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Theory of Rearrangement Collisions

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It is shown that if one omits a certain small term which appears in one of the integral equations describing rearrangement collisions, then that equation becomes inconsistent with the others. This result is of interest because the presence or absence of this term represents the essential formal difference between several recent theories of rearrangement collisions.

IN this note we wish to point out a fact which has apparently not been noticed previously, namely that if one omits a certain small term which appears in one of the integral equations describing rearrangement collisions, then that equation becomes inconsistent with the other equations. This result is of interest because the presence or absence of this term represents the essential *formal* difference between several recent theories[1] of rearrangement collisions. One should of course note that the *practical* consequences of our remark will depend on the role played by the equation in actual (of necessity approximate) calculations (for example all theories yield the same first Born approximation), and that there are several ways in which the equation might conceivably be used in each of the theories. A full discussion of the situation would be beyond the scope of this brief note and will be reserved for a later paper.

For brevity we shall use Lippmann's[1] notation and we refer to his paper for the meaning of the symbols. The equations which we shall need are his Eq. (3.7):

$$\Psi_a{}^{(\pm)\prime}=\Phi_a{}'+\frac{1}{E_a{}'\pm i\epsilon-H_0{}'}V'\Psi_a{}^{(\pm)\prime}, \tag{1}$$

and the analogous equation:

$$\Psi_b{}^{(\pm)\prime\prime}=\Phi_b{}''+\frac{1}{E_a{}'\pm i\epsilon-H_0{}''}V''\Psi_b{}^{(\pm)\prime\prime}. \tag{2}$$

Following Lippmann, one can derive from (1) the equation [Lippmann's (3.11)]

$$\Psi_a{}^{(\pm)\prime}=\frac{\pm i\epsilon}{E_a{}'\pm i\epsilon-H_0{}''}\Phi_a{}'+\frac{1}{E_a{}'\pm i\epsilon-H_0{}''}V''\Psi_a{}^{(\pm)\prime}, \tag{3}$$

and in a similar way one can derive from (2) that

$$\Psi_b{}^{(\pm)\prime\prime}=\frac{\pm i\epsilon}{E_a{}'+i\epsilon-H_0{}'}\Phi_b{}''+\frac{1}{E_a{}'\pm i\epsilon-H_0{}'}V'\Psi_b{}^{(\pm)\prime\prime}. \tag{4}$$

The term in question is the first term on the right-hand side of Eq. (3) [or equally well, the corresponding term in Eq. (4)]. This term, for many problems, is of order ϵ,[2] and therefore, since we are interested in $\Psi_a{}^{(\pm)\prime}$ and $\Psi_b{}^{(\pm)\prime\prime}$ only in the limit $\epsilon\rightarrow 0$ there is a question as to whether or not the term need be kept. We shall now show that if the term is not kept then Eq. (3) becomes inconsistent with the other equations.

From Eq. (3), with the first term on the right-hand side omitted, it follows that if $\Psi_b{}^{(\pm)\prime\prime}$ is a solution of Eq. (2), then so is $\Psi_b{}^{(\pm)\prime\prime}+\alpha\Psi_a{}^{(\pm)\prime}$ for any value of α. That is, Eq. (2) does not have a unique solution. This result is surprising, and moreover it leads to the following contradiction: Since $\Psi_b{}^{(\pm)\prime\prime}+\alpha\Psi_a{}^{(\pm)\prime}$ is a solution of (2), it follows that it must also satisfy (4). [From Lippmann's derivation one sees that *any* function which satisfies (2) will also satisfy (4).] That is, we must have

$$\Psi_b{}^{(\pm)\prime\prime}+\alpha\Psi_a{}^{(\pm)\prime}=\frac{\pm i\epsilon}{E_a{}'\pm i\epsilon-H_0{}'}\Phi_b{}''+\frac{1}{E_a{}'\pm i\epsilon-H_0{}'}V'(\Psi_b{}^{(\pm)\prime\prime}+\alpha\Psi_a{}^{(\pm)\prime}),$$

whence from (4) it follows that

$$\Psi_a{}^{(\pm)\prime}=\frac{1}{E_a{}'\pm i\epsilon-H_0{}'}V'\Psi_a{}^{(\pm)\prime},$$

which is in contradiction with Eq. (1).

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[1] For example, the term is kept by S. Altshuler [Phys. Rev. **92**, 1157 (1953)] and by B. A. Lippmann [Phys. Rev. **102**, 264 (1956)], while it is omitted by S. Altshuler [Phys. Rev. **91**, 1167 (1953)] and by T.-Y. Wu [Can. J. Phys. **34**, 179 (1956)].

[2] This was first shown (for the case of electron-hydrogen scattering) by S. Altshuler, Phys. Rev. **91**, 1167 (1953); see especially Appendix II.