annealing time (Fig. 1) and thus the resistivity-temperature curves for the quenched and annealed specimens cross each other as shown in Fig. 2. It is remarkable that the difference of temperature coefficient of resistivity between the annealed and the quenched sample is appreciable at low temperatures. On the other hand, the residual resistivity of the annealed specimen is only slightly lower than that of the quenched one and this fact is rather surprising. However, the heat treatment of the sample in this case might be still inadequate to obtain the state of highest possible order in this alloy. The present note is intended only to point out the fact that the annealed state of Au_3Cu has lower residual resistivity and, therefore, that Au_3Cu behaves normally in this sense.

However, it is also of some interest to report that the estimated Debye temperature of the annealed state is lower than that of the quenched state by as much as 30°. The Debye temperatures determined from the resistivity curves are 150°K for the quenched state and 119°K for the well-annealed state, though we cannot put too much weight on these absolute values. This behavior is just contrary to that of Cu_3Au and is also rather exceptional.

For the further understanding of this alloy, it is necessary to find a method of obtaining the state of highest possible order and then to determine the electronic structure and phonon spectrum of this system. For the former, irradiation by electrons[4] as well as more adequate heat treatment is being considered, and for the latter, the measurements of Hall effect, low-temperature specific heat, and elastic constants are under way. Discussions with Dr. J. E. Goldman, Dr. B. R. Coles, and Dr. A. W. Overhauser are greatly appreciated.

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Excess Noise in n-Type Germanium*

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An experimental examination of excess noise in n-type single crystal germanium has confirmed that the phenomenon behaves like conductivity modulation caused by carrier density fluctuations. The magnitude and frequency spectrum of the carrier density fluctuations, as measured by the Hall effect, yield good agreement with observed excess noise levels. Majority and minority carrier density fluctuations are correlated in such a way that the density varies percentagewise. It is shown experimentally that noise sources are distributed uniformly along the specimen length and that statistically independent volume elements are smaller than 10^{-5} cm^3. These results are independent of sample shape, resistivity, and surface treatment, which affect only the magnitude of the fluctuations. Results on samples with resistivities ranging from 1.5 to 35 ohm-centimeters show an increase in fluctuations with resistivity.

I. INTRODUCTION

EXCESS noise in semiconductors is characterized by a noise power density having a $1/f$ spectrum. Although it is known to be generated at metal-semiconductor contacts[1,2] or similar electrical discontinuities, it may also be observed in apparently uniform single crystals having "noiseless" electrodes.[3] In this case there is evidence that inhomogeneities on an atomic scale, such as those produced by plastic deformation,[4,5] exert considerable influence. Although surface conditions are known to be experimentally important in excess noise,[5] recent papers are divided on the question of whether the phenomenon should be considered primarily a surface or volume effect.[5,6]

The excess noise power is observed to depend approximately upon the square of the average current. This has been interpreted in terms of conductivity modulation of the semiconductor, and the modulation is ascribed to fluctuations in carrier density. The Hall effect has been used to examine these fluctuations directly,[7,8] with the preliminary conclusion that fluctuations in carrier density have the proper frequency spectrum and approximately the correct magnitude to account for excess noise.

This paper presents a more thorough investigation of excess noise as conductivity modulation, including a rigorous application of the Hall effect to the problem. The conductivity modulation effect is critically examined experimentally and the Hall effect technique is extended to yield more reliable quantitative magnitudes of carrier density fluctuations than previously reported. The measurements have been extended to include

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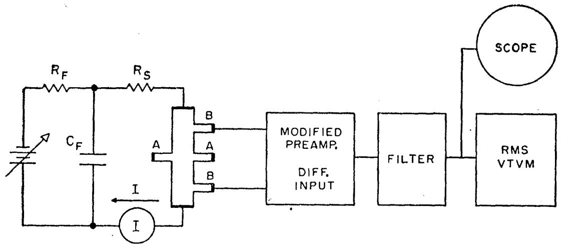

FIG. 1. Block diagram of apparatus used in excess noise work. Probes BB on the sample are used for standard noise studies and probes AA for Hall effect examination.

resistivities ranging from one to thirty ohm-centimeters. All of the work to be described has been carried out on single-crystal, arsenic-doped, germanium.

II. CONDUCTIVITY MODULATION

Although the idea of a conductivity modulation mechanism for excess noise was suggested in even the first paper on the subject[9] and is clearly implicit in most subsequent experimental investigations, a direct experimental study of its existence has not been presented. The technique is very simple and consists essentially of showing experimentally that excess noise is observed under constant current conditions.

A block diagram of the apparatus used for this and subsequent noise studies is shown in Fig. 1. A battery and battery filter R_FC_F supply sample current through a noiseless series resistor. Noise voltages are examined at probes BB of the sample for normal studies and at probes AA for Hall effect studies. The sample is fabricated into the shape indicated by sandblasting and is provided with ohmic, tin-soldered electrodes. The surface is either etched or sandblasted, as required. The noise voltages are amplified by a modified[10] low-noise preamplifier, pass through a standard electronic filter, and are measured on a true rms vacuum-tube voltmeter. The amplifying system is calibrated by measuring Johnson noise of known resistors.

From elementary circuit analysis of the input circuit in Fig. 1, the noise voltage observed at probes BB due to current flow can be expressed as

$$\Delta V^2 = \frac{I^2\langle \Delta R_b^2\rangle}{(1+R_b/R_s)^2} + \frac{\langle \Delta V_E^2\rangle}{(1+R_s/R_b)^2}, \tag{1}$$

where R_b is the average resistance between probes BB, $\langle \Delta R_b^2\rangle$ is the mean square fluctuation in this quantity, and $\langle \Delta V_E^2\rangle$ represents extraneous noise voltages arising from current fluctuations, battery noise, contact noise, etc. If $\langle \Delta V^2\rangle$ is observed for increasing values of R_S, it will approach the constant value $I^2\langle \Delta R_b^2\rangle$ if conductivity fluctuations truly exist. The approach to this value may be either increasing or decreasing, depending upon the relative magnitudes of $I^2\langle \Delta R_b^2\rangle$ and $\langle \Delta V_E^2\rangle$.

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A comparison of the noise voltage observed at probes BB as a function of the series resistance with the behavior predicted by Eq. (1) is shown in Fig. 2. Clearly the agreement is satisfactory. Notice that a true plateau is not reached until the series resistor, which provides constant current conditions, is 30 times the sample resistance, somewhat larger than might be anticipated. Also, it is indicated that $I^2\langle \Delta R_b^2\rangle$ is larger than $\langle \Delta V_E^2\rangle$, a desirable situation. The alternate behavior is occasionally observed and usually is indicative of a noisy current contact. Such samples are not investigated further.

We take this data as experimental evidence that excess noise in single-crystal germanium behaves as expected from a conductivity modulation picture. Furthermore, the experimental technique described provides a direct way of demonstrating that extraneous noise sources, particularly contact noise, can be made negligible. We have made it standard practice to carry out such an examination on every sample used for excess noise studies.

III. HALL EFFECT NOISE

The previous report[7] on noise voltages observed at the Hall probes, AA, showed that two components are present. At zero magnetic field a noise voltage analogous to the usual "unbalance" voltage is observed, and the noise voltage increases linearly with magnetic field in complete agreement with what is expected from the Hall effect and carrier density fluctuations. Typical results of an examination of the noise at probes AA for a 20 ohm-centimeter sample are shown in Fig. 3. The magnetic field used for this work was supplied by a permanent magnet. The noise with magnetic field is independent of the magnetic field direction. The noise level at probes BB is negligibly affected by the addition of the magnetic field.

The noise data in Fig. 3 show the usual $1/f$ behavior for both values of magnetic field. Therefore, the Hall noise voltage, which is the difference between the curves, also has a $1/f$ spectrum. This particular sample had a

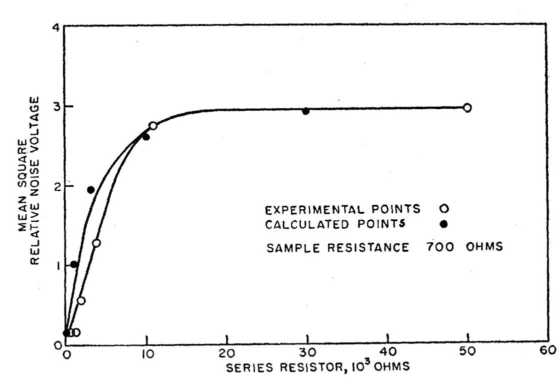

FIG. 2. Comparison of noise voltages observed at probes BB as a function of series resistance with the behavior predicted by Eq. (1). The approach to a plateau is expected for a conductivity modulation phenomenon.

chemically etched surface, but similar results are obtained with other surface conditions, such as sandblasting.

IV. CARRIER DENSITY FLUCTUATIONS

Measurements such as those of Fig. 3 can be interpreted in terms of carrier density fluctuations by the following analysis. The dc voltage which appear at probes AA are the usual dc potential drop and Hall voltage,

$$V_I=R_bI,\quad V_H=R_HIB/t, \tag{2}$$

where R_H is the Hall constant, B is the magnetic field, and t is the sample-thickness. Since the current is constant, voltage fluctuations are ascribed to variations in R_b and R_H. These variations, in turn, are due to fluctuations in carrier densities. If we take the usual two-carrier expressions for R_b and R_H and allow both hole and electron concentrations to vary, Eqs. (2) become, with the approximation for n-type samples, $n_e \gg n_h$:

$$\begin{aligned}-\Delta V_I/V_I&=\Delta n_e/n_e+(1/b)\Delta n_h/n_e,\\ -\Delta V_H/V_H&=\Delta n_e/n_e+\left(\frac{1+2b}{b^2}\right)\Delta n_h/n_e,\end{aligned} \tag{3}$$

where b is the ratio of mobilities.

Equations (3) may be used as independent measures of fluctuations in carrier densities if the correlation existing between Δn_e and Δn_h is known. Three cases have been examined; (1) Δn_e and Δn_h uncorrelated,

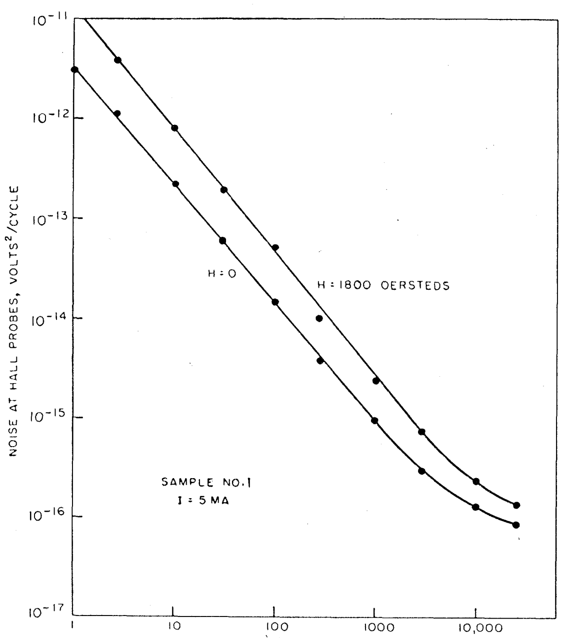

FIG. 3. Excess noise voltages observed at probes AA with and without magnetic field for a 20 ohm-centimeter sample.

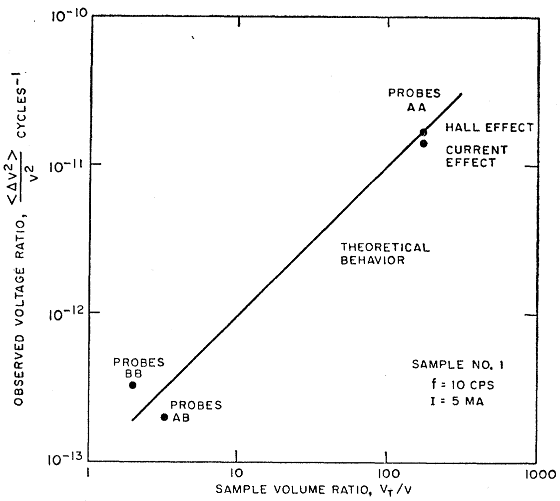

FIG. 4. Excess noise voltages observed as a function of sample length. The straight line represents the expected variation based on statistically independent volume elements.

(2) $\Delta n_e=\Delta n_h$ as in van der Ziel's theory of shot noise,[11] and (3) $\Delta n_e/n_e=-\Delta n_h/n_h$, as suggested by Bess for excess noise.[8] With these assumptions the expressions become:

1. Δn_e, Δn_h uncorrelated

$$\begin{aligned}\langle\Delta V_I^2\rangle/V_I^2&=\langle\Delta n_e^2\rangle/n_e^2+(1/b)^2\langle\Delta n_h^2\rangle/n_e^2,\\ \langle\Delta V_H^2\rangle/V_H^2&=\langle\Delta n_e^2\rangle/n_e^2+[(1+2b)/b^2]^2\langle\Delta n_h^2\rangle/n_e^2.\end{aligned} \tag{4}$$

2. $\Delta n_e=\Delta n_h$

$$\begin{aligned}\langle\Delta V_I^2\rangle/V_I^2&=(1+1/b)^2\langle\Delta n_e^2\rangle/n_e^2,\\ \langle\Delta V_H^2\rangle/V_H^2&=(1+1/b)^4\langle\Delta n_e^2\rangle/n_e^2.\end{aligned} \tag{5}$$

3. $\Delta n_e/n_e=-\Delta n_h/n_h$

$$\langle\Delta V_I^2\rangle/V_I^2=\langle\Delta n_e^2\rangle/n_e^2=\langle\Delta V_H^2\rangle/V_H^2. \tag{6}$$

The above analysis yields values of the magnitude of the carrier density fluctuations per unit volume, but in general experimental specimens are considerably smaller than this. Therefore, the observed noise voltages must be corrected to refer to unit volume. If it is assumed that the fluctuations in each volume element are independent, leaving aside for a moment the size of this element since it is unknown, the correction is merely a multiplicative volume ratio. For the present work, this is conveniently done by using the expression

$$(\langle\Delta V_I^2\rangle/V^2)_{\text{cor}}=(\langle\Delta V^2\rangle/V^2)_{\text{obs}}(V/V_T)W, \tag{7}$$

where V_T is the total potential drop across the sample, W is the total sample volume, and V is the potential drop between probes, as used above.

A comparison of noise voltage observed between various pairs of probes, representing therefore different sample volumes, with the behavior predicted by Eq. (7) is shown in Fig. 4 for a 20 ohm-cm specimen. The

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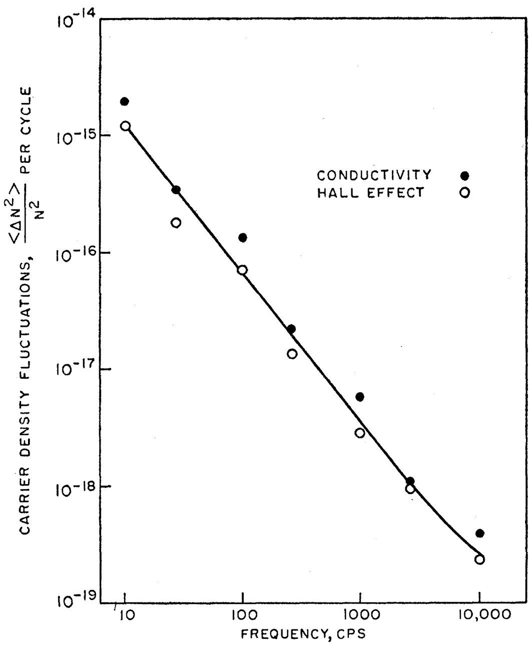

FIG. 5. Frequency spectrum of carrier density fluctuations of the sample of Fig. 3 as computed from Hall and conductivity noise measurements.

expected effect is reasonably well confirmed for variations in sample volume of two orders of magnitude. This is good evidence for a uniform distribution of noise sources along the sample length. Also, since the minimum sample volume represented by probes AA is 5.2×10^{-5} cm³, the size of the volume element for fluctuation correlations must be smaller. For this particular sample cross section, this represents a length of 4.5×10^{-3} cm in the direction of current flow.

Noise data similar to that of Fig. 3 were taken on a number of samples with very similar results and carrier densities calculated from Eqs. (4)–(6). Table I summarizes the results for three n-type specimens which range in resistivity from 1.5 to 35 ohm-centimeters, in volume from 4×10^{-3} cm³ to 0.2 cm³, and for both chemically etched (20 ohm-cm sample) and sandblasted (1.5 and 35 ohm-cm samples) surfaces. The measured potential fluctuations are given for a frequency of 100 cps and are quite representative since $1/f$ behavior was observed in all cases.

Using these data to compute carrier density fluctuations by means of Eqs. (4)–(6) results in negative values under the no-correlation assumption, which is physically meaningless. It appears, therefore, that this assumption is not correct. The shot-noise assumption results in a disparity of a factor of three between the carrier density fluctuations obtained from the two measurements, as illustrated in the table. The agreement is considerably better when one uses the third assumption, which confirms the findings of Bess.[8] The residual error in this case may be attributed in part to simplifications in the derivation of Eqs. (4)–(6) and the usual experimental difficulties in measurements of random noise. However, these factors do not seem large enough to completely account for the disagreement and a small unknown effect must be present in addition to those considered. One possibility is the change in carrier trajectories in the presence of the magnetic field, which may alter the noise characteristics slightly.

By using the data of Fig. 3, the assumption of Eq. (6), and the volume correction of Eq. (7), the carrier density fluctuations for a 20 ohm-centimeter specimen are plotted in Fig. 5. These show the expected $1/f$ behavior over the frequency interval examined. Similar results are obtained with other crystals, with the magnitude of the fluctuations increasing with resistivity as illustrated in the table.

TABLE I. Carrier density fluctuations.

			$\left(\frac{\langle\Delta n_e^2\rangle}{n_e^2}\right)_I \Big/ \left(\frac{\langle\Delta n_e^2\rangle}{n_e^2}\right)_H$	
Resistivity ohm-cm	$\langle\Delta V_I^2\rangle/V_I^2$ (cps)$^{-1}$ (f=100 cps)	$\langle\Delta V_H^2\rangle/V_H^2$ (cps)$^{-1}$ (f=100 cps)	$\Delta n_e=\Delta n_h$ (shot noise)	$\Delta n_e/n_e=-\Delta n_h/n_h$ ($1/f$ noise)
35	4.9×10^{-16}	3.8×10^{-16}	2.8	1.3
20	1.2×10^{-16}	0.77×10^{-16}	3.4	1.5
1.5	1.0×10^{-19}	0.71×10^{-19}	3.1	1.4

V. SUMMARY

This experimental examination of excess noise in n-type germanium crystals has shown explicitly that the phenomenon behaves as a conductivity modulation caused by fluctuations in carrier density. The experimental technique used eliminates contributions from extraneous noise sources and shows when this condition is obtained. The Hall effect technique has been extended to account for various types of carrier density fluctuations and it has been confirmed that equal percentage fluctuations of majority and minority carriers provide the most satisfactory description of excess noise. Noise sources are uniformly distributed throughout the volume (or length) of these single crystal samples and the fluctuations per volume element must be independent in volumes as small as 10^{-5} cm³. For future work, it is now possible to obtain reliable measures of the magnitude of carrier density fluctuations by using IR-drop noise voltages alone without the necessity of resorting to Hall studies. These general conclusions are independent of sample shape, volume, resistivity, and surface treatments, although each of these influence the magnitude of the fluctuations observed.