those of Malan and Schonland[6] that there is no observable stepped electrostatic field change during the downward course of a nearby stepped leader. This leads one to the conclusion that the charge q lowered per cm path during the step-flash must be too small to produce detectable steps in the electrostatic field change. This view was already put forward by Schonland[3] and has been reiterated in his recent article.[7] Following Schonland, if it is supposed that the charge lowered per cm during the step-flash is 1/16th the charge per cm carried by the pilot streamer and if the velocity of the step-flash is taken as 2×10^9 cm/sec, the stepped-leader current comes out to be $(0.8\times10^{-5}\times2\times10^9)/16$ or 1000 amperes. It is to be noted that with the value of the velocity of the step-flash equal to 2×10^9 cm/sec, the duration of the step-flash would be 1–10 μsec for a step-length of 20–200 meters.

The pilot streamer was considered by Schonland[3] as responsible for the observed electrostatic field change but his estimate of the pilot streamer current, $i=320$ amperes, is based merely on the observation that a charge of 4 coulombs is lowered by the leader in a model time of 0.0125 sec. In the present communication, it has been shown that the same value for the pilot leader current, *prior* to the step-flash, may be obtained from Loeb's streamer mechanism formula, $i=qv_t$, by taking the velocity v_t of the streamer tip to be 4×10^7 cm/sec which is considerably less than the velocity of the step-flash. A theoretical basis according to streamer theory is thus given for the pilot leader current which is consistent with the observed electrostatic field change.

[6] D. J. Malan and B. F. J. Schonland, Proc. Roy. Soc. (London) **A171**, 485 (1947).

[7] B. F. J. Schonland, *Encyclopedia of Physics*, edited by S. Flugge (Springer-Verlag, Berlin, 1956), Vol. 22. See article by Schonland on "Lightning Discharge."

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Inherent Noise of Quantum-Mechanical Amplifiers*

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The noise figure or limiting sensitivity for both traveling-wave and resonant-cavity quantum-mechanical amplifiers, sensitive to one direction of propagation, is derived with spontaneous emission as the limiting noise. The concept of effective temperature is introduced as an analytical parameter; thus negative temperatures appear in a natural fashion. It is pointed out that the results of this calculation can be considered the solution to the problem of linear counting of coherent particles. In this case the least count is one and the signal-to-noise ratio (for constant photon flux) increases as the reciprocal of the band width. The limiting temperature sensitivity of properly designed quantum-mechanical amplifiers is given as $h\nu/k$ degrees because of the drastic difference between negative and positive temperatures.

I. INTRODUCTION

Interest in quantum-mechanical amplifiers has demonstrated that there are available many methods for making such devices.[1–4] Of the many properties of quantum-mechanical amplifiers—gain, band width, stability—their limiting sensitivity or noise figure is of great importance. This paper will be devoted to an investigation of the noise figure for quantum-mechanical amplifiers of either traveling-wave or resonant-cavity design. In later papers we shall discuss the problems that enter into the actual design of quantum-mechanical amplifiers.

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[2] Scovil, Feher, and Seidel, Phys. Rev. **105**, 762 (1957).

[3] M. W. P. Strandberg, Proc. Inst. Radio Engrs. **45**, 92 (1957); M. W. P. Strandberg, Bull. Am. Phys. Soc. Ser. II, **2**, 36 (1957).

[4] Gordon, Zeiger, and Townes, Phys. Rev. **99**, 1264 (1955).

II. TRAVELING-WAVE AMPLIFIER NOISE FIGURE

We shall be considering the situation shown in Fig. 1—a piece of transmission line which has within it and coupled to it energy levels that can be prepared for operation as a quantum mechanical amplifier. In this paper we are not particularly interested in the actual design; thus the amplifier that we visualize will be a directionally sensitive amplifier, that is, it will amplify waves unidirectionally. We merely state that it is possible to design such a directionally sensitive amplifier. This allows our amplifier to be insensitive to the output-load temperature and also eliminates regenerative amplification arising from reflections.

The construction of such a directionally sensitive

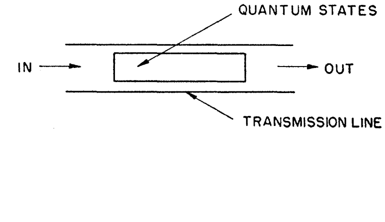

Fig. 1. Traveling-wave amplifier.

device is based on the Faraday effect, with circular polarization of the signal radiation.[5] The Faraday effect of the amplifying energy levels or of auxiliary paramagnetic or ferrite crystals may be used. Our work on the development of directionally sensitive devices for both low- and high-Q structures will be reported later.

The physics that we have to use at this point is easily summarized. Induced transition probabilities are proportional to the radio-frequency energy per unit volume per frequency interval.[6] We know also, from radiation theory, that the spontaneous-emission probability is the same that we would have if we had a radiation density of as many photons per frequency interval per unit volume as the number of modes per frequency interval per unit volume.[6] This means that the equivalent radiation energy density for spontaneous emission is the photon energy divided by the effective cross-sectional area of the transmission line and the group velocity of the radiation. The signal-power flow per frequency interval is simply the radiation density multiplied by the group velocity and the effective cross-sectional area.

We shall also require that, in the equilibrium condition, a net balance exist between the emission from the transmission-line walls and the energy which they absorb. The conditions outlined above, then, lead us to our first equation for the rate of change of the power per frequency interval in the section of the transmission guide:

$$\frac{dp_\nu}{dx}=\frac{Ap_\nu(n_2-n_1)h\nu}{v_g}+\frac{Ah\nu ah\nu n_2}{av_g}+\alpha_c p_\nu+\alpha_c p_\nu(T_c). \quad (1)$$

The symbols in this equation are apparent: A is a quantity related to the coupling between the quantum mechanical levels and the electromagnetic radiation, n_2 and n_1 are the number of energy states per unit volume in the upper and lower states, h is Planck's constant, α_c is the guide attenuation constant, and $p_\nu(T_c)$ is the characteristic thermal-noise distribution function for the coupling line, given in terms of its temperature T_c, and p_ν is the total energy density, which is x dependent.

In thermal equilibrium the left-hand side of Eq. (1) is zero; this requirement is met by having an equality in magnitude between the last two terms on the right-hand side of the equation. The first two terms are also in proper form. In thermal equilibrium, the ratio n_1/n_2 is just the Boltzmann distribution, and we are led to the conventional form for the thermal-radiation power density, which is given by

$$p_\nu(T)=\frac{h\nu}{\exp(h\nu/kT)-1}. \quad (2)$$

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[6] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, p. 105.

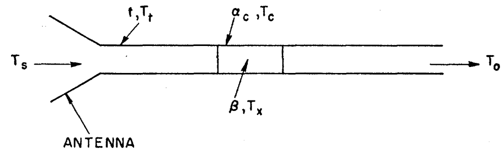

FIG. 2. Symbolic system diagram.

The coefficient multiplying the radiation power density in the first term on the right-hand side of Eq. (1) is apparently the quantum-mechanical gain of the system. If we use β for this term, the equation can be simplified to more symmetrical form as

$$dp_\nu/dx=\beta p_\nu-\beta p_\nu(T_x)-\alpha_c p_\nu+\alpha_c p_\nu(T_c). \quad (3)$$

We have introduced in Eq. (3) the idea of a temperature for the quantum-mechanical amplifier by defining $n_1/n_2=\exp(h\nu/kT_x)$. The concept of the quantum-mechanical temperature can have no greater meaning than that it is a convenient analytical parameter. We shall retain the Planck form of the thermal-radiation density given in Eq. (2), since there will be occasion to use temperatures that will not make the exponent of e small compared with 1. Equation (3) can now be integrated along the length of the amplifier section, and the output radiation power density can be given as

$$(p_\nu)_{\text{out}}=(p_\nu)_{\text{in}}g^2+\frac{\beta p_\nu(T_x)-\alpha_c p_\nu(T_c)}{(\beta-\alpha_c)}(1-g^2), \quad (4)$$

where g^2 is the gain of the quantum-mechanical amplifier. The interpretation of Eq. (4) is obvious. The effective noise radiated from the amplifying section comes from the spontaneous emission in the quantum mechanical system itself, and from the thermal-noise radiation of the transmission lines. The kind of weighting factor, expressed by the last term in Eq. (4), between two noise sources coexisting at different temperatures, is well known, and our calculation has merely demonstrated that the spontaneous emission in the quantum-mechanical amplifier gives rise to a noise that is equivalent to the noise expected from a properly defined resistor.

We are now in a position to evaluate the limiting sensitivity or noise figure of a complete system. We shall choose a system and a notation that are indicated in Fig. 2. The source is defined by an effective temperature T_s (see Fig. 2), the transmission line has a power-loss factor t and a temperature T_t, and the output load of the amplifier has an excess noise power $p_\nu(T_0)$. By conventional calculations, we obtain the noise figure of a traveling-wave quantum-mechanical amplifier, which is

$$\text{noise figure}=\frac{(p_\nu)_{\text{out}}}{tg^2p_\nu(T_s)}=1+\frac{1}{tp_\nu(T_s)}\Bigg\{(1-t)p_\nu(T_t)+(1-g^{-2})\left[\frac{\alpha_c p_\nu(T_c)-\beta p_\nu(T_x)}{\beta-\alpha_c}\right]+g^{-2}p_\nu(T_0)\Bigg\}. \quad (5)$$

Equation (5) may be expressed in a more familiar form if we allow kT to be appreciably larger than $h\nu$. In this case the radiation-density terms can be represented by their parametric temperatures. It is quite obvious, from Eq. (5), that a noise figure approaching unity can be obtained if reasonable gains ($g\approx 30$) are used, and if the transmission-line and coupling-line losses are kept low. For example, if the transmission line is operated at 300°K and the effective source temperature is 3°K, a transmission-line loss of one percent between antenna and amplifier would result in doubling the noise figure. The transmission-line coupling section with a loss factor α_c will probably be run at or near $|T_x|$, so that, for any reasonable gain, the contribution of the coupling section will always be negligible.

III. RESONANT-CAVITY AMPLIFIER NOISE FIGURE

We shall develop the noise figure for a quantum mechanical amplifier that acts in a high-Q cavity system, by using a procedure similar to that used in Sec. II. Again, we use as our model a directionally sensitive system, so that the amplifier is shielded from the load temperature, and the regenerative gain of the device is stabilized by isolating it from load variations. The stimulating radiation density, ρ_ν, and the Q's, which are descriptive of the resonator losses and the external losses, will be used. Spontaneous emission takes place, as we observed in Sec. II, as if it were induced by a radiation density of as many photons per frequency interval per unit volume as the number of modes per frequency interval per unit volume. With a resonant system the number of modes is one, the characteristic volume is that of the cavity, and the frequency width is the effective power width of the amplifier, which is just $\frac{1}{2}\pi\Delta\nu$. In conventional notation, the necessary power balance in the system is given by

$$\frac{d\rho_\nu}{dt}=A\rho_\nu(n_2-n_1)V_xh\nu+\frac{2Ah\nu}{\pi\Delta\nu V_c}n_2V_xh\nu-\frac{\omega\rho_\nu}{Q_0}-\frac{\omega\rho_\nu}{Q_e}+\frac{p_{\nu\,\mathrm{ext}}}{V_c}+\frac{p_{\nu\,\mathrm{wall}}}{V_c}. \quad (6)$$

In the steady state the left-hand side of Eq. (6) is zero. We use the coefficient of ρ_ν in the first term on the right-hand side of Eq. (6) to define an amplifier Q, and use this amplifier Q_x in the second term on the right-hand side of Eq. (6), as we did in Sec. II, to define, again, an effective amplifier temperature T_x in terms of a population distribution, through the use of Eq. (2). A straightforward rearrangement of terms then leads to

$$\frac{\omega V_c\rho_\nu}{Q_L}=\frac{4p_\nu(T_x)Q_L}{Q_x}+\frac{4p_\nu(T_c)Q_L}{Q_0}+\frac{4p_\nu(T_s)Q_L}{Q_e}, \quad (7)$$

where

$$Q_L^{-1}=Q_x^{-1}+Q_0^{-1}+Q_e^{-1}.$$

The substitutions in the last two terms in Eq. (7) for the power from the external circuits and the power from the walls may be rationalized from a consideration of matching or, much more simply, they may be justified, at this point, from arguments of thermal equilibrium. In thermal equilibrium all components will be at the same temperature; whence Eq. (7) reduces to an obvious identity. If we define the gain of a cavity quantum mechanical amplifier as the ratio between the incident and the reflected power, we have, in conventional notation, the expression for this gain.

$$P_r/P_c=g^2;\quad g=2(Q_L/Q_e)-1. \quad (8)$$

One form that the noise figure takes is

$$\text{noise figure}=1+\frac{1}{tp_\nu(T_s)}\left[(1-t)p_\nu(T_t)+\frac{(g+1)^2}{g^2}\left\{p_\nu(T_s)+\frac{Q_e}{Q_0}[p_\nu(T_c)-p_\nu(T_x)]-\left(\frac{g-1}{g+1}\right)p_\nu(T_x)\right\}\right]. \quad (9)$$

The observations to be made on Eq. (9), with respect to transmission loss, cavity temperature, and so forth, are fairly obvious and we dispense with them here. The gain relationship displayed in Eq. (8) does, however, demonstrate a particular property of regenerative amplifiers. Since the loaded Q and the square root of the gain are linearly related, we have the property that the square root of the gain multiplied by the band width of the system is essentially a constant. In other words, increased gain is paid for by decreased band width, although, since the band width is decreased as the square root of the gain, the price is not hard to pay.

IV. SUMMARY

We have tried to present a rational, sound analysis of the limiting sensitivity of quantum-mechanical amplifiers. Since the *raison d'être* for a quantum-mechanical amplifier is its high sensitivity or low noise figure, it is essential that these calculations be available, in order that we may evaluate the worth whileness of research along these lines. Although our calculations may still contain errors, it is hoped that they are minor from the viewpoint of our present understanding of the physical processes involved. At least they have put the role played by spontaneous emission in these devices in proper perspective, and they have given a natural, confident foundation to the concept of temperature in these devices.

We may now see why such considerations of noise are of interest in microwave systems that operate at room temperature. At high temperature, say room temperature, the net absorption (or net emission, depending upon which dominates) is nearly canceled by the induced emission (or absorption). Many photons must be absorbed (or emitted) to achieve *one* photon of net absorption (or emission). These net photons must complete with the spontaneous-emission photons. Thus, as T_{eff} is lowered, fewer photons need be handled for the same signal-to-noise ratio. Here the tremendous difference between so-called positive and negative temperatures (our T_{eff}) is apparent. For the absorption case, the noise-power distribution $p_\nu(T)$ approaches zero as T_{eff} approaches $+0$. However, it approaches $-h\nu$ as T_{eff} approaches -0. This is intuitively satisfying, since it means that spontaneous-emission noise actually acts as least-count noise in a net emission system. To put it otherwise, if we have n photons per frequency interval per second from the amplifier, the least count is one photon and this is just the spontaneous-emission noise. We are dealing here with phase-coherent photons, however, so the signal-to-noise ratio is as the reciprocal band width, instead of as the square root of the reciprocal band width, which is the case when incoherent photons (or particles) are counted. We have essentially solved the problem of the statistical noise for a linear system with coherent particles.

For those who like a simple, appealing, albeit inaccurate, explanation of quantum-mechanical noise, we offer the following suggestions that have grown out of our work. At high effective temperatures, the noise is high, since the least-count effect (shot effect) becomes large, because the net emission is small on account of interfering absorption. As the effective temperature is lowered, the number of photons to be amplified can be linearly lowered and the same least count, i.e., the same signal-to-noise ratio, can be maintained. The limit as T_{eff} approaches 0 will always be photon shot noise.

Neglecting, then, many practical details that are solely within the realm of engineering ingenuity (for which we hold high regard), we have shown that the limiting sensitivity of quantum-mechanical amplifiers is given in a readily achievable limit by the effective quantum-mechanical noise power density. This noise power density is given parametrically by an effective temperature. The essential and drastic difference between negative and positive temperatures is demonstrated by this function, in that, as T approaches -0, this function approaches $(-h\nu)$ and, as T approaches $+0$, this function approaches 0. This means that in the region where $h\nu<kT_{s,x}$ the noise figure can be represented, essentially, as the ratio of the quantum-mechanical temperature and the source temperature. With the equality sign reversed, the noise figure becomes large. For 1-cm radiation, this turning point is at 1.5°K. At any frequency, we may say that the limiting temperature sensitivity for a quantum-mechanical amplifier is, essentially, $h\nu/k$.

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Information Theory and Statistical Mechanics

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Information theory provides a constructive criterion for setting up probability distributions on the basis of partial knowledge, and leads to a type of statistical inference which is called the maximum-entropy estimate. It is the least biased estimate possible on the given information; i.e., it is maximally noncommittal with regard to missing information. If one considers statistical mechanics as a form of statistical inference rather than as a physical theory, it is found that the usual computational rules, starting with the determination of the partition function, are an immediate consequence of the maximum-entropy principle. In the resulting "subjective statistical mechanics," the usual rules are thus justified independently of any physical argument, and in particular independently of experimental verification; whether or not the results agree with experiment, they still represent the best estimates that could have been made on the basis of the information available.

It is concluded that statistical mechanics need not be regarded as a physical theory dependent for its validity on the truth of additional assumptions not contained in the laws of mechanics (such as ergodicity, metric transitivity, equal *a priori* probabilities, etc.). Furthermore, it is possible to maintain a sharp distinction between its physical and statistical aspects. The former consists only of the correct enumeration of the states of a system and their properties; the latter is a straightforward example of statistical inference.

1. INTRODUCTION

THE recent appearance of a very comprehensive survey[1] of past attempts to justify the methods of statistical mechanics in terms of mechanics, classical or quantum, has helped greatly, and at a very opportune time, to emphasize the unsolved problems in this field. Although the subject has been under development for many years, we still do not have a complete and satisfactory theory, in the sense that there is no line of argument proceeding from the laws of microscopic mechanics to macroscopic phenomena, that is generally regarded by physicists as convincing in all respects. Such an argument should (a) be free from objection on mathematical grounds, (b) involve no additional arbi-

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