Proton Polarizability Correction to Electron-Proton Scattering*

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The contribution to observed electron-proton scattering cross sections of the electron-induced polarization of the proton is estimated and found to be small for electron energies <500 Mev.

I. INTRODUCTION

IN analyses of the scattering of high-energy electrons by protons[1] the electric-charge and magnetic-moment distributions of the proton have been treated as if rigid. This ignores effects on the scattering due to the electron-induced polarization of this charge and current cloud. This polarization is responsible for the nuclear Compton effect. Although it contributes only corrections of order e^2 to the main term of the electron-proton scattering amplitude, the proton polarizability can be anomalously large near those frequencies for which photomeson production is big, i.e., $\hbar\omega$ from 200 to 400 Mev. These are also the significant electromagnetic-field frequencies felt by the proton in small-impact-parameter, large-angle electron scatterings which are sensitive to the charge and current distribution of the proton. In this work the polarization contribution to the observed scattering is estimated. For electrons scattered through 90° it is found to be $\lesssim\frac{1}{2}\%$ at 400 Mev and an increasing function of energy. The smallness of this result has a classical interpretation.

II. CALCULATION

The contribution of proton polarization to the electron-proton scattering is represented in the Feynman diagram, Fig. 1.

The relevant scattering amplitude is

$$f(\mathbf{p},\mathbf{K})=\left\langle f\left| e^2 \int\int d\omega d\mathbf{k}\gamma_\mu[p-k-m]^{-1} \times\gamma_\nu F_{\mu\nu}(\omega,\mathbf{k},\mathbf{K})(\omega^2-k^2)(\omega^2-K^2-k^2+2\mathbf{k}\cdot\mathbf{K})^{-1}i\right.\right\rangle. \quad (1)$$

$F_{\mu\nu}$ is the amplitude for the scattering of a (virtual) photon of polarization μ, momentum $\mathbf{k}$, into a state of polarization ν, momentum $\mathbf{k}-\mathbf{K}$, but with no change in frequency ω. For photons on the energy shell ($\omega=k$) and for $\mathbf{K}=0$ this amplitude is related to the photon-proton total cross section through dispersion relations.

For an estimate of the polarization scattering, Eq. (1) is integrated directly, with the photon scattering amplitude, $F_{\mu\nu}$, approximated as follows:

(1) $F_{\mu\nu}$ is replaced by the scattering amplitude of photons on the energy shell, written as a function of the photon frequency ω and scattering angle θ.

(2) Following the work of Gell-Mann, Goldberger, and Thirring,[2] the dispersion relations are used to relate $F_{\mu\nu}(\omega,\theta=0)$ to the total cross section for photons on protons; the predominant contribution arises from meson production.

(3) For simplicity, the photon scattering amplitude thus obtained is assumed to be isotropic and is multiplied by a factor 2.5. Since we are seeking an upper limit for the polarizability effect, the factor 2.5 is introduced to correspond to indications from reference 2, the work of Yamagata *et al.*,[3] and the calculations of Matthews,[4] that large-angle scattering of photons may be more important. This is at best a rough guess and an alternate method of calculation is developed below to help tie it down.

(4) $F_{\mu\nu}$ is replaced by an isotropic tensor, $F\delta_{\mu\nu}$; spin flip and inclusion of longitudinal virtual photon contributions are neglected. With Eq. (1) approximated in

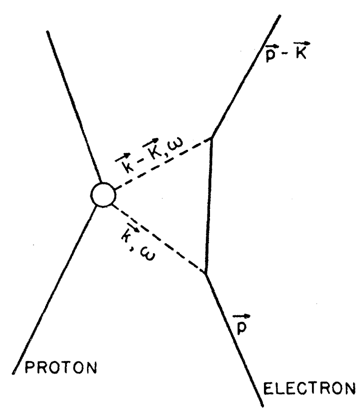

FIG. 1. Feynman diagram for polarization contribution to electron-proton scattering.

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[1] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).

[2] Gell-Mann, Goldberger, and Thirring, Phys. Rev. **95**, 1612 (1954). The analysis in this reference is supplemented by subsequent California Institute of Technology data on meson photoproduction. See R. L. Walker *et al.*, Phys. Rev. **99**, 210 (1955); and A. V. Tollestrup *et al.*, Phys. Rev. **99**, 220 (1955).

[3] T. Yamagata *et al.*, Bull. Am. Phys. Soc. Ser. II, **1**, 350 (1956).

[4] Jon Matthews, post-deadline paper at the Monterey meeting of the American Physical Society, December 27–29, 1956; see also Pugh, Gomez, Frisch, and Janes, Phys. Rev. **105**, 982 (1957).

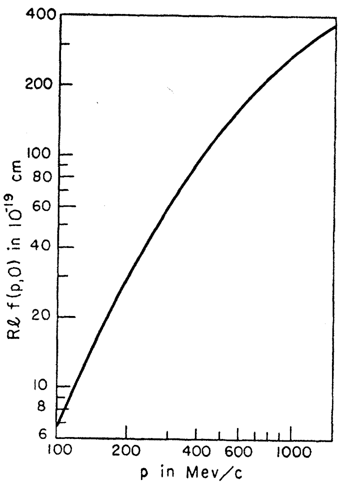

FIG. 2. Contribution of polarization to forward scattering amplitude of electrons by protons.

this way, the calculation can be reduced to a one-parameter numerical integration. The final result contributes an additive correction to the measured cross section of between $\frac{1}{3}\%$ and $\frac{1}{2}\%$ for 400-Mev electrons scattered through 90° by protons.[1]

An alternative estimate of the polarization scattering follows from a combination of the dispersion relation with the method of virtual quanta. The dispersion relation which relates the forward electron-proton scattering amplitude is complicated by the infinite cross section and infinite number of bound states for an electron in a Coulomb field. However, since only nonstatic mesonic corrections of order e^4 are pertinent it is simpler to write the dispersion relation for the difference between the true elastic forward scattering amplitude and amplitude for scattering by a fixed point charge[5]:

$$f(p,0)=f(0,0)+\frac{p^2}{2\pi^2}\int_0^\infty dk'\frac{\bar{\sigma}_e(k')}{k'^2-p^2+i\delta}, \qquad (1')$$

where

$$\bar{\sigma}_e=\sigma(e+p\rightarrow\pi^+)+\sigma(e+p\rightarrow\pi^0).$$

The constant $f(0,0)$ can be estimated from Eq. (1) with $\mathbf{K}$ and $\mathbf{p}=0$. It vanishes in that approximation which neglects the electron mass and can be ignored next to the integral in Eq. (1′) for $p>10^2$ Mev/c. Although $\bar{\sigma}_e$ has not yet been extensively measured it can be inferred in an approximate way from the cross sections for photomeson production. From a Weiszsäcker-Williams approximation Dalitz and Yennie[6] find

$$\bar{\sigma}_e(p)=\int^p N_e(p,k)\bar{\sigma}_\gamma(k)dk/k, \qquad (2')$$

where

$$N_e(p,k)=\frac{\alpha}{\pi}\left\{\frac{\epsilon^2+\epsilon'^2}{p^2}\ln\frac{\epsilon\epsilon'+pp'+m^2}{m(\epsilon-\epsilon')}-\frac{(\epsilon+\epsilon')^2}{2p^2}\ln\left(\frac{p+p'}{p-p'}\right)-\frac{p'}{p}\right\},$$

and

$$\epsilon=(m^2+p^2)^{\frac{1}{2}},\quad \epsilon'=(m^2+p'^2)^{\frac{1}{2}},\quad \mathbf{p}'=\mathbf{p}-\mathbf{k}.$$

$\bar{\sigma}_\gamma(k)$ is the total cross section for meson production (π^+ and π^0) from protons by photons of momentum $\mathbf{k}$. Figure 2 is a plot of $f(p,0)$ calculated from Eqs. (1′) and (2′) and the observed photomesonic cross sections.

To estimate the angular distribution it is useful to examine the contribution to $f(p,0)$ from the different angular momentum states of the electron and proton. An energetic electron with impact parameter b gives a photon spectrum over the proton:

$$n(b,k)dk=\frac{\alpha m^2kdk}{4p^2}\left\{H_1^{(1)}\left(\frac{ibkm}{p}\right)\right\}^2. \qquad (3')$$

The partial cross section for an impact parameter r is then

$$d\sigma=2\pi rdr\int_0^p n(r,k)\bar{\sigma}_\gamma(k)dk. \qquad (4')$$

In a WKB approximation such a $d\sigma$ and the resulting angular distribution for $\mathrm{Im}f(p,K)$ are the same as that from an imaginary potential whose spatial dependence is $r^{-2}d\sigma/dr$. The approximation leading to Eq. (3′) implies $r>a=h/p$. The imaginary part of the polarization electron-proton scattering amplitude has an energy ($\epsilon\sim p$) and momentum transfer (K) dependence given in Born approximation by

$$\mathrm{Im}f(p,K)\propto\int d\mathbf{r}\exp(i\mathbf{K}\cdot\mathbf{r})r^{-2}d\sigma/dr. \qquad (5')$$

For any fixed momentum transfer K, it has been shown[7] that the real and imaginary parts of $f(p,K)$ are still related by the usual dispersion relation. For a nonzero K, $\mathrm{Im}f(p,K)$ is obtained from Eqs. (3′), (4′), and (5′). It is convenient to introduce the cutoff a by multiplying $d\sigma/dr$ by $[1-\exp(-r/a)]$. Then for large momentum transfers ($K\gg1$ Mev/c) these equations give

$$\mathrm{Im}f(p,K)=G(p,K)\ \mathrm{Im}f(p,0), \qquad (6')$$

where

$$G(p,K)=\frac{(p/K)\tan^{-1}(K/p)+\frac{1}{2}\ln(1+p^2/K^2)}{2\ln(p/mc)-\int_0^p\bar{\sigma}(\omega)\ln\omega d\omega\Big/\int_0^p\bar{\sigma}(\omega)d\omega}. \qquad (7')$$

[5] In the derivation of Eq. (1′) it is assumed that σ is the same for electrons and positrons.

[6] R. Dalitz and D. Yennie, Phys. Rev. **105**, 1598 (1957).

[7] R. H. Capps and G. Takeda, Phys. Rev. **103**, 1877 (1956).

Above meson production threshold $\text{Im}f(p,0)$ has the approximate energy dependence p which varies much more rapidly with p than $G(p,K)$. Therefore if $\text{Im}f(p,K)$ of Eq. (6′) is used in the integrand of a dispersion relation, an estimate of $\text{Re}f(p,K)$ is obtained by assuming $G(p',K)$ constant and evaluating it at the zero of the energy denominator ($p'=p$). Thus for large K the real and imaginary parts of $f(p,K)$ have approximately the same angular distribution so that

$$\text{Re}f(p,K)\simeq G(p,K)\,\text{Re}f(p,0). \tag{8'}$$

For a given angle K/p is independent of p. G has only a logarithmic decrease with increasing p so that $\text{Re}f(p,K)$ for a fixed angle may even increase with p, according to (7′) and (8′), in distinction to the e^2 scattering amplitude which decreases like p^{-2}. The polarization and Coulomb scattering amplitudes have constructive interference for all K/p. For 90° scattering and 200-Mev electrons the ratio $\text{Re}f(p,K)$ to the Coulomb scattering amplitude is found to be $\sim+1/20\%$; at 400 Mev (90°) this ratio is found to be $\sim\frac{1}{2}\%$. These corrections are quite negligible in agreement with the earlier conclusion.

In view of the smallness of the polarizability correction and of the agreement between the two different methods of calculating it, no attempt has been made to obtain a more quantitatively reliable result. Such a calculation may be desirable for the analysis of electron-proton scattering in the energy range above 1 Bev. The approximations which lead to (7′) and (8′) are no better than a WKB approximation and therefore are not valid for large momentum transfers. However we may note that at a fixed angle the ratio of the polarization to the Coulomb scattering increases rapidly with p, even if we ignore the form factors which cut down electron scattering.[1]

III. DISCUSSION

On the basis of the analysis presented here, it seems safe to ignore polarizability contributions in the interpretation of a scattering in the 500-Mev energy range and below. This result can be understood in terms of a simple classical picture. Consider the proton as a charged oscillator with resonant frequency ν_r and damping constant γ (corresponding to the resonant energy and width of the meson-nucleon interaction). An incident monochromatic light wave, $\cos\omega t$, will be scattered with no change in phase by the charge of the proton. In "driving" the proton, the light wave will also induce a dipole moment, giving rise to dipole radiation proportional to

$$\frac{(\nu_r^2-\omega^2)\cos\omega t+\gamma\sin\omega t}{(\nu_r^2-\omega^2)^2+\gamma^2\omega^2}.$$

The first term in the above expression is in phase with and interferes with the direct charge-scattered term; it gives rise to the polarizability correction of interest. This contribution is expected to be $\sim e^2=1/137$ except in the neighborhood of the resonance frequency $\omega\sim\nu_r$, in which case it will be enhanced. However the resonant, in-phase contribution is seen to be odd about the resonance frequency. It is thus largely cancelled in the integral over the virtual photon spectrum of the scattered electron although it gives rise to a resonance peak in the proton Compton effect.

It appears then that below 500 Mev the scattering of electrons and of photons are complementary and exclusive tools for the study of the structure of a nucleon, the electrons measuring the form factors of the electric charge and magnetic moment, and the photons the polarizability of the "structure." However, the polarizability contribution to electron scattering may be important as one explores the proton structure with more energetic electrons.

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