Final-State Interactions in the Total Cross Section for Deuteron Photodisintegration

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We have estimated the role of final-state interactions in the theory of the total cross section for photodisintegration of the deuteron at high energies. The Feshbach-Lomon model of nucleon-nucleon scattering was employed. Within the context of this model, final-state effects were found to enhance the total cross section by about 15 to 20% over the usual electric dipole calculation for photon energies around 60 Mev.

I. INTRODUCTION

IT is well known[1] that for photon energies as low as 60 Mev there exists a discrepancy between the measured total cross section for the reaction $\gamma+d\rightarrow n+p$ and a theoretical electric dipole transition calculation based on a Hulthén wave function for the deuteron $^3S_1$ state and plane waves for the final two-nucleon state. In particular, the measured cross section at 60 Mev is greater than 109 μb (Whalin *et al.* give 109 μb for 65-Mev photon energies[1]) and the simple dipole theory gives 82 μb for the same energy, a discrepancy of the order of 25 or 30%. At higher energies the measured cross section falls off *much* less rapidly than the theory as described above and even begins to increase above 100 Mev. However, in this note we shall only be concerned with the theoretical understanding of the 60-Mev total-cross-section data.

We begin by noting that there are at least 6 ways in which the simple calculation described above may be improved: (a) We may include higher electric moments. (b) We may include magnetic moments. (c) We may change the form of the deuteron $^3S_1$ state. (d) We may include the effect of the $^3D_1$ part of the deuteron ground state. (e) We may consider mesonic current effects in the interaction of the deuteron with the photon. (f) We may consider interactions in the final two-nucleon states.

It is generally agreed that the modifications introduced by (a), (b), and (d) in the total cross section are only of the order of a percent or two at this energy. The magnetic-moment terms seem to be entirely negligible.[1] If one estimates the 60-Mev electric quadrupole contribution, again assuming a Hulthén deuteron wave function and plane wave final states, one finds that this term is about 1% of the electric dipole element calculated in the same way. Austern[2] has shown that the introduction of the $^3D_1$ state cannot be expected to contribute more than a few percent to the total cross sections at the energies of interest.

Variations in the $^3S_1$ wave function can in principle cause large uncertainties in the total cross section calculation. For example, if one naively uses an exponential form with the effective-range normalization,

$$\psi_0=\frac{e^{-\alpha r}}{(4\pi)^{\frac{1}{2}}r}\left(\frac{2\alpha}{1-\alpha r_t}\right),$$

down to the origin, then the electric dipole term at 60 Mev changes from 82 μb with the Hulthén, to 121 μb with the exponential.

An accurate estimate of the meson current contributions is complicated by all of the difficulties of a strong-coupling field theory and no very convincing calculation of these has been made to date. In particular, it is not obvious that they are negligible even at 60 Mev.[3]

This brings us to the subject of the present work: The effect of final-state interactions in the total cross section for photodisintegration. To compute final-state interactions one would need to know the continuum wave functions for the interacting neutron-proton system. These are not known except in the asymptotic region where the wave functions are characterized by the spherical Bessel functions appropriate to a given angular momentum state and the scattering phase shifts for the state. To find the scattering states in all detail, one would need to solve the meson-nucleon problem in a manner which seems beyond reasonable present expectation. Hence it seemed to us worthwhile to employ in this study a simple two-nucleon phenomenological model, that of Feshbach and Lomon,[4] which gives a fit to the scattering data in the relevant energy region, and which makes an explicit assumption about the continuum wave functions for all radial distances. In fact it is supposed that these functions take on their asymptotic values down to a core radius which depends on the angular momentum and spin state of the two-

[1] See especially Whalin, Schriever, and Hanson, Phys. Rev. **101**, 377 (1956), where references are given to earlier experimental work. For the details of the electric dipole calculation see, for example, J. F. Marshall and E. Guth, Phys. Rev. **78**, 738 (1950) or L. I. Schiff, Phys. Rev. **78**, 733 (1950).

[2] N. Austern, Phys. Rev. **85**, 283 (1952). We are also pleased to acknowledge several stimulating and informative conversations with Dr. Austern on photoeffect problems.

[3] For an interesting phenomenological treatment of meson effects see the paper of R. R. Wilson, Phys. Rev. **104**, 218 (1956). It should be noted that the effect of virtual meson processes is not correctly estimated by Wilson for photon energies below the meson threshold. In particular he assumes that virtual photo P-wave meson emission from single nucleons in the deuteron vanishes below threshold, as would not be the case in any meson theory.

[4] H. Feshbach and E. Lomon, Phys. Rev. **102**, 891 (1956). In that work no explicit assumption is made about the wave functions inside the core. However the $^3P_0$-state force is shown to be essentially a repulsive hard sphere and in this spirit we have put the wave functions equal to zero inside the core radius.

nucleon system and in general on the two-nucleon energies. Inside the core the functions shall be assumed to be zero. In an electric dipole transition from the $^3S_1$ state, the relevant final states are the $^3P_0$, $^3P_1$, $^3P_2$ states and for the energies of interest the core radii needed to fit the scattering data in this model turn out to be, respectively, 1.32, 0.88, and 0.88 (in units of 10^{-13} cm). Within the context of the same theory, the deuteron wave function is taken to be its asymptotic form down to a core radius of 1.32×10^{-13} cm and zero within. The deuteron wave function is not normalized to unity in the usual way but we have fixed the scale of our results by normalizing the electric dipole term calculated at low energies with the Feshbach-Lomon theory to fit the experimentally measured total cross section at $(\hbar\omega-\epsilon)/\epsilon=2$. Here ϵ is the deuteron binding energy, and the choice of 2 in the parameter $(\hbar\omega-\epsilon)/\epsilon$ corresponds to a photon energy of 6.68 Mev (in the lab system). The normalization of the ground state computed this way turns out to be $N^2=0.92\times10^{13}$ cm^{-1} as compared to the usual effective-range normalization $2\alpha/(1-\alpha r_t)=0.77\times10^{13}$ cm^{-1}.

Our choice of 60-Mev photon energy is motivated by the fact that the two nucleons then have an outgoing kinetic energy of about 120 Mev in the laboratory. For this energy the Feshbach-Lomon model yields phase shifts which are at least in qualitative agreement with the scattering and polarization experiments. At higher energies the fit becomes more tenuous. It is in any case characteristic of the fit that a strong repulsive force is needed in the $^3P_0$ state. In fact, at 120 Mev the phase shifts are given as

$$\delta_0=-38°8',$$
$$\delta_1=2°14',$$
$$\delta_2=8°48'.$$

However, it turns out that even such strong final-state forces do not change the total cross section appreciably in the 60-Mev photoeffect, at least when the latter is computed with the Feshbach-Lomon model. An interesting and much more difficult way of posing the problem would be to take the phase shifts as simply given and to explore how various modifications of the wave functions near the origin change the total photodisintegration cross section. Some crude attempts at this indicate that sensible modifications of the wave functions do not change the order of magnitude of the final-state interactions as given by the Feshbach-Lomon calculation.

We now proceed to that calculation.

II. CALCULATION

Our starting point is the Rarita-Schwinger[5] formula for the total electric dipole photodisintegration cross section:

$$\sigma=\frac{\pi}{3}\frac{e^2}{\hbar c}\frac{M\omega}{\hbar k}\frac{1}{9}(I_0^2+3I_1^2+5I_2^2). \quad (1)$$

Here M is the nucleon mass, k the relative nucleon wave number, ω the photon frequency, and

$$I_0=\int_0^\infty rv_0(u-\sqrt{2}w)dr,$$
$$I_1=\int_0^\infty rv_1\left(u+\frac{1}{\sqrt{2}}w\right)dr, \quad (2)$$
$$I_2=\int_0^\infty rv_2\left(u-\frac{1}{5}\frac{1}{\sqrt{2}}w\right)dr.$$

u and w are the deuteron radial S and D functions and the v_i are the continuum nucleon functions which must be further specified. The numbers 1, 3, and 5 are the statistical weights $2J+1$ appropriate to the three angular momentum states.

In making the calculation we drop the D-state contributions and in the spirit of the Feshbach-Lomon model suppose that

$$\begin{aligned} u&=e^{-\alpha r}N, \quad r\geqslant1.32\times10^{-13}\text{ cm}\\ &=0, \quad r<1.32\times10^{-13}\text{ cm}\end{aligned} \quad (3)$$

where N is evaluated as indicated in the introduction. The v_i are given by

$$\begin{aligned} v_i&=\frac{\sin(kr+\delta_i)}{kr}-\cos(kv+\delta_i), \quad r\geqslant r_i\\ &=0, \quad r<r_i\end{aligned} \quad (4)$$

where δ_i are the relevant phase shifts and the r_i are as given above. With these wave functions the integrations become elementary and at 60 Mev give a cross section of 96 μb to be compared with the 82 μb obtained with the Hulthén and plane-wave calculation. Hence the Feshbach-Lomon model gives an enhancement over the usual theory of about 15% at 60 Mev. We have applied the same ideas to estimate the cross sections at higher energies where the discrepancies with the ordinary theory are much worse and obtain results of about the same order of magnitude. Needless to say, this cannot be regarded as a definitive statement about the role of final-state interactions in the total deuteron photodisintegration cross sections. But coupled with the knowledge that meson effects must be included at higher photon energies, our results would seem to indicate that these effects may be important in understanding the data even in the 60-Mev region.

III. ACKNOWLEDGMENT

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[5] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 and 556 (1941).