Diffuse-Surface Optical Model Analysis of Elastic Scattering of 17- and 31.5-Mev Protons*

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Differential cross sections for the elastic scattering of 17- and 31.5-Mev protons by various nuclei have been analyzed according to the diffuse-surface optical model of the nucleus using a central interaction potential of the form $-(V+iW)/\{1+\exp[(r-R)/a]\}$. Calculated curves of the ratio of the differential to the Rutherford cross sections are presented and compared with experimental values. The adopted values of the four parameters of the model are those which either give best agreement with experiment or, insofar as ambiguities exist, permit the interaction radius R to be expressed as $R_0A^{\frac{1}{3}}$ with R_0 constant throughout most of the periodic table. The average values of the parameters chosen in this way were found at 17 Mev to be as follows: real part of the nuclear potential $V=47$ Mev, imaginary part of the nuclear potential $W=8.5$ Mev, nuclear radius constant $R_0=1.33\times10^{-13}$ cm, surface rounding parameter $a=0.5\times10^{-13}$ cm. A similar analysis at 31.5 Mev gave $V=35.5$ Mev, $W=15.5$ Mev, $R_0=1.33\times10^{-13}$ cm, $a=0.55\times10^{-13}$ cm. Ambiguities in the quoted values due to compensating effects of changes in V and R_0 are discussed. For intermediate and heavy nuclei like Ag and Pt, good agreement can be obtained at 17 Mev for values of R_0 ranging from 1.2 to 1.4×10^{-13} cm, but for lighter elements like Co the range of R_0 is much more restricted. Theoretical curves of the reaction cross section are presented at 17 and 31.5 Mev based on quoted values of the model parameters. It is pointed out that accurate experimental measurements of this quantity may possibly resolve the ambiguity.

1. INTRODUCTION

THE introduction of the optical model of the nucleus by Fernbach, Serber, and Taylor[1] was motivated by the transparency of nuclei to high-energy nucleons. The model has since been widely used for the analysis of the polarization,[2] and the reaction, differential elastic, and total cross sections[3] of high-energy nucleons. After some initial uncertainties, the results have been generally satisfactory but the parameters have not yet been very precisely determined. In particular, the effects of rounding the nuclear surface and the question of simultaneously fitting the four types of available experimental data have not yet been fully explored.

The optical model has been extended by Feshbach, Porter, and Weisskopf[4] to intermediate- and low-energy neutron scattering in order to account for the variation of total neutron cross sections with energy and mass number as observed by Barschall.[5] Early analysis with a complex square well successfully reproduced this behavior, at least qualitatively, but the square well model was less successful in accounting for differential elastic and reaction cross sections.[4,6] The use of a diffuse surface,[7] however, seems to have eliminated most of the difficulties[8] and the optical model is now proving quite satisfactory in the analysis of low- and intermediate-energy neutron scattering.

A complex square well was used even earlier in analysis of intermediate-energy proton scattering. The model exhibited at first a modicum of success[9]; more extensive analyses, however, indicated serious difficulties, especially for the heavier elements.[7,10,11] Again, the situation has been considerably improved by the introduction of a rounded well.[7,11,12] Indeed, this model, commonly referred to as the diffuse-surface optical

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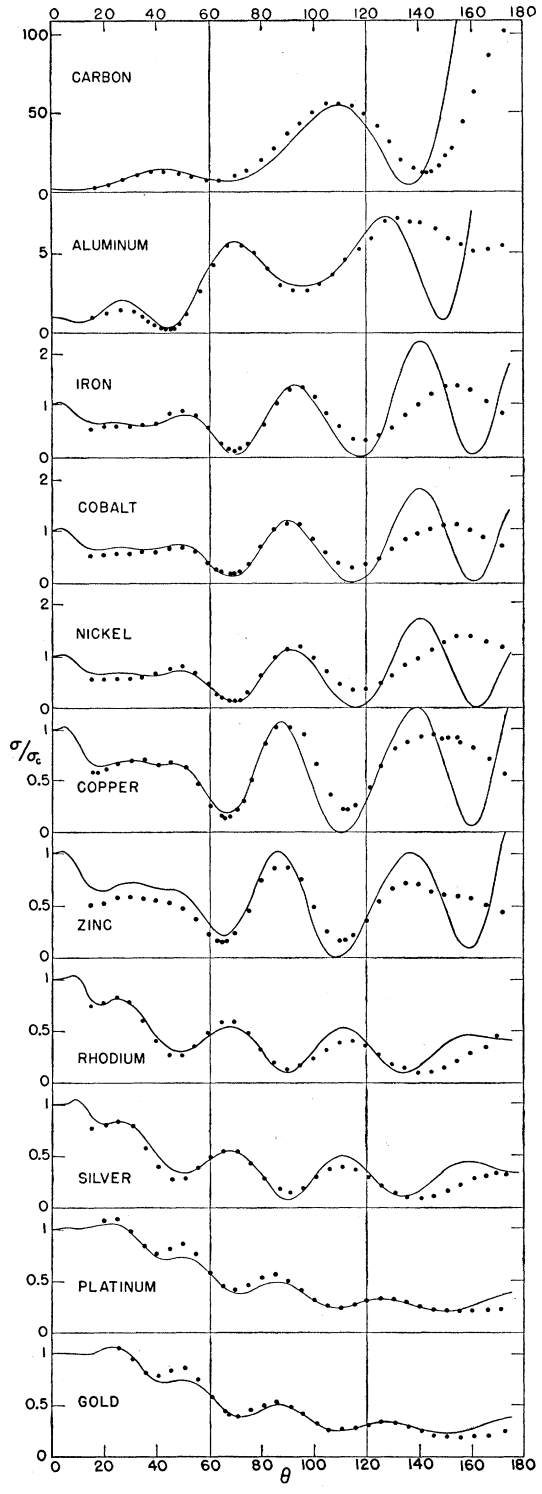

FIG. 1. Ratio of differential cross section to Rutherford for elastic scattering of protons against various nuclei at 17 Mev (center-of-mass energy). · · · Experimental points of Dayton and Schrank. ——— Theoretical curves.

model, was first introduced to overcome just this difficulty with the scattering of protons by heavy elements.

Numerous attempts have been made to provide a firm theoretical basis for the optical model. Most successful, perhaps, has been the high-energy multiple-scattering treatment of Watson and his collaborators.[13] The relation of the optical model parameters to two-body interactions, including the spin-orbit terms necessary to account for high-energy polarization, have been established, at least roughly.[14] The situation at low energies and the extrapolation to bound-state interactions, concerned as it is with a reasonably exact treatment of the many-body problem, is less satisfactory and many discrepancies remain to be worked out.[15]

Generally speaking, it thus appears that the optical model, no matter how sketchy its theoretical basis, can account for a large mass of experimental results. The parameters of the model are, however, still in doubt. This uncertainty is primarily due to the following:

(1) The model is, after all, only an approximation and accordingly it is difficult to fit the experimental data with any precision; the less precise the agreement with experiment, the greater the latitude in choice of parameters which are equally acceptable.

(2) The results obtained from a single type of analysis such as that of elastic angular scattering do *not* lead to an unambiguous assignment of values to the parameters. We shall elaborate this point further.

Apart from these considerations, experimental difficulties sometimes add to the uncertainty. Thus, for example, it becomes difficult to try to fit elastic scattering data at low energy because of the presence of appreciable compound elastic scattering which experimentally cannot be separated from the elastic scattering.

The main purpose of this paper is to report on an attempt to fit experimental data on the differential cross section for elastic scattering of protons by various nuclei at 17 and 31.5 Mev, based on the diffuse-surface optical model. The results of this analysis should help to fix the values of the model parameters. Attention is

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called to further experiments which might help to establish them on a firmer basis.

In part 2 we briefly describe the computational procedure. In part 3 the results are presented and described. In part 4 we discuss the procedure reported previously by us[7,11,12] for fitting the experimental data. In part 5 the significance of the results is discussed and compared with theoretical predictions. It is shown that the analysis does not fix the parameters unambiguously, but rather defines a certain acceptable region in the parameter space. The final values of the parameters are then somewhat arbitrarily chosen by fixing the nuclear density in such a way as to fit most of the periodic table at 17 Mev. In part 6 we present some results on the reaction cross section and point out the importance of this quantity in resolving the ambiguity that may still remain.

2. COMPUTATIONAL PROCEDURE

In order to compute the required cross sections, it is first of all necessary to choose a potential for the Schrödinger equation of the incident particle. The potential chosen here is that used previously by us and consists of a rounded complex nuclear potential plus a Coulomb potential corresponding to a constant charge density within the nucleus:

$$V=V_N+V_C, \tag{1}$$

where

$$V_N=(V+iW)/\{1+\exp[(r-R)/a]\}, \tag{2}$$

and

$$\begin{aligned} V_C&=(Ze^2/2R)(3-r^2/R^2) \quad \text{for} \quad r\leqslant R,\\ &=Ze^2/r \quad\quad\quad\quad\quad\ \text{for} \quad r\geqslant R, \end{aligned} \tag{3}$$

and[16]

$$R=R_0A^{\frac{1}{3}}\times 10^{-13}\ \text{cm}. \tag{4}$$

The resulting Schrödinger equation cannot be solved analytically. The partial wave radial equations are solved by numerical integration; standard techniques are used: the numerical integration is carried out until the nuclear potential vanishes and the resulting numerical wave function is joined smoothly to the proper Coulomb function, thus yielding the complex phase shifts used to compute the cross section as a function of the scattering angle θ. Calculations were carried out on the SWAC (Numerical Analysis Research, Department of Mathematics, University of California, Los Angeles) using a code prepared by Woods.[17] This code was so constructed that it is limited to $L\leqslant 13$. This necessarily determines the maximum energy for which calculations can be carried out for a given nucleus.

[16] The problem was originally formulated in such a way that the charge radius and the radius of the nuclear potential could be independently assigned. It turned out, however, that the elastic scattering at the energies considered was insensitive to moderate changes in the charge radius alone and hence a common value was used.

[17] See reference 7. The numerical integration is carried out by using a Milne 5-points predictor-corrector scheme. The interval of integration is automatically adjusted to minimize the error without excessive time consumption, by requiring both a minimum and a maximum correction at each iteration. The numerical integrations are repeated for each partial wave either until the phase shifts become negligible or until L reaches 13. The average time for a run is about 15 minutes.

TABLE I. Parameters of the diffuse-surface optical model which were used to fit the differential cross section for elastic scattering of protons against various nuclei at 17 and 31.5 Mev.

	E (Mev)	V (Mev)	W (Mev)	R_0 (10^{-13} cm)	a (10^{-13} cm)
C[a]	17[b]	50	7	1.30	0.43
Al[a]	17[b]	40	9	1.30	0.73
Fe, Co, Ni, Cu,[a] Zn	17[b]	46	9	1.33	0.48
Rh, Ag[a]	17[b]	46	9	1.33	0.50
Pt,[a] Au	17[b]	48	8	1.33	0.50
Li, Be, B, C,[a] N, O, F	31.5[c,d]	44	11	1.28	0.50
Mg, Al,[a] Si, P, S, Cl, A	31.5[c]	35	9	1.33	0.60
Ti, V, Fe, Ni, Cu,[a] Ga, Zr, Ag[a]	31.5[c]	35	15	1.33	0.50
Ta	31.5[c]	36	20	1.33	0.55
Au,[a] Pb	31.5[c]	36	16	1.33	0.55

[a] These elements were individually analyzed by varying the parameters.
[b] In the center-of-mass system.
[c] In the laboratory system.
[d] The incident energy was about 1 Mev lower for C, N, and O but this difference was found to be unimportant.

3. RESULTS

The results of the analysis are presented in Figs. 1 and 2 and in Table I. Figure 1 shows the theoretical fits to the experimental data of Dayton and Schrank.[18] The elements C, Al, Fe, Ag, Au were investigated individually by varying the parameters V, W, a, and R_0, and the values of the parameters chosen for each of these elements were then used for neighboring elements.

Figure 2 shows the theoretical fits to some of the experimental results available around 31.5 Mev. The data obtained from Kinsey[19] are more numerous but extend over a smaller angular region than Wright's[20] and Leahy's.[21] Hence, we have chosen to specifically fit the latter for C, Al, Cu, Ag, Pt, Ta, and Au. The values of the parameters obtained in fitting these elements were then again used for their neighbors. Differences in the results obtained by the various investigators, together with the fact that we have specifically tried to fit Wright's and Leahy's data, probably account for discrepancies in the magnitudes of the theoretical curves and Kinsey's experimental results. Closer agreements between these could be achieved by decreasing the value of W, and by allowing V and W to vary more smoothly with atomic number for the light elements.

We have chosen to present the results by plotting the ratio of the differential to Rutherford cross section against angle. We regard such plots as more sensitive

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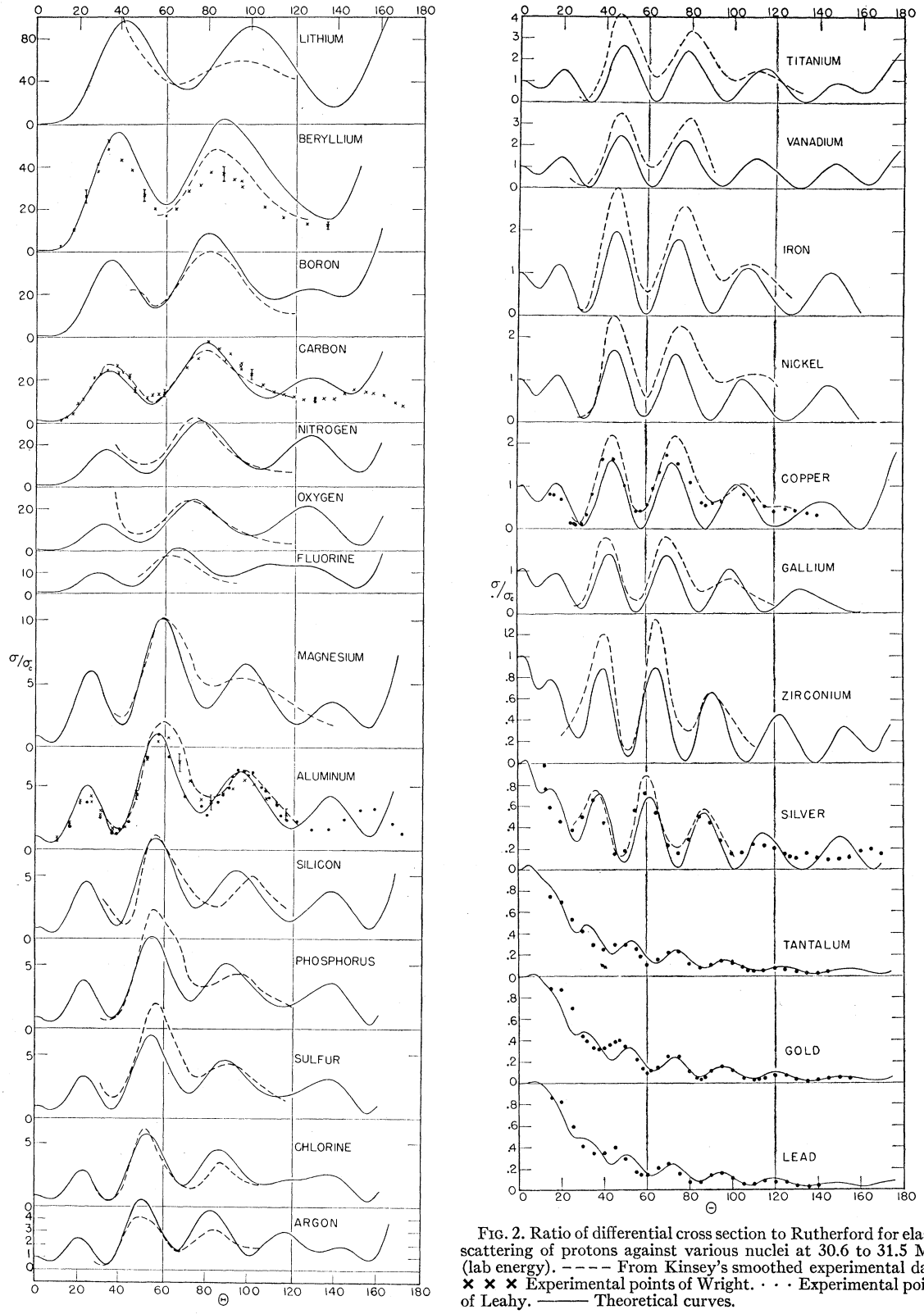

FIG. 2. Ratio of differential cross section to Rutherford for elastic scattering of protons against various nuclei at 30.6 to 31.5 Mev (lab energy). – – – – From Kinsey's smoothed experimental data. × × × Experimental points of Wright. · · · Experimental points of Leahy. ——— Theoretical curves.

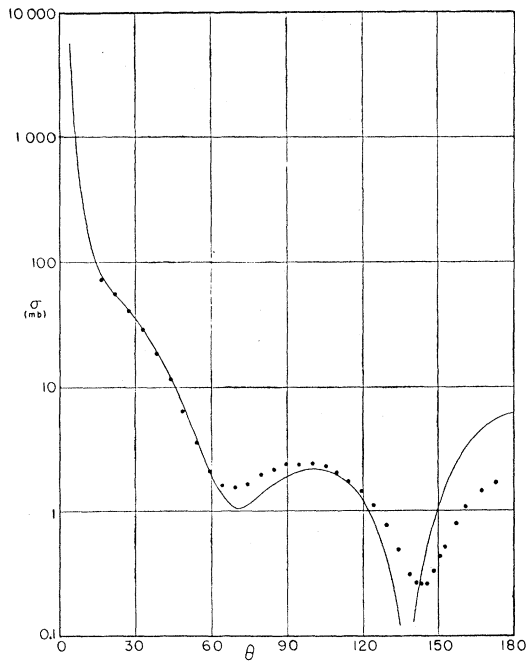

FIG. 3. Differential cross section (in millibarns) for elastic scattering of 17-Mev protons against C. · · · Experimental points by Dayton and Schrank. ——— Theoretical curve, $V=50$ Mev, $W=7$ Mev, $R_0=1.30\times10^{-13}$ cm, $a=0.425\times10^{-13}$ cm.

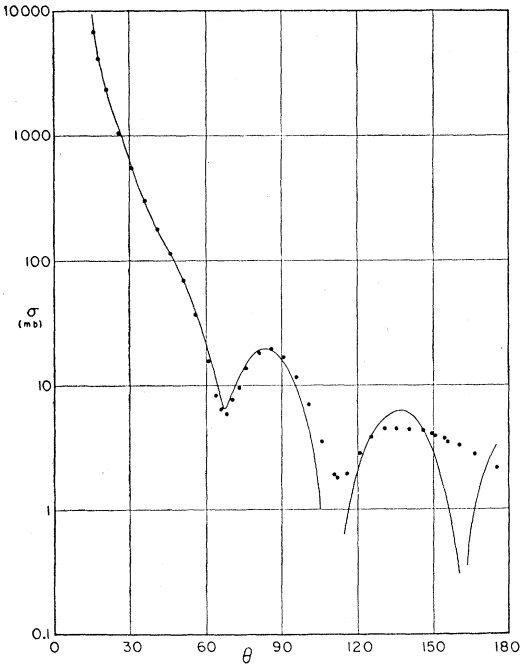

FIG. 4. Differential cross section (in millibarns) for elastic scattering of 17-Mev protons against Cu. · · · Experimental points of Dayton and Schrank. ——— Theoretical curve, $V=46$ Mev, $W=9$ Mev, $R_0=1.33\times10^{-13}$ cm, $a=0.475\times10^{-13}$ cm.

than logarithmic plots of the absolute cross section. Evidently any graphical presentation (even a linear plot of the absolute cross section) is bound to distort the results in some sense. Our particular presentation is characterized by the following features: (1) emphasis of small-angle scattering and de-emphasis of large angle scattering for heavy elements; (2) de-emphasis of deep minima such as occur for intermediate elements; (3) emphasis of large-angle scattering and de-emphasis of small angle scattering for light elements. These features may be verified by comparing the curves for C, Cu, and Au in Fig. 1 with the logarithmic plots of the absolute cross sections for these elements in Figs. 3, 4, and 5. Note also that the small-angle oscillations show up clearly in Fig. 1, but are difficult to distinguish at all on the logarithmic plots.

4. METHOD OF FITTING THE EXPERIMENTAL DATA

Two main difficulties arise in this type of analysis: (1) the difficulty inherent in deciding qualitatively about the best fit to the experimental data; (2) the impracticability of carrying out a complete exploration of the four-parameter space.

The first of these might perhaps be resolved by suitable choice of a quantitative criterion such as a least-squares analysis.[22] Our fits are sufficiently crude, however, that such an analysis would not be practical without an arbitrary assignment of weights to various regions, thus restoring the original difficulty. Such an assignment of weight would for instance have to de-emphasize the large-angle region for light elements in order to permit any kind of fit at all to be achieved. The second difficulty is somewhat tempered by the need for physically acceptable values of the parameters; nevertheless there exists a real danger of missing an important region and considerable effort is required to avoid this. The actual method used for fitting the experimental data is somewhat as follows:

(1) A broad physically acceptable region of the four-parameter space is located by trial and error, such that the number and approximate positions of the maxima match the experimental data. This region, of course, need not be unique.

(2) The parameters are varied independently to ascertain their individual effects on the theoretical curves. This is illustrated in Figs. 6(a), (b), (c), and (d). The effect of increasing a (i.e., increasing the rounding) as shown in Fig. 6(a), is to lower the whole curve, rotating it about its initial point. The effect of increasing W, shown on Fig. 6(b), is to smooth out the curve without affecting it otherwise. The effects of increasing V and R_0, indicated, respectively, on Figs. 6(c) and 6(d), are very similar; in both cases the curve moves left. The effects described here are fairly well marked and, except for R_0 and V, show independent and almost

[22] Such a method was used by Hill, Freeman, and Ford in fitting the angular distribution of high-energy electrons scattered by nuclei; see *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1955), Vol. 5.

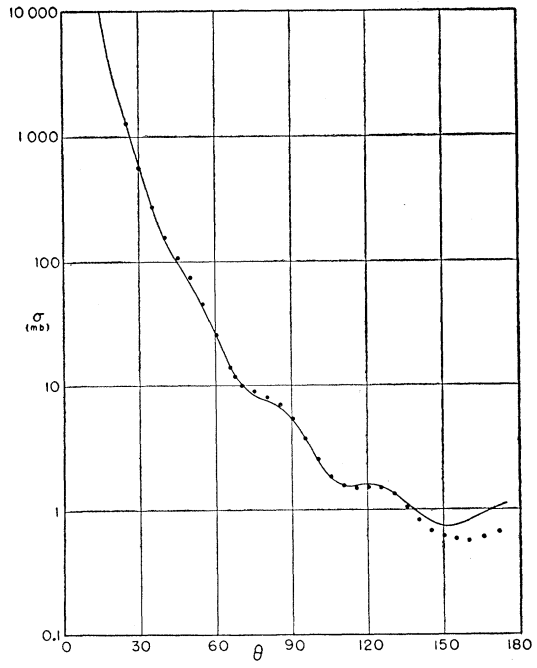

FIG. 5. Differential cross section (in millibarns) for elastic scattering of protons against Au. · · · Experimental points of Dayton and Schrank. ——— Theoretical curve, $V=48$ Mev, $W=8$ Mev, $R_0=1.33\times10^{-13}$ cm, $a=0.5\times10^{-13}$ cm.

linear behavior. Unfortunately, this is not always the case; thus, for light elements and at lower energies these effects are considerably less predictable, thereby increasing the labor required to obtain acceptable fits.

(3) The parameters are changed so as to try to fit the magnitudes and positions of the minima and especially the maxima. In the course of this manipulation, certain regions of the curve are sometimes abandoned as hopeless, such as the backward region for light and intermediate elements, where a better fit can only be achieved at the cost of seriously misfitting the rest of the data.

TABLE II. Values of the parameters which yield for given values of R_0, best fits to differential scattering data on Pt, Ag, and Co at 17 Mev.

	R_0 (10^{-13} cm)	V (Mev)	W (Mev)	a (10^{-13} cm)
Pt	1.07	81	11	0.58
	1.20[a]	62	10	0.55
	1.33[a]	48	8	0.50
	1.45	37	7	0.48
Ag	1.07	78	13	0.63
	1.20[a]	58	11	0.58
	1.33[a]	46	9	0.50
	1.45	36	8	0.48
Co	1.20	58	11	0.58
	1.33[a]	46	9	0.48
	1.45	36	9	0.50

[a] Indicate values of R_0 which gave the best fits to the experimental data. (See Fig. 7.)

(4) In general there still remains ambiguity about the best parameter values but we select those which fit the largest number of elements over the periodic table. This final point will be elaborated upon in the following section.

5. DISCUSSION OF RESULTS

Over-All Fit

The over-all fit to the experimental data is fairly good. The heights and positions for most of the maxima and at least the positions of most minima are fairly well reproduced. Indeed it is rather remarkable how faithfully the experimental effect of varying A and Z is reproduced by the theoretical curves between 10 and 130 degrees at 17 Mev. On the other hand, several difficulties still remain:

(1) The theoretical curves oscillate too fast in the backward region, especially for the light and intermediate elements. We did not find any region in the parameter space where this effect could be eliminated. It might perhaps be attributed to a breakdown in the model for large momentum transfers.

(2) At small angles the theoretical curves oscillate too fast for the heavy elements at 31.5 Mev. However, this effect probably does not represent a genuine discrepancy. The cross sections are falling so rapidly with angle in this region that it is difficult to interpret the experimental data. Indeed, this difficulty is almost imperceptible on a logarithmic plot of the cross section.

(3) Generally the theoretical curves show stronger oscillation than manifested by experimental results especially at 31.5 Mev. Such an effect is responsible for the large holes which appear on the logarithmic curve. However, the smoothness of the experimental curves is partially due to a spread in θ, E, and A; still, this cannot account for the entire difference.

$V-R_0$ Ambiguity

The similarity of the effects of increasing V and R_0 mentioned in Sec. 4, creates a considerable ambiguity in choosing the best set of parameters for elements from Ag on up. The situation is illustrated schematically in Fig. 7. This figure was obtained as follows: For a given element several fixed values of R_0 were chosen. For each of these, the remaining three parameters were varied until the best fit could be obtained. The more precise the agreement between calculated and measured cross sections, the more precisely are these parameters determined and conversely. In Fig. 7, the uncertainty in these parameter values is indicated schematically by the thickness of the shaded regions. Thus the thinnest region indicates the best fit. It may be observed that for Pt and Ag an almost similar fit may be obtained by using $R_0=1.2$ to 1.37. The other parameters would change of course, especially V. Table II indicates the value of the parameters for best fit at other values of R_0. While this large range is available for Ag and Au,

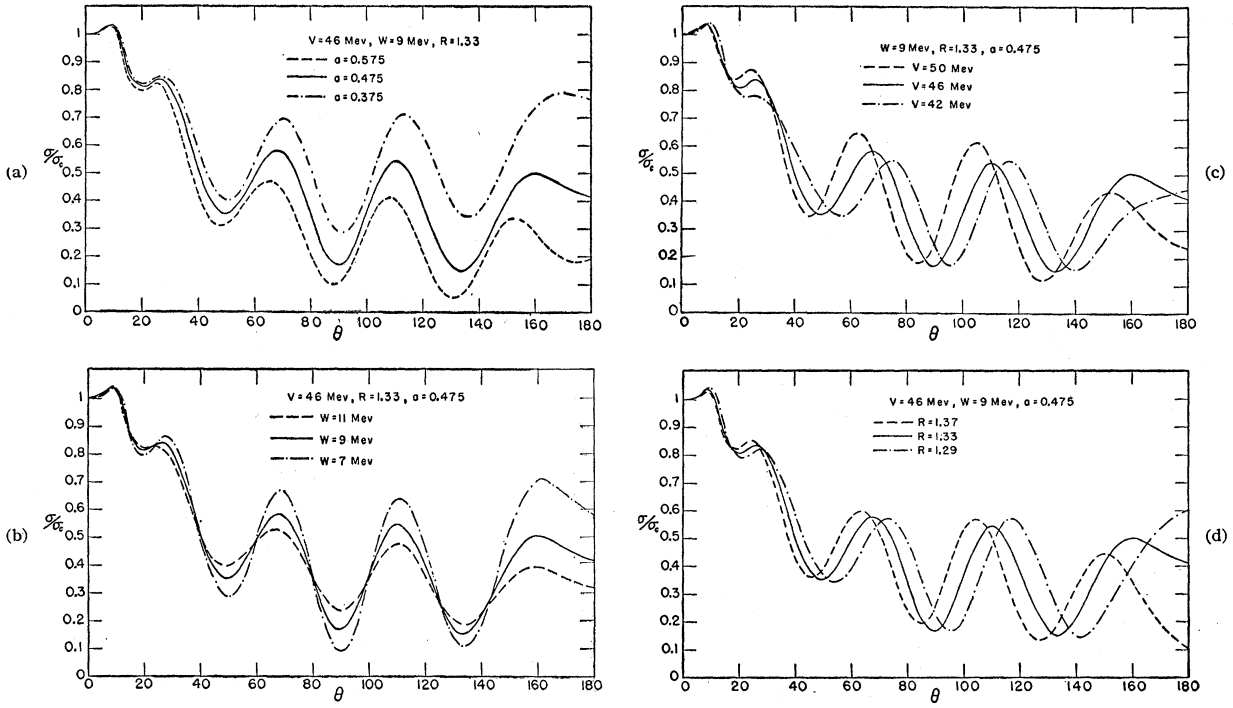

FIG. 6. Ratio of differential cross section to Rutherford for elastic scattering of 17-Mev protons against Ag. (a) Effect of varying a, the surface thickness. (b) Effect of varying W, the imaginary part of the nuclear potential. (c) Effect of varying V, the real part of the nuclear potential. (d) Effect of varying R_0.

considerably less freedom is permissible for Co. The fit (never as good as for the heavier elements) rapidly deteriorates on either side of $R_0=1.33$, especially for smaller values of R_0. Thus if one wishes, however arbitrarily, to use the same value of R_0 throughout the periodic table, a value of $R_0=1.3$ to 1.35 is indicated. We have made such a choice and the values quoted in Table I as well as the curves shown in Figs. 1 and 2 represent a deliberate attempt to keep $R_0=1.33$. It will be noted, however, that for the very light elements this was not generally possible.

As may be verified from Table II, the ambiguity in the optical model parameters can be summarized roughly as follows: equally good agreement with experiment can be obtained if V and R_0 are varied over a considerable interval according to the law $VR_0^n=$ constant, with $2<n<3$, and with relatively small changes in W and a. This behavior is fairly reasonable since Born approximation would give $n=3$ and the bound state limit would give $n=2$.

Comparison of Parameter Values with Theoretical Prediction

It is rather gratifying to find that throughout the periodic table, except for the lighter elements, the parameters are fairly constant at a given energy. The empirically determined energy dependence of the parameters has been discussed[23] and seems reasonable. The values of the potential depths are close to theoretically predicted values.[14,15] The real potential depth V is 5 to 10 Mev larger than that found for neutrons. While this is in agreement with recent analysis of bound-state and shell structure,[24] the potentials could be made equal by appropriate choice of R_0. It should be noted at this point that the parameters describing the optical model for neutrons have recently been determined[25] from an analysis which includes the reaction and total cross section as well as the differential

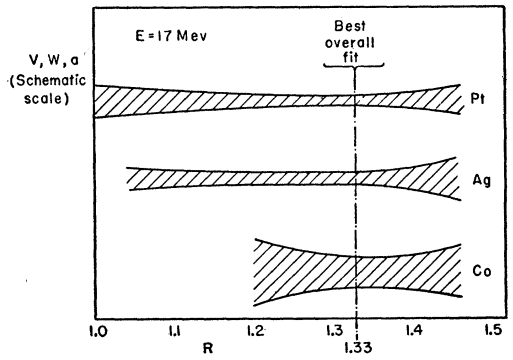

FIG. 7. Schematic representation of optical model parameters giving about equal fits to the experimental differential cross section for elastic scattering of 17-Mev protons. The thicker hashed regions describe poorer fits.

[23] Melkanoff, Moszkowski, Nodvik, and Saxon, see reference 12.

[24] Ross, Mark, and Lawson, Phys. Rev. **102**, 1613 (1956).

[25] Beyster, Walt, and Salmi, see reference 8.

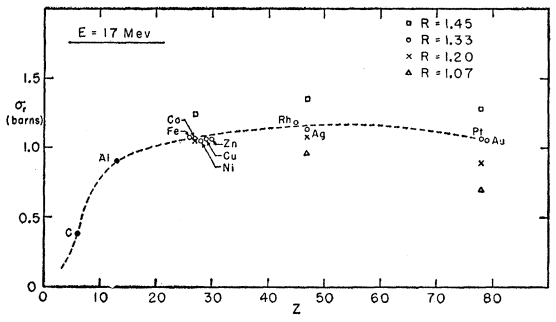

FIG. 8. Reaction cross sections for 17-Mev (c.m.) protons computed on the basis of the diffuse surface optical model. The effect of varying R_0 is indicated for Co, Ag, and Au. The solid points were obtained from computations with the optical model parameters giving the best fits to the elastic differential cross sections. The open points were obtained from computations with the optical model parameters of the nearest "fitted" neighbor.

scattering cross section; these parameters are therefore more definite than the proton parameters. The value of a is also quite reasonable and agrees with the results of electron-scattering analysis.[26] Theoretical calculations of this quantity depend on the type of analysis and the assumed form factor; thus while some estimates[27,28] are in agreement with the value quoted here, others differ considerably.[29]

In order to discuss the value of the nuclear radius, it is important to distinguish the various quantities found in the literature; we shall define:

(1) R_e, the electromagnetic charge radius[30] as measured by electron scattering, μ-mesonic atoms, and other electromagnetic data,

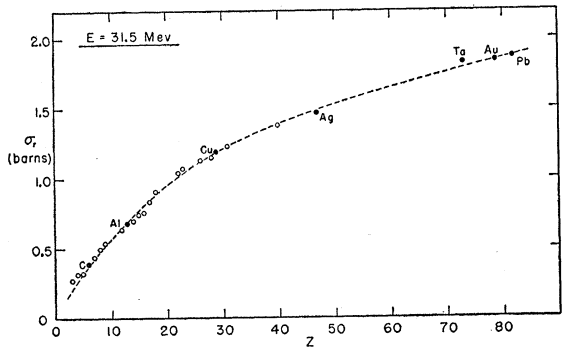

FIG. 9. Reaction cross sections for 30.6 to 31.5 Mev (lab) protons. The solid points were obtained from computations with the optical model parameters giving the best fits to the elastic differential cross sections. The open points were obtained from computations with the optical model parameters of the nearest "fitted" neighbor.

(2) $R_{\rho p}$ and $R_{\rho n}$, the nuclear density radii for neutrons and protons respectively which enter, for example in the Weiszäcker formula,

(3) R_{Vp} and R_{Vn}, the nuclear potential radii for neutrons and protons, respectively, as determined from nuclear scattering experiments.

It is, of course, quite possible that all of these have different values; there are therefore five quantities R_e, $R_{\rho n}$, $R_{\rho p}$, R_{Vn}, and R_{Vp}, to be considered, the last of which we called simply R in Eqs. (2), (3), and (4). Comparison of various results is further complicated by the use of a variety of form factors. The value of R_{Vp} as found in the present analysis is about 20% larger than R_e, in agreement with R_{Vn} as found from neutron scattering data.[25] The difference between R_e and R_{Vp} is in agreement with most theoretical predictions[28] if it is assumed that $R_e \cong R_{\rho p}$ as is most reasonable. If it is further assumed that $R_{Vn}-R_{\rho n} \cong R_{Vp}-R_{\rho p}$, then the agreement between R_{Vp} and R_{Vn} computed with the same form factor is in agreement with some calculations,[31] but contrary to most theoretical predictions[32] which yield $R_{\rho n}>R_{\rho p}$. The value of R_{Vp} is, however, not too definite owing to the $V-R$ ambiguity.

For light elements, the situation is not too clear. Since the optical model is not expected to be valid for nuclei containing only a few particles, the surprise is not that the parameter values fluctuate considerably for such elements, but rather that the agreement is as good as it is. It might be mentioned here that we had considerable difficulty in fitting aluminum and that this could only be done by raising a to 0.725 as indicated in Table I.

TABLE III. Theoretical reaction cross section computed from the phase shifts used to fit the differential elastic scattering cross sections.

	σ_r (mb)		σ_r (mb)		σ_r (mb)
		E=17 Mev (c.m.)			
C	380	Ni	1045	Ag	1132
Al	895	Cu	1063	Pt	1068
Fe	1070	Zn	1060	Au	1050
Co	1063	Rh	1177		
		E=31.5 Mev (lab)			
Li	268	Si	703	Cu	1200
Be	318	P	749	Ga	1238
B	325	S	762	Zr	1392
C	390	Cl	832	Ag	1470
N	442	A	906	Ta	1830
O	496	Ti	1040	Au	1843
F	540	V	1075	Pb	1870
Mg	640	Fe	1130		
Al	685	Ni	1150		

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[30] An excellent discussion of the electromagnetic charge radius is given by K. W. Ford and D. L. Hill, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1955), Vol. 5.
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6. REACTION CROSS SECTIONS

The total reaction cross section is easily obtained from the phase shifts; it has been calculated and is listed in Table III. It is also shown in Figs. 8 and 9 at 17 and 31.5 Mev as a function of Z. The points are all theoretical values and the curves are the best curves drawn through these points.

In view of the ambiguity still existing for individual elements, the reaction cross section acquires special importance as it may differentiate between various sets of parameters equally acceptable for the differential elastic scattering curves. This is indicated on Fig. 8 where predicted values of the reaction cross sections are indicated for various values of R_0. For heavier elements the differences are sizable, and it would therefore be desirable to have accurate experimental determinations of the reaction cross section.

Preliminary information on the (p,n) cross section at 10 Mev[33] shows remarkable differences from one isotope to the next. Since the (p,n) cross section presumably gives the main contribution to the reaction cross section, this should be reflected by similar changes in the differential elastic scattering cross sections. Experimental results on mixed isotopes do not show such an effect, but it would be of interest to search for it by measuring the differential elastic scattering cross section for individual isotopes.

The reaction cross section at 31.5 Mev was only calculated for single sets of parameters. If, however, effects similar to those at 17 Mev are present as expected, the resulting ambiguity in R_0 will again emphasize the importance of the reaction cross section.

TABLE IV. Geometrical parameters which may be used to describe the reaction cross section at 31.5 Mev.

	r_0 (10^{-13} cm)	$\lambda\!\!\!^{-}$ (10^{-13} cm)
C	1.18	0.81
Al	1.28	0.81
Cu	1.35	0.81
Zr	1.29	0.81
Ag	1.25	0.81
Pt	1.19	0.81

[33] H. A. Howe (private communication).

TABLE V. Average values of the parameters of the diffuse surface optical model.

E (Mev)	V (Mev)	W (Mev)	R_0 (10^{-13} cm)	a (10^{-13} cm)
17	47	8.5	1.33	0.49
31.5	35	15.5	1.33	0.53

On the basis of a simple geometrical interpretation, reaction cross sections are frequently represented by expressions of the form

$$\sigma_r=\pi(r_0A^{\frac{1}{3}}+\lambda\!\!\!^{-})^2;$$

the values of r_0 determined from this expression are listed in Table IV and are seen to decrease from 1.35 for the lighter elements (excluding the very light elements such as C and Al) to 1.19 for the heavier ones. Such representation at 17 Mev is not meaningful because of the sizable effect of the Coulomb barrier.

CONCLUSION

We have analyzed the differential cross section for elastic scattering of 17- and 31.5-Mev protons against various nuclei on the basis of the diffuse-surface optical model. The average values of the parameters giving the over-all best fits are given in Table V. These values are in reasonable agreement with theoretical estimates and seem to indicate a radius about 20% larger than the electromagnetic radius and a real potential depth 5 to 10 Mev larger than the real potential for neutrons. Ambiguity in the parameter values especially in V and R_0 still exist and might be eliminated if adequate experimental reaction cross sections become known.

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