic field (spontaneous emission), producing noise which could not be predicted by classical theory.

These considerations have been treated from a phenomenological point of view by Shimoda, Takahasi, and Townes.[4] The present study is based on a theory of noise proposed by Callen and Welton[5] and developed further by Weber[6] and by Ekstein and Rostoker.[7]

II. NOISE THEORY OF NONEQUILIBRIUM SYSTEMS

A circuit capable of supporting a normal mode of frequency ω can be represented by a harmonic oscillator whose Hamiltonian is

$$H=\tfrac{1}{2}(p^2+\omega^2q^2). \quad (1)$$

We consider the circuit coupled to two dissipative media one of which is in thermal equilibrium; the Hamiltonian of the system is[6]

$$H=\tfrac{1}{2}(p^2+\omega^2q^2)+H_{R1}+H_{R2}+\frac{p}{\sqrt{C}}(Q_1+Q_2), \quad (2)$$

where H_{R1}, H_{R2} are the Hamiltonians of the unperturbed media, Q_1, Q_2 are functions of the coordinates and momenta of the particles in the media, and C is the capacity of the circuit.

The total transition probability for the exchange of energy between the circuit and the dissipative media, if initially the circuit is in a state of energy E_F, and the media in states of energies E_{R1} and E_{R2}, is

$$\begin{aligned}W_R=\frac{2\pi}{\hbar}\{&\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle^2\\ &\times[\rho(E_{R1}+\hbar\omega)\langle E_{R1}|Q_1|E_{R1}+\hbar\omega\rangle^2\\ &+\rho(E_{R2}+\hbar\omega)\langle E_{R2}|Q_2|E_{R2}+\hbar\omega\rangle^2]\\ &+\langle E_F|p/\sqrt{C}|E_F+\hbar\omega\rangle^2\\ &\times[\rho(E_{R1}-\hbar\omega)\langle E_{R1}|Q_1|E_{R1}-\hbar\omega\rangle^2\\ &+\rho(E_{R2}-\hbar\omega)\langle E_{R2}|Q_2|E_{R2}-\hbar\omega\rangle^2]\}, \quad (3)\end{aligned}$$

where $\rho(E)$ is the density-in-energy of the states of the media.

Now the information about the energy state of medium 1 is statistical in all cases, while the information about medium 2 may be either statistical or exact. The first of these alternatives arises, for example, when the medium is made active by inverting the state populations from equilibrium, as in some of the proposed paramagnetic masers; the second is characteristic of a process such as is used in the ammonia maser, in which the upper and lower state populations are considered to be known exactly. We will deal with both alternatives here and show that they are substantially equivalent.

The average transition probability due to medium 1 is obtained by averaging over an ensemble of similar systems. This calculation has been carried out by

* Work supported by the U. S. Army Signal Corps.
[1] Gordon, Zeiger, and Townes, Phys. Rev. **99**, 1264 (1955); referred to hereafter as GZT.
[2] A. Honig and J. Combrisson, Phys. Rev. **102**, 917 (1956); G. Feher and E. A. Gere, Phys. Rev. **103**, 501 (1956); N. Bloembergen, Phys. Rev. **104**, 324 (1956).
[3] N. F. Ramsey, Phys. Rev. **103**, 20 (1956).
[4] Shimoda, Takahasi, and Townes (private communication). The author is indebted to Professor Townes for furnishing him with a manuscript of this work before publication.
[5] H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).
[6] J. Weber, Phys. Rev. **90**, 977 (1953); **94**, 211, 215 (1954); **101**, 1619, 1620 (1956).
[7] H. Ekstein and N. Rostoker, Phys. Rev. **100**, 1023 (1955).

Weber[6] but we reproduce it here because we will utilize a similar procedure later.

$$\langle W_{R1}\rangle=\frac{2\pi}{\hbar}\Big\{\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle^2\int_0^\infty\rho(E_{R1}+\hbar\omega)\times\langle E_{R1}+\hbar\omega|Q_1|E_{R1}\rangle^2\rho(E_{R1})f(E_{R1})dE_{R1}+\langle E_F|p/\sqrt{C}|E_F+\hbar\omega\rangle^2\int_{\hbar\omega}^\infty\rho(E_{R1}-\hbar\omega)\times\langle E_{R1}-\hbar\omega|Q_1|E_{R1}\rangle^2\rho(E_{R1})f(E_{R1})dE_{R1}\Big\}$$
$$=\frac{2\pi}{\hbar}\Big\{\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle^2\int_0^\infty\rho(E_{R1}+\hbar\omega)\times\langle E_{R1}+\hbar\omega|Q_1|E_{R1}\rangle^2\rho(E_{R1})f(E_{R1})dE_{R1}+\langle E_F|p/\sqrt{C}|E_F+\hbar\omega\rangle^2\int_0^\infty\rho(E_{R1})\times\langle E_{R1}|Q_1|E_{R1}+\hbar\omega\rangle^2\rho(E_{R1}+\hbar\omega)\times[f(E_{R1}+\hbar\omega)/f(E_{R1})]f(E_{R1})dE_{R1}\Big\}. \quad (4)$$

Here $f(E)$ is the statistical weighting factor, and

$$f(E_{R1}+\hbar\omega)/f(E_{R1})=\exp(-\hbar\omega/kT). \quad (5)$$

If we introduce the quantity

$$S_1=\frac{2\pi}{\hbar}\int_0^\infty\rho(E_{R1}+\hbar\omega)\langle E_{R1}|Q_1|E_{R1}+\hbar\omega\rangle^2\times\rho(E_{R1})f(E_{R1})dE_{R1}, \quad (6)$$

the transition probability becomes

$$\langle W_{R1}\rangle=S_1[\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle^2+\langle E_F|p/\sqrt{C}|E_F+\hbar\omega\rangle^2\exp(-\hbar\omega/kT)]. \quad (7)$$

Now consider the first alternative discussed above for medium 2, i.e., that of an active material produced by state inversion (system 1). Then the calculation used for medium 1 applies to medium 2, with the modification

$$f(E_{R2}+\hbar\omega)/f(E_{R2})=\exp(+\hbar\omega/kT), \quad (8)$$

so that

$$\langle W_{R2}\rangle=S_2[\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle^2+\langle E_F|p/\sqrt{C}|E_F+\hbar\omega\rangle^2\exp(+\hbar\omega/kT)]. \quad (9)$$

If the state populations are known exactly (system 2), say n_a and n_b for the upper and lower state respectively, then no averaging is necessary and

$$\langle W_{R2}\rangle=\frac{2C}{\hbar\omega}[\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle^2 b_2'+\langle E_F|p/\sqrt{C}|E_F+\hbar\omega\rangle^2 a_2'], \quad (10)$$

where a_2' and b_2' are proportional to the matrix element between the levels, and to n_a and n_b, respectively.

We can now calculate the time-dependent behavior of the circuit by collecting the terms which induce upward and downward transitions. For system 1 the energy U stored in the circuit is governed by the equation

$$\frac{dU}{dt}=\frac{(\hbar\omega)^2}{2C}\Big\{S_1\Big[(n+1)\exp\Big(-\frac{\hbar\omega}{kT}\Big)-n\Big]+S_2\Big[(n+1)\exp\Big(+\frac{\hbar\omega}{kT}\Big)-n\Big]\Big\}, \quad (11)$$

where we have put in the values of the harmonic oscillator matrix elements. We define the quantities

$$G_1=\tfrac{1}{2}\hbar\omega S_1[1-\exp(-\hbar\omega/kT)],$$
$$G_2=\tfrac{1}{2}\hbar\omega S_2[1-\exp(+\hbar\omega/kT)]. \quad (12)$$

We can solve Eq. (11) by using the fact that $U=(n+\frac{1}{2})\hbar\omega$, obtaining

$$U=\tfrac{1}{2}\hbar\omega\Big[G_1\frac{1+\exp(-\hbar\omega/kT)}{1-\exp(-\hbar\omega/kT)}+G_2\frac{1+\exp(\hbar\omega/kT)}{1-\exp(\hbar\omega/kT)}\Big]\times\Big[\frac{1-\exp[-(G_1+G_2)t/C]}{G_1+G_2}\Big]+U_0\exp[-(G_1+G_2)t/C]. \quad (13)$$

This has the proper classical behavior, if we identify the quantities G_1 and G_2 as the classical conductances. We note that $G_2<0$ as would be expected of the conductance of an active element.

If $G_1+G_2>0$, the system is stable and has a steady state energy (after subtracting the zero-point energy) of

$$U(\infty)=\frac{\hbar\omega}{G_1+G_2}\Big[\frac{G_1\exp(-\hbar\omega/kT)}{1-\exp(-\hbar\omega/kT)}+\frac{G_2\exp(\hbar\omega/kT)}{1-\exp(\hbar\omega/kT)}\Big]. \quad (14)$$

With the approximation $\hbar\omega\ll kT$, Eq. (14) becomes

$$U(\infty)\cong\Big(\frac{G_1-G_2}{G_1+G_2}\Big)kT, \quad (15)$$

which indicates that at ordinarily encountered temperatures the noise contribution of a negative conductance is given by the Nyquist noise[8] appropriate to its absolute value.

[8] H. Nyquist, Phys. Rev. 35, 110 (1928).

The energy equation of system 2 is

$$\frac{dU}{dt}=\frac{(\hbar\omega)^2}{2C}S_1[(n+1)\exp(-\hbar\omega/kT)-n] +\hbar\omega[(n+1)a_2'-nb_2'], \quad (16)$$

which is solved by

$$U=\tfrac{1}{2}\hbar\omega\left[\frac{G_1}{C}\left(\frac{1+\exp(-\hbar\omega/kT)}{1-\exp(-\hbar\omega/kT)}\right)+a_2'+b_2'\right] \times\left[\frac{1-\exp\{-[(G_1/C)+b_2'-a_2']t\}}{(G_1/C)+b_2'-a_2'}\right] +U_0\exp\{-[(G_1/C)+b_2'-a_2']t\}, \quad (17)$$

and which, in the stable case, has the steady-state energy (less zero-point energy) of

$$U(\infty)=\frac{\hbar\omega a_2'+(G_1/C)\hbar\omega/[\exp(\hbar\omega/kT)-1]}{(G_1/C)+b_2'-a_2'}. \quad (18)$$

The meaning of Eqs. (13) and (17) becomes more apparent if we note that

$$G_k/C=\omega/Q_k \quad (19)$$

is the net probability per unit time of absorption (or emission if G_k, $Q_k<0$) due to the loss represented by Q_k and that b' and a' have the same meanings for the exactly specified active medium of system 2. Thus we can write

$$G_k/C=b_k'-a_k'=\begin{cases} b_k \\ -a_k \end{cases}, \quad (20)$$

where we use b_k for a net absorption probability and a_k for a net emission probability.

If we define an "effective temperature" for the active medium in system 2 by the relation

$$a_2'/b_2'=\exp(\hbar\omega/kT_e), \quad (21)$$

where it is understood that this definition does not imply thermal equilibrium, then from Eqs. (20) and (21)

$$a_2'=\frac{b_2'-a_2'}{\exp(-\hbar\omega/kT_e)-1}=\frac{a_2\exp(\hbar\omega/kT_e)}{\exp(\hbar\omega/kT_e)-1}. \quad (22)$$

With this new notation we can rewrite, for example, Eq. (14):

$$U(\infty)=\frac{\hbar\omega}{b_1-a_2}\left[\frac{b_1}{\exp(\hbar\omega/kT)-1}+\frac{a_2\exp(\hbar\omega/kT)}{\exp(\hbar\omega/kT)-1}\right], \quad (23)$$

and Eq. (18):

$$U(\infty)=\frac{\hbar\omega}{b_1-a_2}\left[\frac{b_1}{\exp(\hbar\omega/kT)-1} +\frac{a_2\exp(\hbar\omega/kT_e)}{\exp(\hbar\omega/kT_e)-1}\right]. \quad (24)$$

Alternatively, by noting that for the medium in thermal equilibrium

$$a_1'/b_1'=\exp(-\hbar\omega/kT), \quad (25)$$

we have, for both Eqs. (14) and (18),

$$U(\infty)=\hbar\omega\left(\frac{a_1'+a_2'}{b_1-a_2}\right). \quad (26)$$

Thus it is quite apparent that systems 1 and 2 are fully equivalent and that there is no further need for separate developments.

A result equivalent to Eq. (26) has been deduced by Shimoda, Takahasi, and Townes[4] on the basis of a stochastic equation for the emission and absorption processes. The present Eq. (26) and the theory leading to it differs from that result in that the noise energy from the active and passive media are treated separately. This feature makes the present theory suitable for the direct calculation of the noise figure of an amplifier, since the concept of noise figure is based on the comparison of excess noise with thermal noise.

III. APPLICATION TO THE REGENERATIVE MASER AMPLIFIER

A simple kind of molecular amplifier is shown in Fig. 1. This regenerative amplifier was discussed originally by GZT on a classical basis. We deal first with the stable amplifier for which $b_1>a_2$.

For the system of Fig. 1,

$$b_1=b_i+b_o+b_c, \quad (27)$$

where the subscripts i, o, c refer to the input coupling, output coupling, and cavity walls, respectively.

In terms of this notation the gain and noise factor of the amplifier as given by GZT appear as

$$\mu=\frac{4b_ib_o}{(b_1-a_2)^2}, \quad (28)$$

$$F=\frac{a_2}{b_i}+\frac{1}{\mu}. \quad (29)†$$

Equation (29) does not take into account any noise generated in the active medium. In order to include this effect, we obtain from the theory of Sec. II the amount of energy stored in the cavity in the steady

† *Note added in proof.*—Eq. (29) does not conform to the conventional definition of noise figure, because it is based on the signal-to-noise ratio in the wave traveling away from the output which contains some (but not all) of the noise generated in the load. The conventional noise figure is $F=a_2/b_i$; if all the noise from the load is assessed against the amplifier, the noise figure is $F=b_1/b_i$. This last equation is based on the signal-to-noise ratio which would be measured immediately following the amplifier and thus is most meaningful in practice. All these equations are equivalent when $\mu\gg1$. I am indebted to the referee and to Professor E. T. Jaynes for this point.

state due to this source:

$$U_2=\frac{\hbar\omega a_2'}{b_1-a_2}. \qquad (30)$$

Equation (30) is an expression for the total energy integrated over all frequencies. In order to obtain a noise figure we need the spectral density of this energy. The spectral density can be obtained readily by an extension of the theory of Sec. II which has been carried out for media in thermal equilibrium by Weber[6] and Ekstein and Rostoker.[7] Their procedure can be carried out with no essential modification for nonequilibrium systems and leads to the following expression of the spectral density of a noise current generator equivalent to the medium:

$$\begin{aligned}\langle G_I(\omega)\rangle&=\frac{2|G_2|}{\pi}\left[\frac{\hbar\omega}{2}+\frac{\hbar\omega a_2'}{a_2'-b_2'}\right]\\&=\frac{2|G_2|}{\pi}\left[\frac{\hbar\omega}{2}+\frac{\hbar\omega}{\exp(\hbar\omega/kT_e)-1}\right].\end{aligned} \qquad (31)$$

In Eq. (31) the quantities G_2, a_2', b_2' are of course functions of frequency; their variation is given essentially by the line shape of the transition and thus depends on the details of the amplifier design. It has been shown, however,[1] that if the gain of the amplifier is large, its band width is small compared with the line width, so that over the band width of the amplifier we may consider $\langle G_I(\omega)\rangle$ to be constant.

We can now compute the noise figure of the amplifier by extending the method of GZT:

The amount of thermal noise power incident on the maser cavity in a frequency interval $\Delta\omega$ due to the input signal generator impedance is

$$P_i(\omega)\Delta\omega=\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\left(\frac{\Delta\omega}{2\pi}\right). \qquad (32)$$

The net thermal noise power transmitted into the cavity is given by

$$P_{\mathrm{tr}}(\omega)\Delta\omega=\left(\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right)\left(\frac{\Delta\omega}{2\pi}\right)\times\left(\frac{4b_i(b_1-b_i-a_2)}{(b_1-a_2)^2+4(\omega-\omega_c)^2}\right), \qquad (33)$$

where ω_c is the resonant frequency of the cavity in the presence of the active medium. The energy stored in the cavity due to the generator's thermal noise is

$$U_i(\omega)\Delta\omega=\left(\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right)\left(\frac{\Delta\omega}{2\pi}\right)\times\left(\frac{4b_i}{(b_1-a_2)^2+4(\omega-\omega_c)^2}\right). \qquad (34)$$

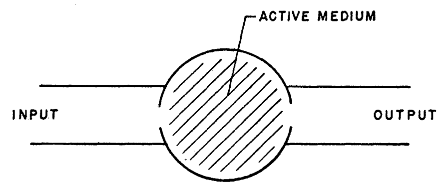

FIG. 1. A cavity maser.

It might be noted that Eq. (34) which for reasons of convenience we have deduced from classical circuit theory, also follows from Eqs. (23) of (24), Eq. (31), and the discussion of band width.

By the same argument the energy stored in the cavity due to thermal noise from the walls is

$$U_c(\omega)\Delta\omega=\left(\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right)\left(\frac{\Delta\omega}{2\pi}\right)\times\left(\frac{4b_c}{(b_1-a_2)^2+4(\omega-\omega_c)^2}\right), \qquad (35)$$

and the energy stored due to noise from the active medium is

$$U_2(\omega)\Delta\omega=\hbar\omega\frac{\Delta\omega}{2\pi}\left(\frac{4a_2'}{(b_1-a_2)^2+4(\omega-\omega_c)^2}\right), \qquad (36)$$

so that

$$\int_{-\infty}^{\infty}U_2(\omega)d\omega=U_2. \qquad (37)$$

From Eqs. (34), (35), and (36) we find, for the noise power in the output from sources other than the load impedance,

$$P_0'(\omega)\Delta\omega=\left(\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right)\left(\frac{\Delta\omega}{2\pi}\right)\times\left(\frac{4b_0(b_i+b_c)}{(b_1-a_2)^2+4(\omega-\omega_c)^2}\right)+\hbar\omega\frac{\Delta\omega}{2\pi}\left(\frac{4b_0a_2'}{(b_1-a_2)^2+4(\omega-\omega_c)^2}\right). \qquad (38)$$

The total noise power in the output wave guide, including noise power generated in the load and reflected at the output coupling, is

$$P_0(\omega)\Delta\omega=\left(\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right)\left(\frac{\Delta\omega}{2\pi}\right)\times\left[\frac{4b_0a_2}{(b_1-a_2)^2+4(\omega-\omega_c)^2}-1\right]+\hbar\omega\frac{\Delta\omega}{2\pi}\left(\frac{4b_0a_2'}{(b_1-a_2)^2+4(\omega-\omega_c)^2}\right), \qquad (39)$$

so that the noise figure becomes

$$F=\frac{a_2}{b_i}+\frac{1}{\mu}+\frac{a_2'}{b_i}[\exp(\hbar\omega/kT)-1]. \qquad (40)$$

Thus the noise from the active medium has a negligible effect on the sensitivity of a maser amplifier, if the absolute value of effective temperature $|T_e|$ is small compared with the ambient and input temperature T. However it has been proposed that maser amplifiers employing materials at liquid helium temperatures be used, say for radio-astronomical applications. In such applications the noise figure would be based on the temperature of the sky, which is of the order of 1°K. Under these circumstances the correction embodied in Eq. (40) might amount to several decibels at the higher microwave frequencies. When $T_e\sim T$, which will be true under these conditions, and perhaps at ordinary temperatures for system 1, discussed above, the last term on the right-hand side of Eq. (40) will not be negligible. From Eq. (28).

$$b_1-a_2=(4b_ib_0/\mu)^{\frac{1}{2}},$$

and using Eqs. (20), (21), and (27), we obtain

$$\frac{a_2'}{b_i}=\left(\frac{1}{1-\exp(-\hbar\omega/kT_e)}\right)\left[1+\frac{b_0+b_c}{b_i}-\left(\frac{4b_0}{b_i\mu}\right)^{\frac{1}{2}}\right].$$

If we substitute this expression in Eq. (40) and use Eq. (22), we obtain

$$F=\left[1+\frac{b_0+b_c}{b_i}-\left(\frac{4b_0}{b_i\mu}\right)^{\frac{1}{2}}\right]\times\left[1+\exp(\hbar\omega/kT)\left(\frac{1-\exp(-\hbar\omega/kT)}{1-\exp(-\hbar\omega/kT_e)}\right)\right],$$

with the following special cases: If $\hbar\omega\ll kT$, $\hbar\omega\ll kT_e$, $\mu\gg1$,

$$F\cong\left[1+\frac{b_0+b_c}{b_i}\right]\left[1+\frac{T_e}{T}\right].$$

If $T_e=T$ (system 1), $\mu\gg1$,

$$F\cong\left[1+\frac{b_0+b_c}{b_i}\right][1+\exp(\hbar\omega/kT)].$$

From these equations we can conclude that for a regenerative maser it is preferable to obtain the active medium by selective focusing or optical pumping rather than by state inversion.

IV. SUPERREGENERATIVE AMPLIFIER

If in the system of Fig. 1 the quantity of active material is increased until $a_2>b_1$, the amplifier is no longer stable and the energy in the cavity will increase exponentially as predicted by Eq. (13) or (17). If some appropriate means of quenching the oscillation is provided, the system can be used as a super-regenerative amplifier.

It is not possible to predict the noise figure or gain of such an amplifier without knowing the details of its design. It is possible, however, to calculate the excess of noise above its classical value due to emission from the active material. For this purpose we make the following assumptions:

(1) At the start of the exponential rise the cavity is in thermal equilibrium, so that

$$U_0=\hbar\omega/[\exp(\hbar\omega/kT)-1]. \qquad (41)$$

(2) The "gain" is large, that is to say, the signal is allowed to build up long enough so that

$$\exp[-(b_1-a_2)t]\gg1. \qquad (42)$$

With these assumptions Eq. (17) becomes, after subtracting the zero-point energy,

$$U=\frac{\hbar\omega}{2}\left(\frac{1}{b_1-a_2}\right)\left\{\left[\frac{2b_1}{\exp(\hbar\omega/kT)-1}+\frac{2}{\exp(\hbar\omega/kT)-1}\right]+a_2'\right\}\exp[-(b_1-a_2)t]. \qquad (43)$$

The classical thermal noise is due to the term in square brackets in Eq. (42); the excess noise is due to the second term. The contribution arising from U_0 is negligible since $b_1\gg1$, and thus the correction to the classical noise figure is

$$F=F_{\mathrm{cl}}\{1+(a_2'/b_1)[\exp(\hbar\omega/kT)-1]\}, \qquad (44)$$

where F_{cl} is the noise figure that is obtained when the noise generated by the active medium is neglected.

V. CONCLUSION

It has been shown, by an extension of the quantum theory of noise to systems not in thermal equilibrium, that the active material in a molecular amplifier makes a finite contribution to the noisiness of the system. The applications treated include the stable regenerative maser and the superregenerative maser. The correction due to this noise source is small at the usual microwave frequencies and at ordinary temperatures, but becomes significant at frequencies and temperatures for which $\hbar\omega\sim kT$, a condition which may be met in some proposed devices.

The author is indebted to Dr. W. L. Beaver, Dr. J. C. Helmer, and to Professor M. Chodorow for several stimulating discussions.