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Relation between Scattering and Absorption in the Pais-Piccioni Phenomenon*

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The expressions for the θ_1 and θ_2 amplitudes in a beam of neutral θ mesons traversing an absorber are put in terms of forward-scattering amplitudes. It is discovered that a phase-shift term as well as an absorption term is needed to describe the regeneration of θ_1 mesons in the unscattered beam.

A simple relation is derived between the intensities of the above process and of the θ_1 mesons regenerated by scattering. Experimental verification of the relation may give additional information about the nature of the neutral K mesons.

INTRODUCTION

THE "particle-mixture" hypothesis of Gell-Mann and Pais[1] led to the prediction by Pais and Piccioni[2] of a startling phenomenon, the regeneration of short-lived θ_1 mesons in an absorber placed in a beam of long-lived θ_2 mesons. There seem to be two mechanisms of regeneration: (a) regeneration by scattering, in which the θ^0 and $\bar{\theta}^0$ components of the θ_2 scatter differently, thus giving rise to a θ_1 component in the scattered beam; and (b) regeneration by absorption, in which the composition of the unscattered beam changes with depth in the absorber, so that an unscattered θ_1 component develops. This note is a discussion of the interrelation of these two phenomena.

It is found that there is a simple relation between them, at least for small absorber thicknesses. The concept of the complex index of refraction of the absorber for the θ^0 and for the $\bar{\theta}^0$ proves useful. (This is the index of the absorber as a whole, in the sense of slow-neutron optics or light optics, rather than the index of the interior of the nucleus, as in the "optical model" of nuclear scattering.)

DESCRIPTION OF UNSCATTERED BEAM

First, we need a description of the composition of the unscattered beam.

A recent paper by Case[3] has developed the equations governing the time dependence of the amplitudes α_1 and α_2 of the θ_1 and θ_2 states in traversing an absorber. We refer to Case's paper for the details of the derivation; we shall deal only with the differences between his treatment and ours.

In treatment of strong interactions, the θ^0, $\bar{\theta}^0$ representation is the more useful. If α, α' are the amplitudes for θ^0, $\bar{\theta}^0$, i.e.,

$$\psi=\alpha\theta^0+\alpha'\bar{\theta}^0,$$

then the spatial behavior of these amplitudes in the absorber is determined, as far as the strong interactions are concerned, by their forward-scattering amplitudes, just as the amplitude of the electric field of a plane wave in matter is determined in ordinary optics[4]:

$$\frac{\partial\alpha}{\partial x}=i\left(\frac{2\pi N}{k^2}A(0)+1\right)k\alpha,\qquad \frac{\partial\alpha'}{\partial x}=i\left(\frac{2\pi N}{k^2}A'(0)+1\right)k\alpha', \tag{1}$$

where x = coordinate along the beam, N = number of nuclei per cm^3, k = wave number in free space, and $A(\phi)$, $A'(\phi)$ = complex scattering amplitude for θ^0, $\bar{\theta}^0$ scattering through angle ϕ. ($\phi=0$ for forward scattering.) These equations are equivalent to saying that there is a complex index of refraction for θ^0 and for $\bar{\theta}^0$:

$$n=1+(2\pi N/k^2)A(0),\quad n'=1+(2\pi N/k^2)A'(0), \tag{2}$$

so that we have

$$\partial\alpha/\partial x=ink\alpha,\quad \partial\alpha'/\partial x-in'k\alpha'. \tag{3}$$

The imaginary part of the forward-scattering amplitude is related to the total cross section by the "optical theorem,"

$$\sigma_{\text{total}}=\frac{4\pi}{k}\text{Im}[A(0)],\quad \sigma'_{\text{total}}=\frac{4\pi}{k}\text{Im}[A'(0)]. \tag{4}$$

Thus the imaginary part of n (n') describes the attenuation of the beam, as governed by the total cross section (not the absorption cross section).

The real part of n (n') describes the de Broglie oscillations of the waves. For n (n') = 1, this gives the free-space wave number, so that the real part of $n-1$ ($n'-1$), or the real part of $A(0)$ [$A'(0)$], describes the phase shift of the wave relative to its behavior in free space.

In general, the real and imaginary parts of $A(0)$ [$A'(0)$] are of the same order of magnitude. Therefore, the phase shifts involved are on the order of radians per nuclear mean free path, or radians per inch. Ordinarily, one would not be interested in such a small phase shift; however, we are here concerned with a

* This work was done under the auspices of the U. S. Atomic Energy Commission.

[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[2] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

[3] K. Case, Phys. Rev. **103**, 1449 (1956).

[4] See, for instance, M. Lax, Revs. Modern Phys. **23**, 287 (1951). The scattering amplitudes involved in Eq. (1) are those for *elastic* scattering only.

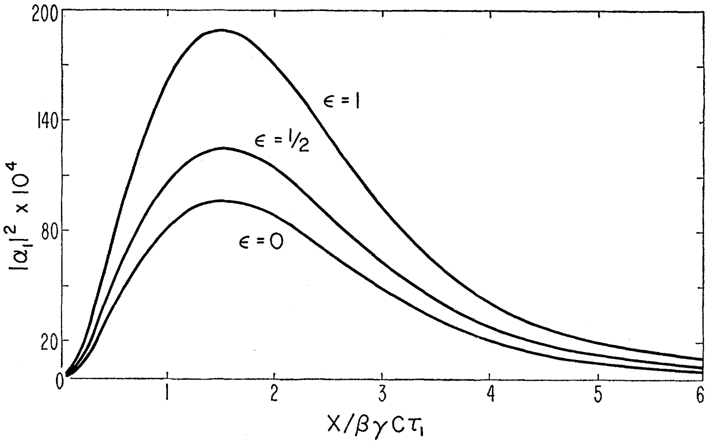

FIG. 1. θ_1 intensity *vs* distance for θ_2 beam incident on absorber. $\epsilon=\mathrm{Re}[A(0)-A'(0)]/\mathrm{Im}[A(0)-A'(0)]=0, \frac{1}{2}, 1$, $\omega_2{}^0-\omega_1{}^0=1/\tau_1$, $\sigma_{\text{total}}=\frac{1}{3}\sigma'_{\text{total}}$, $N\sigma'_{\text{total}}=1/\beta\gamma c\tau_1$.

coherent linear combination of two states. This small phase shift affects the coherence, and therefore is important.

Including the effect of the weak interactions, and changing to the θ_1, θ_2 representation in the same manner as Case, we obtain, for the equations of motion of α_1, α_2,

$$\frac{d}{dt}\alpha_1=i\beta ck\left[\left(\frac{n+n'}{2}\right)\alpha_1-i\left(\frac{n-n'}{2}\right)\alpha_2\right]-\left(i\omega_1+\frac{1}{2\gamma\tau_1}\right)\alpha_1,$$

$$\frac{d}{dt}\alpha_2=i\beta ck\left[\left(\frac{n+n'}{2}\right)\alpha_2+i\left(\frac{n-n'}{2}\right)\alpha_1\right]-\left(i\omega_2+\frac{1}{2\gamma\tau_2}\right)\alpha_2, \tag{5}$$

where βc = velocity of particle, t = laboratory time, $d/dt=(\partial/\partial t)+\beta c\partial/\partial x$ = total time derivative evaluated at a moving point, $\gamma=(1-\beta^2)^{-\frac{1}{2}}$, $\omega_{1,2}$ = de Broglie frequencies of θ_1, θ_2 particles, $(\hbar\omega_{1,2})^2=(\hbar kc)^2+(m_{1,2}c^2)^2$, and $\tau_{1,2}$ = proper mean life for decay of θ_1, θ_2. The cross terms linking the α_1 and α_2 equations depend on the difference in index of refraction (or difference in forward-scattering amplitude) for the θ^0, $\bar{\theta}^0$ components.

This treatment makes it clear that the regeneration phenomenon depends on the difference in total cross section and on the difference in phase shift of the θ^0, $\bar{\theta}^0$ waves, rather than on the difference in absorption cross section.

The solution of these equations, in Case's notation, is

$$\begin{pmatrix}\alpha_1(t)\\ \alpha_2(t)\end{pmatrix}=\left[\frac{\alpha_1(0)-R\alpha_2(0)}{1-R^2}\right]e^{-\lambda_1 t}\begin{pmatrix}1\\ R\end{pmatrix}+\left[\frac{\alpha_2(0)-R\alpha_1(0)}{1-R^2}\right]e^{-\lambda_2 t}\begin{pmatrix}R\\ 1\end{pmatrix}, \tag{6}$$

where

$$\lambda_1=\omega+\Delta,\quad \lambda_2=\omega-\Delta,$$

$$R=\frac{\beta ck(n-n')}{-i\left(\dfrac{\omega_2{}^0-\omega_1{}^0}{\gamma}\right)-\dfrac{1}{2\gamma}\left(\dfrac{1}{\tau_2}-\dfrac{1}{\tau_1}\right)+2\Delta}=-\left[\frac{i\left(\dfrac{\omega_2{}^0-\omega_1{}^0}{\gamma}\right)+\dfrac{1}{2\gamma}\left(\dfrac{1}{\tau_2}-\dfrac{1}{\tau_1}\right)+2\Delta}{\beta ck(n-n')}\right],$$

$$\omega=\frac{1}{2}\left[i(\omega_2+\omega_1)+\frac{1}{2\gamma}\left(\frac{1}{\tau_2}+\frac{1}{\tau_1}\right)-i\beta ck(n+n')\right],$$

$$\Delta=+\frac{1}{2}\left\{\left[\frac{i(\omega_1{}^0-\omega_2{}^0)}{\gamma}+\frac{1}{2\gamma}\left(\frac{1}{\tau_1}-\frac{1}{\tau_2}\right)\right]^2+[i\beta ck(n-n')]^2\right\}^{\frac{1}{2}}.$$

These expressions are the same as given by Case,[3] with the following exceptions:

(1) The relativistic time dilation, which "slows down" the θ_1 decay and the $\theta_1-\theta_2$ mass-difference frequency, has been included.[5] It has the effect of increasing Case's parameter β[6] by a factor γ. Because the smallness of β makes the Pais-Piccioni experiment difficult, this factor can be important in practical situations.

(2) The effects of the total cross section and of the phase shift produced by the real part of the forward scattering are included; Case's model included only the $\bar{\theta}^0$ absorption cross section.

[5] The term $\omega_2-\omega_1$, which appears in the solution, has been replaced by $(\omega_2{}^0-\omega_1{}^0)/\gamma$ (where $\hbar\omega_{1,2}{}^0$ is the rest energy of $\theta_{1,2}$) for the following reason: The de Broglie frequencies are given by $(\hbar\omega_{1,2})^2=(\hbar kc)^2+(m_{1,2}c^2)^2$, so that, to first order, we have $\hbar^2\omega\delta\omega=(mc^2)\delta(mc^2)$ and $\hbar\delta\omega=(mc^2/\hbar\omega)\delta(mc^2)=\delta(mc^2)/\gamma=\hbar(\omega_2{}^0-\omega_1{}^0)/\gamma$. (This result follows even if we have $k_1\neq k_2$, so long as the difference is of order $\delta m/m$.)

[6] Case's β is essentially the ratio of absorption to decay.

Figure 1 shows the probability $|\alpha_1|^2$ *vs* x for a pure θ_2 beam incident on the absorber, and for $\omega_2^0-\omega_1^0=(1/\tau_1)$, $N\sigma'_{\text{total}}=(1/\beta\gamma c\tau_1)$, $\sigma_{\text{total}}=\frac{1}{3}\sigma'_{\text{total}}$. Curves are plotted for the real part of the forward scattering equal to zero, one-half, and one times the imaginary part. The phase-shift term is seen to have an appreciable effect.

REGENERATION BY SCATTERING

We can now discuss the relation between the regeneration of θ_1's in the unscattered beam and the regeneration by scattering. The composition of the unscattered beam is

$$\psi=\alpha(x)\theta^0+\alpha'(x)\bar{\theta}^0=\alpha_1(x)\theta_1+\alpha_2(x)\theta_2,$$

where the α's are obtained from Eq. (6) with $x=\beta ct$ and $\alpha_2(0)=1$, $\alpha_1(0)=0$ (i.e., pure θ_2 beam incident on the absorber).

The probability of seeing a θ_1 in the unscattered beam is given by

$$n_{un}=|\alpha_1(x)|^2=\left|\frac{[\alpha(x)+\alpha'(x)]}{\sqrt{2}}\right|^2, \tag{7}$$

while that of seeing a scattering through angle ϕ followed by a θ_1 decay is, in thickness dx of the scatterer, and in solid angle $d\Omega$,

$$dn_s=\left|\frac{\alpha(x)A(\phi)+\alpha'(x)A'(\phi)}{\sqrt{2}}\right|^2 Ndxd\Omega. \tag{8}$$

The unscattered regenerated θ_1 mesons reflect the composition of the unscattered beam, while the scattered ones reflect the composition times the scattering matrix. However, for small thicknesses of absorber (small here means $|\Delta|x/\beta c\ll 1$), α_1 is small, and $\alpha_2\cong 1$. Thus we obtain

$$\alpha_1(x)\simeq k\left(\frac{n-n'}{2}\right)x$$

by integrating Eq. (5) for small x, and

$$\alpha_1(x)\simeq\frac{2\pi N}{k}\left(\frac{A(0)-A'(0)}{2}\right)x; \tag{9}$$

in these circumstances we have

$$\alpha'\simeq-\alpha=i/\sqrt{2},$$

and thus

$$n_{un}\simeq(\pi^2N^2/k^2)|A(0)-A'(0)|^2x^2, \tag{10}$$

$$dn_s\simeq\tfrac{1}{4}|A(\phi)-A'(\phi)|^2Ndxd\Omega. \tag{11}$$

If one observes decays in a cloud chamber that is placed in a beam of θ_2 mesons behind a thickness of absorber δx that is small compared with $\beta c/|\Delta|$, he should see an angular distribution of θ_1 decays as in Fig. 2. We can now, from Eqs. (10) and (11), give the ratio of the number of events, n_{un}, in the peak, to the number of events, δn_s, in the background under it in some finite angular interval $\delta\phi$ at $\phi=0$.

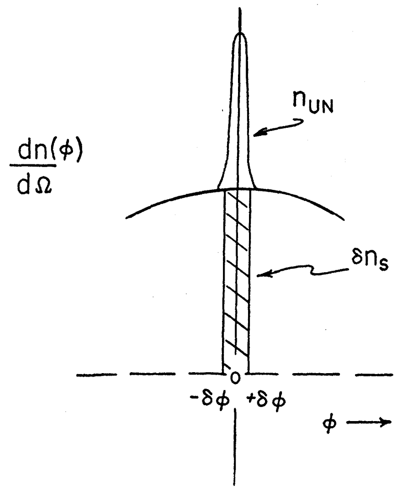

FIG. 2. Distribution in angle of regenerated θ_1 mesons near $\phi=0$. The group δn_s refers only to *elastically* scattered θ mesons. In general, there will also be inelastically scattered θ mesons, which would have to be rejected in checking the predictions of Eq. (12).

(If we choose $\delta\phi$ equal essentially to the angular resolution of the apparatus, this ratio is the ratio of peak height to the background height at $\phi\approx 0$.) If we set $d\Omega=2\pi(1-\cos\delta\phi)\approx\pi(\delta\phi)^2$, $dx=\delta x$ in dn_s, $x=\delta x$ in n_{un}, then we have

$$n_{un}/\delta n_s\simeq 4\pi N(\delta x)/[k^2(\delta\phi)^2]. \tag{12}$$

The scattering amplitudes cancel out, and the ratio is dependent only on the geometry and the wavelength. (In order to see a large peak, one needs fairly large thicknesses, low energy, and high angular resolution.)

For $\delta\phi=1°$, 100-Mev kinetic energy θ's and a 0.5-cm lead plate, $n_{un}/\delta n_s=2.5$.

The simple relation given by Eq. (12), which is independent of all nuclear parameters, should serve as an interesting check on our understanding of the process.

However, we need to examine carefully its limits of validity. (The analysis so far has implicitly assumed, for instance, zero spin both for θ^0 mesons and for the scattering nucleus, inasmuch as the scattering of θ^0 mesons has been described by a single scattering amplitude.)

This examination is outlined in the Appendix. It is concluded that, if one performs an experiment to test the relation between scattered regenerated θ_1 mesons observed at small angles and unscattered regenerated θ_1 mesons, as given by Eq. (12), then (a) one should use an isotopically pure element for a target; (b) if the spin of the target nucleus is zero, Eq. (12) should unconditionally be obeyed; (*c*) if the target nucleus spin is $\frac{1}{2}$, lack of agreement with Eq. (12) signifies nonzero spin for the θ^0.

It is possible, then, that careful observation of the two types of Pais-Piccioni regeneration may give information about the nature of the neutral K mesons beyond what was expected originally.

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APPENDIX: LIMITS OF VALIDITY OF EQ. (12)

We need to consider the following possible complications.

1. *Nonidentical Scattering Nuclei*

The forward peak, n_{un}, is caused by constructive interference of large numbers of nuclei.[7] The scattered group, δn_s, involves scattering from just one nucleus. Because of this, if there are two kinds of scatterers present, one averages their scattering amplitude and then forms the squares, in calculating n_{un}; whereas, in calculating δn_s, one squares first and then averages. This spoils the cancellation of scattering amplitudes that led to Eq. (12).

Equation (12), then, should in general hold only for an absorber that is a chemical element, and (in principle at least) is a single isotope of that element.

2. *Nonzero Spin of Scattering Nucleus*

A detailed analysis shows that the intuitive result is correct, namely that the presence of $2s+1$ spin substates of the scatterer is the same as having $2s+1$ kinds of nuclei present. Equation (12) does not, in general, hold for nonzero scattering nucleus spin.

3. *Nonzero Spin of the* θ^0

If we neglect spin-flip processes temporarily, each spin substate of the θ^0 scatters separately, and the intensities for each substate obey Eq. (12) separately. The total intensity is the sum of the intensities of the substates, and therefore Eq. (12) is obeyed, in general, for nonzero θ^0 spin, so long as the spin of the nucleus is zero.

[7] The treatment given does not make this very obvious, since the index of refraction is usually thought of as the result of interference of the individual scattered wavelets with the incident wave, rather than with one another. Suppose, however, that one does not go through the intermediate step of deriving an index of refraction for θ^0, $\bar{\theta}^0$, but rather describes the problem of a θ_2 wave incident on a thin scatterer directly in terms of θ^0 and $\bar{\theta}^0$ wave functions, each consisting of an incident wave and N scattered wavelets. By adding these two wave functions, one obtains the θ_1 wave function directly. The incident-wave term falls out, since there are no incident θ_1 mesons, and only the sum of the θ_1 scattered wavelets, $\Sigma_i(\theta_1)_i$, remains. Upon squaring, the terms in $i=j$ lead to the ordinary scattered intensity, δn_s, while the terms for $i\neq j$ give rise to a constructive interference in the forward direction, which is just n_{un}. One ends up with Eq. (12), but the treatment makes evident the cooperative nature of the forward peak.

4. *A Special Case:* θ^0 *Particle Spin Zero, Nuclear Spin* $\frac{1}{2}$

In this case, there are but two scattering amplitudes, and both are equal. The order in which one performs the operations of averaging and squaring is then unimportant, and therefore Eq. (12) holds, for this special case.

(If the θ^0 particle spin is not zero and the target nucleus spin is $\frac{1}{2}$, the scattering for target-nucleus spin parallel and antiparallel to the θ^0 spin might well be different, so that the remarks of Point 2 apply.)

5. *Spin-Flip Processes*

(a) If the nuclear spin state changes in a scattering, the process is incoherent and cannot contribute to the forward peak. Such an effect would "spoil" Eq. (12).

(b) On the other hand, if either of the spins is zero, the spin-flip amplitude must vanish in the forward direction, because of conservation of angular momentum.

(c) Because in all those cases enumerated above, in which Eq. (12) should hold, one or the other of the spins is zero, we can conclude that spin-flip processes do not affect the validity of Eq. (12).

6. *Parity-Exchange Scattering*

If a neutral τ^0 exists and can turn into a neutral θ^0 upon elastic scattering, the situation is obviously more complex than was assumed in the derivation of Eq. (12).

In the forward direction, however, the situation is simplified; any spin-flip amplitudes vanish in the cases of interest, as in Point 5 above.

Any non-spin-flip $\tau^0\rightarrow\theta^0$ scattering, on the other hand, is coherent. The incident τ^0 particles form a coherent forward peak of θ_1 mesons just as the incident θ_2 mesons do, and the θ_1 contributions from such an effect obey Eq. (12) whenever the $\theta_2\rightarrow\theta_1$ scattering does.

This result may be shown to be independent of the relative amplitudes and relative phases of the τ_1 and τ_2 postulated to be in the beam.

Thus parity-exchange scattering, curiously enough, does not affect the applicability of Eq. (12).

We may conclude that (a) for target spin zero and an isotopically pure target, Eq. (12) should unconditionally hold; (b) for target spin ($\frac{1}{2}$) (isotopically pure), deviations from Eq. (12) would signify nonzero spin of the neutral K mesons.

Note added in proof.—It now appears that charge conjugation is not conserved.[8] The implications of this for the $\theta_1-\theta_2$ problem in vacuum have been examined

[8] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

by Lee, Yang, and Oehme,[9] Gatto,[10] and Gell-Mann.[11] The results are that if the product CP (charge conjugation times parity) is conserved, one obtains again the Gell-Mann and Pais $\theta_1-\theta_2$ scheme; but if CP is not conserved, the eigenstates of the system (i.e., the particles with definite mass and definite lifetime) are, in vacuum as follows:

$$K_+{}^0=\frac{1}{C}(pK^0+qK^0),\qquad K_-{}^0=\frac{1}{C}(pK^0-qK^0), \tag{13}$$

[9] Lee, Yang, and Oehme, Phys. Rev. (to be published).
[10] R. Gatto, "The K^0 decay modes and the question of time reversal of weak interactions," UCRL-3658, January, 1957; also Phys. Rev. (to be published).
[11] M. Gell-Mann, Nuovo cimento (to be published).

where $C=[|p|^2+|q|^2]^{\frac{1}{2}}$, and the term K^0 implies a single particle, with θ^0 and τ^0 decay modes.

We must now ask the following question: what is the behavior, in this most general case, of the neutral K-meson complex in an absorber?

If we use Eq. (13) for the eigenstates in rederiving the relations stated in this paper, the equations of motion of a_+ and a_- (the amplitudes of $K_+{}^0$ and $K_-{}^0$) turn out to be precisely Eq. (5) again, except that a_1 is replaced by a_+, a_2 by (ia_-), and ω_1, ω_2, λ_1, λ_2 by ω_+, ω_-, λ_+, λ_-.

It follows that the regeneration of the short-lived component (say $K_+{}^0$) from the long-lived one ($K_-{}^0$) would then be identical in detail to that predicted in the original Gell-Mann and Pais scheme; in particular, Eq. (12) would hold here wherever it did there.

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Relation of Isotopic Spin Space to Space-Time*

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The customary form of physical space as the direct product of space-time and isotopic spin space can be replaced by a certain fusion of these two spaces into one space if one widens the group from the Lorentz group to the conformal space-time group which endows particles with an intrinsic finite size. It is shown that the familiar space-time transformations continue to induce their familiar Dirac spinor transformations, while the new space-time transformations in general induce isotopic spin transformations as well. The detailed correspondence is worked out for a theory of $T=1$ bosons coupled to $T=\frac{1}{2}$ fermions.

In elementary particle theory, physical space has long been considered as simply the direct product of space-time and of a 3-dimensional vector space called isotopic spin space. This is physically not very satisfying; for example, the operational prescriptions which take one from one frame to an equivalent one in space-time are at the foundations of special relativity, but the corresponding prescriptions for a change of frame in isotopic spin space are lacking. This note suggests a possible fusion of the two spaces on the basis of the conformal sphere geometry in physics. This group, containing the conventional inhomogeneous Lorentz group, permits a covariant description of finite-size sources in both classical and quantum theories,[1,2] as one of its interesting features.

1. DETAILS

Six homogeneous coordinates X^μ ($\mu=0, 1, \cdots 5$) are linked to space-time x^m ($m=1, \cdots 4$) and to the coordinate of finite size λ by[3]

$$\tau X^0=1,\quad \tau X^m=x^m,\quad \tau X^5=\tfrac{1}{2}(\boldsymbol{x}^2+\lambda^2),\qquad (\boldsymbol{x}^2\equiv g_{mn}x^mx^n), \tag{1.1}$$

where g_{mn} (signature $+++-$) is the Lorentz metric. Taking τ to have the dimensions of length like the x^m (we write $\tau\sim L$), we note that the X^μ have different dimensions: $X^0\sim L^{-1}$, $X^m\sim 1$, $X^5\sim L$. The points ($\lambda=0$) of space-time are then given by the locus

$$G_{\mu\nu}X^\mu X^\nu\equiv \boldsymbol{X}^2-2X^0X^5=0, \tag{1.2}$$

where italic boldface shall always refer to the four space-time components. The transformations of the group are now simply rotations $\bar{X}^\mu=L^\mu{}_\nu X^\nu$ ($L^\mu{}_\nu$= constants) with respect to the metric $G_{\mu\nu}$ of signature $(+++-, +-)$. The corresponding spinor algebra is generated by six 8×8 γ matrices Γ^μ satisfying

$$\Gamma^\mu\Gamma^\nu+\Gamma^\nu\Gamma^\mu=2G^{\mu\nu}1, \tag{1.3}$$

where 1 means the 4×4 spinor identity.

We now assert that the fusion of space-time and

* This research was partially supported by the USAF, Contract No. AF18(600)1315.
[1] R. Ingraham, Phys. Rev. **101**, 1411 (1956).
[2] R. Ingraham and J. Ford (to be published).
[3] See, for example, R. Ingraham, Nuovo cimento **12**, 825 (1954).